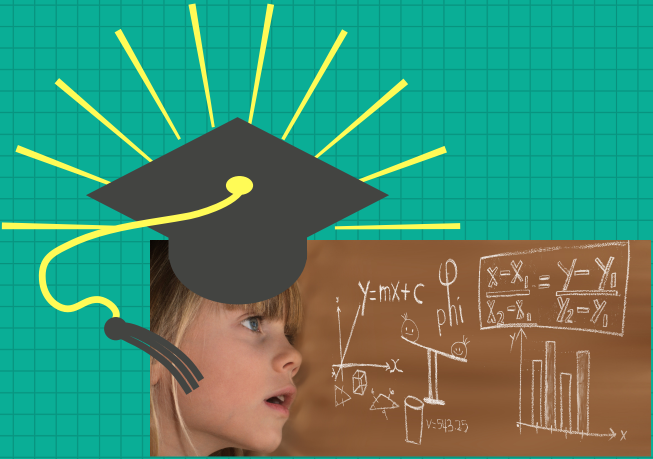
POLITEKNIK MALAYSIA

KEMENTERIAN PENGAJIAN TINGGI

KOLEJ KOMUNITI MALAYSIA

PBM2014

BASIC MATHEMATICS 2



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PBM2014

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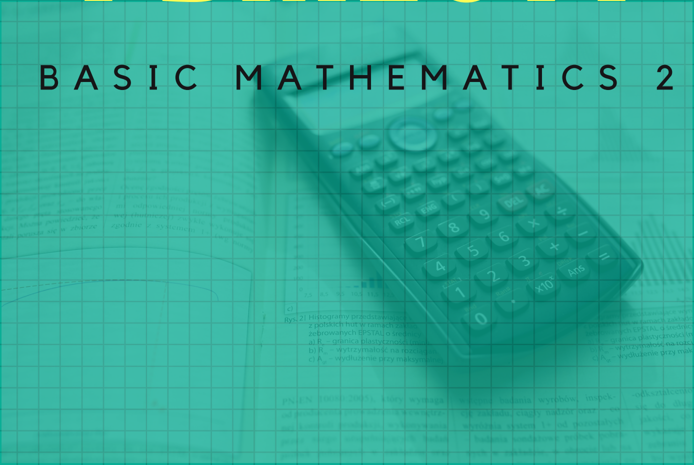

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Preface

Alhamdulillah, praise to Allah SWT, with His grace and mercy, the First Edition of e-book PBM2014 Basic Mathematics 2 has finally completed. We hope that this e-book will be helpful as a guideline in their learning process. This e-book is developed as a guide and reference for lecturers also. Special thanks also to those who were directly or indirectly involved in the completion of this e-book. Any positive feedback mostly welcomed and appreciated.

Sinopsis

This e-book is designed for Polytechnic Pre-diploma of Science intake. The content of this book has been designed to meet the syllabus requirements of the polytechnic in order to equip students with basic courses preparing them to further studies at higher level. This e-book contains five topics which is trigonometry, circular measure, vector, inequality and matrices. Each topic was include with the explanation, example, tutorial and solution. It also enhanced with examples of applications in daily life. We hope that these books will help them to engage and capture their interest.

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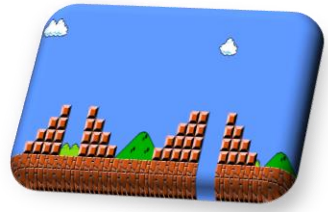
TOPIC 1
TRIGONOMETRY

Real life application

Navigation	Aviation	Engineering
Astronomy	Construction	Physic
Video game	Criminology	Cartography
Archaeologist	Satellite	Architecture
	Marine biology	



Do You Know?

Al- Battani or Muhammad Ibn Jabir Ibn Sinan Abu Abdullah is the father of trigonometry.

Subtopic

1.1 Define Pythagoras Theorem.

1.2 Define Trigonometric Ratios.

1.3 Evaluate Trigonometric Functions of any angle.

1.4 Understand Special Triangles.

1.5 Solve problems related to triangles.

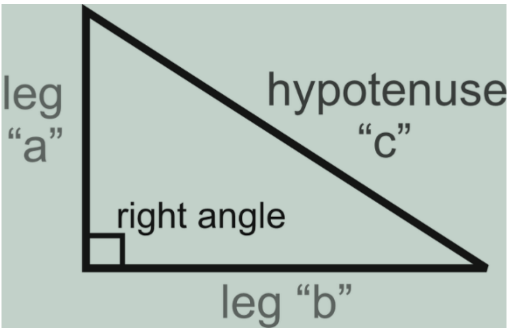
i. Sine rule.

ii. Cosine rule.

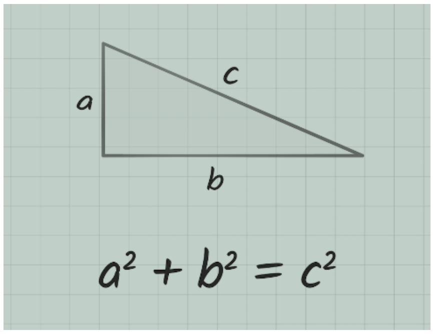
iii. Area of triangles.

1.1 Pythagoras Theorem

Pythagoras Theorem describes the relationship between the lengths of the sides of right angle triangle.

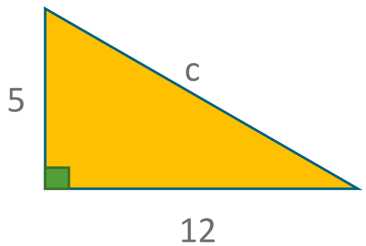


In Pythagoras Theorem its stated that if a right triangle has legs of lengths a and b, and hypotenuse of length c, then :



Example 1.1 a

Find the value of c



$$c^2 = a^2 + b^2$$

$$c^2 = 5^2 + 12^2$$

$$c^2 = 25 + 144$$

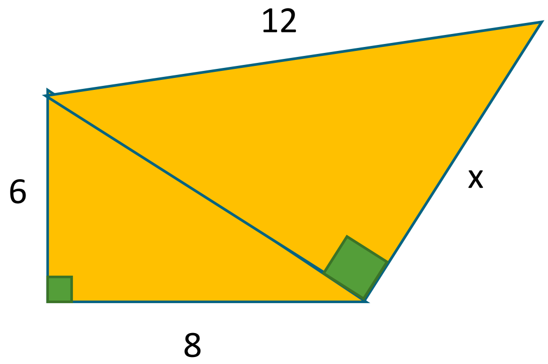
$$c^2 = 169$$

$$c = \sqrt{169}$$

$$c = 13$$

Example 1.1 b

What is the length of x?



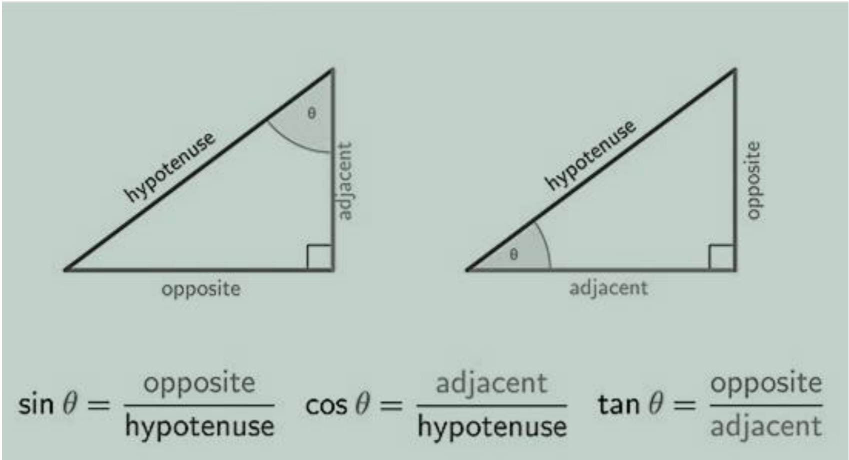
$$c^2 = a^2 + b^2$$
$$c^2 = 6^2 + 8^2$$
$$c^2 = 36 + 64$$
$$c^2 = 100$$
$$c = \sqrt{100}$$
$$c = 10$$

$$c^2 = a^2 + b^2$$
$$12^2 = c^2 + x^2$$
$$12^2 = 10^2 + x^2$$
$$144 = 100 + x^2$$
$$144 - 100 = x^2$$
$$x^2 = 44$$
$$c = \sqrt{44}$$
$$c = 6.63$$

1.2 Trigonometric Ratios

- The trigonometric ratios are special measurements of a right angle triangle

- Trigonometric ratios provide relationships between the sides and angles of a right angle triangle. There are three basic trigonometric ratios: Sine, Cosine and Tangent.



Remember!!!

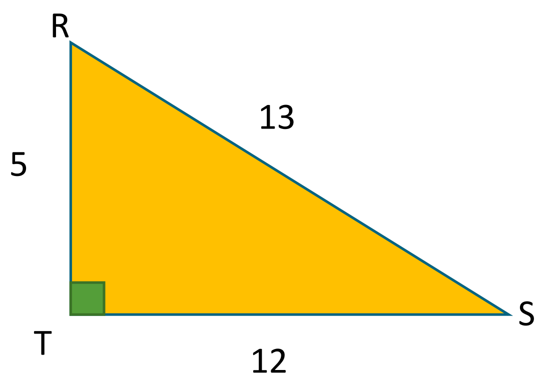
- An adjacent side is always next to the angle, θ
- An opposite side is opposite to the angle, θ

The reciprocal functions of the first three trigonometric ratios are the inverse function of the ratios.

basic		reciprocal
$\sin(\theta) = \frac{o}{h}$	cosecant	$\csc(\theta) = \frac{1}{\sin(\theta)} = \frac{h}{o}$
$\cos(\theta) = \frac{a}{h}$	secant	$\sec(\theta) = \frac{1}{\cos(\theta)} = \frac{h}{a}$
$\tan(\theta) = \frac{o}{a}$	cotangent	$\cot(\theta) = \frac{1}{\tan(\theta)} = \frac{a}{o}$

Example 1.2 a

Given triangle RST as below.



i. Find the sine, cosine and tangent of the indicated angle RST.

$$\sin S = \frac{O}{H} = \frac{5}{13} = 0.3846$$

$$cos\ S = \frac{A}{H} = \frac{12}{13} = 0.9231$$

$$\tan S = \frac{O}{A} = \frac{5}{12} = 0.4167$$

ii. Find the sine, cosine and tangent of the indicated angle SRT.

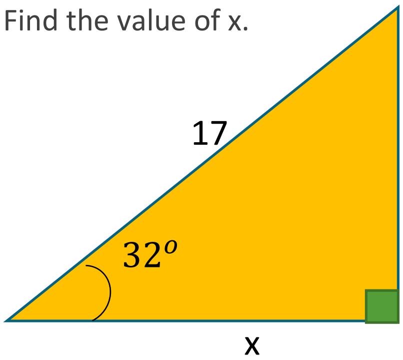
$$\sin S = \frac{O}{H} = \frac{12}{13} = 0.9231$$

$$cos\ S = \frac{A}{H} = \frac{5}{13} = 0.3846$$

$$\tan S = \frac{O}{A} = \frac{12}{5} = 2.4$$

Example 1.2 b

Find the value of x.



$$\cos\theta = \frac{A}{H}$$

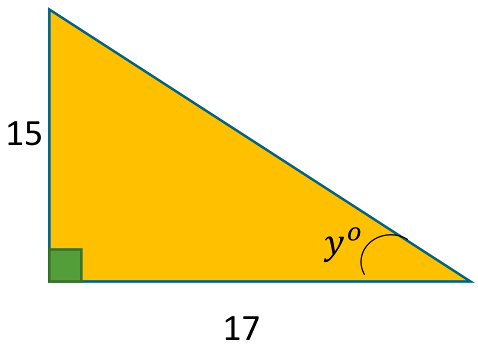
$$\cos 32^0 = \frac{x}{17}$$

$$x = \cos 32\,(17)$$

$$x = 14.42$$

Example 1.2 c

Find the value of y.



$$tan\,\theta = \frac{O}{A}$$

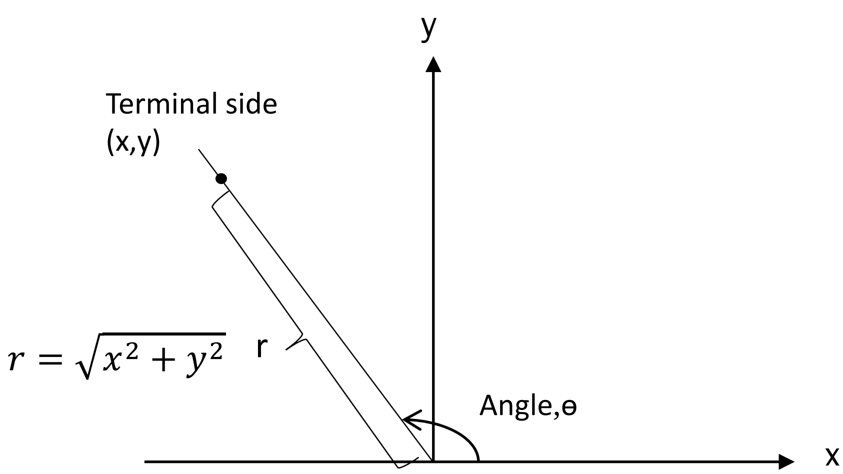
$$\tan y^o = \frac{15}{17}$$

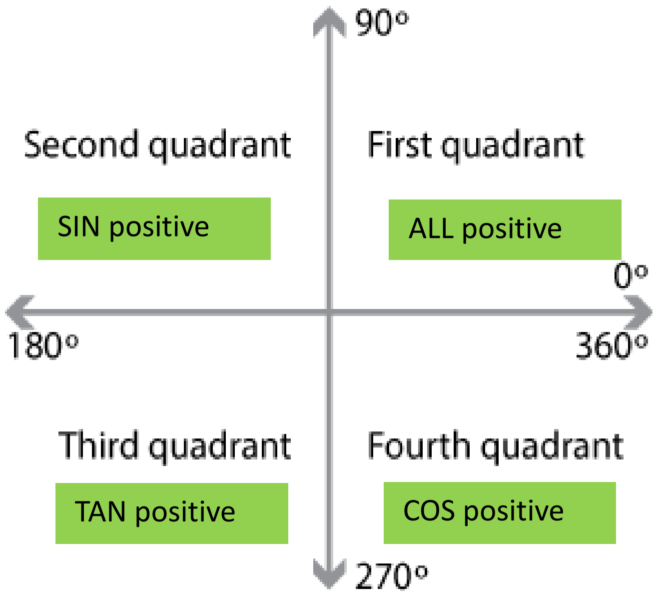
$$\tan y^o = 0.882$$

$$y^o = \quad tan^{-1}\,0.882$$

$$y = 41.42^o$$

1.3 Evaluating Trigonometric Function

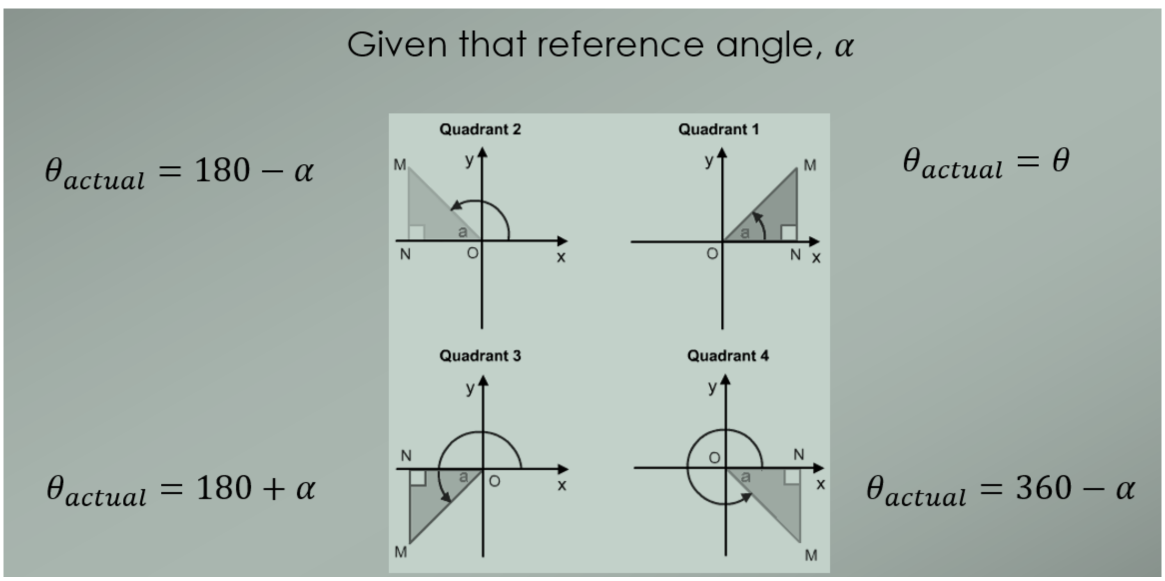




How to remember?

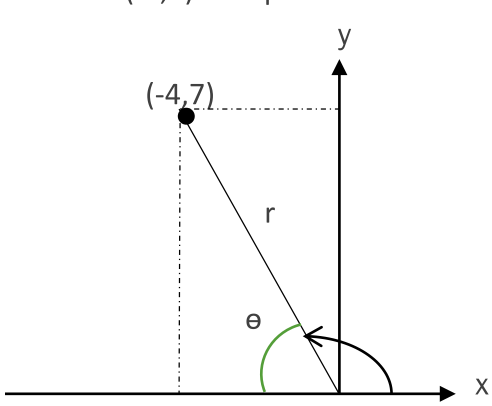
All Science Teacher Cute

Add Sugar To Coffee



Example 1.3 a

Let (-4,7) be a point on the terminal side of θ. Find the sine, cosine and tangent of θ.



$$r = \sqrt{x^2 + y^2}$$

$$r = \sqrt{(-4)^2 + 7^2}$$

$$r = \sqrt{16 + 49}$$

$$r = \sqrt{65}$$

$$r = 8.06$$

$$\sin\theta = \frac{O}{H} = \frac{y}{r} = \frac{7}{8.06}$$

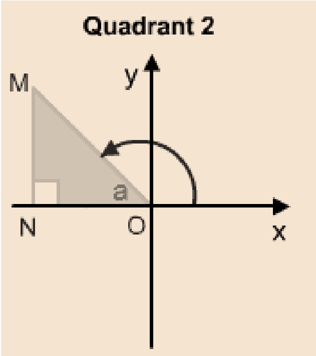
$$\cos\theta = \frac{A}{H} = \frac{x}{r} = \frac{-4}{8.06}$$

$$\tan\theta = \frac{O}{A} = \frac{y}{x} = \frac{7}{-4}$$

Example 1.3 b

Find the reference angle for the following angles.

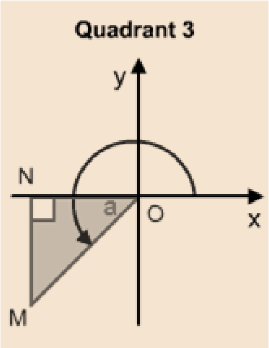
i) 145^o



reference angle,
$\alpha = 180^o - 145^o$

$\alpha = 35^o$

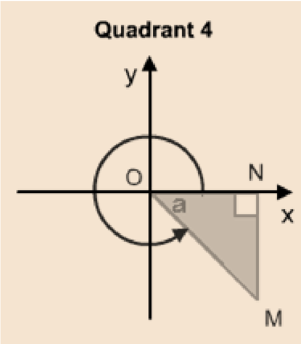
ii) 210^o



reference angle,
$\alpha = 210^o - 180^o$

$\alpha = 30^o$

iii) 325^o



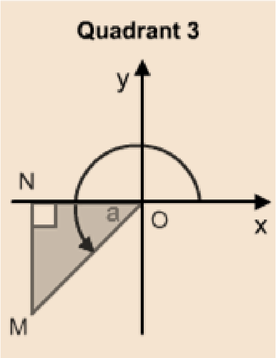
reference angle,
$\alpha = 360^o - 325^o$

$\alpha = 35^o$

Example 1.3 c

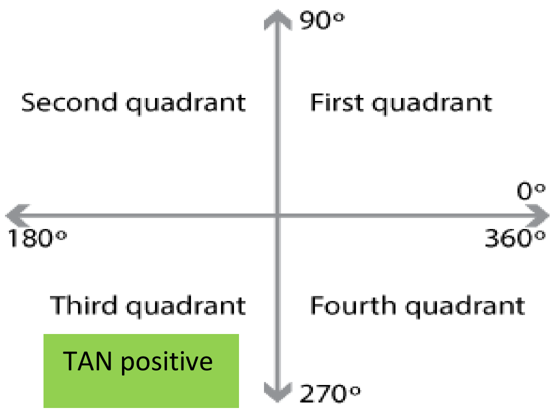
Evaluate each of the trigonometric functions.

i) $\sin 264^o$



reference angle,
$\alpha = 264^o - 180^o$

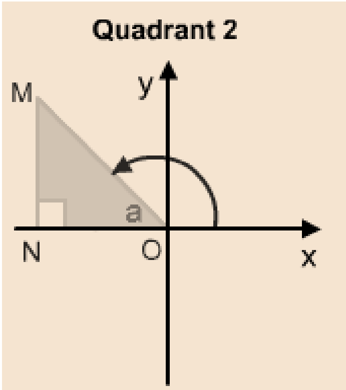
$\alpha = 84^o$

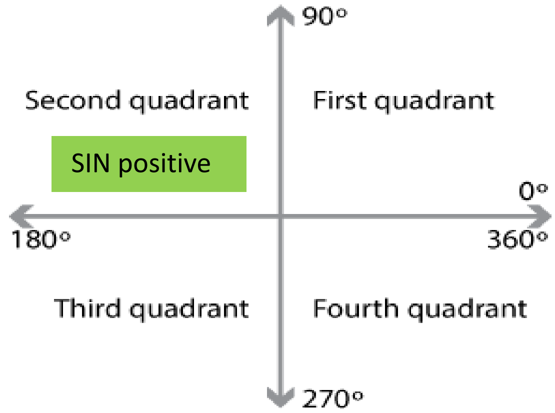


$\therefore \sin 264^o = \sin 84^o$

$= -0.995$

ii) $\cos 140^o$





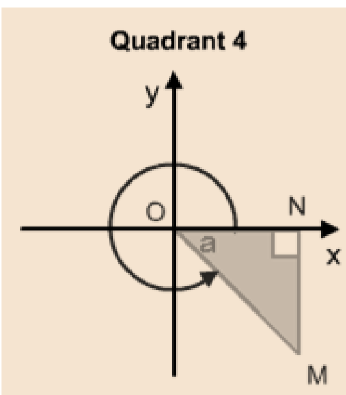
reference angle,
$\alpha = 180^o - 140^o$

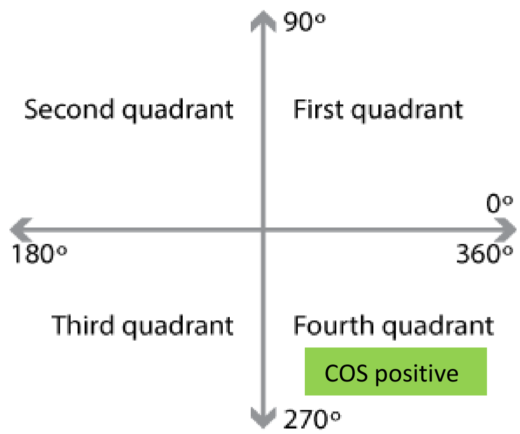
$\alpha = 40^o$

$\therefore \cos 140^o = \cos 40^o$

$= -0.766$

iii) $\tan 315^o$



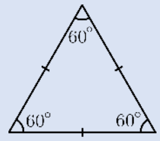
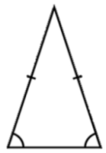
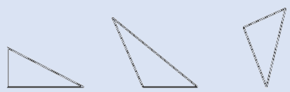


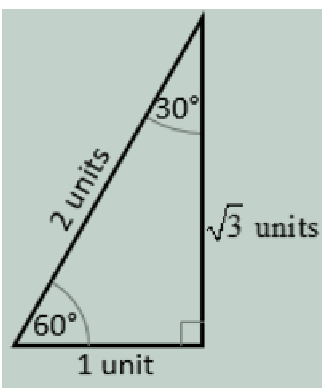
reference angle,
$\alpha = 360^o - 315^o$
$\alpha = 45^o$

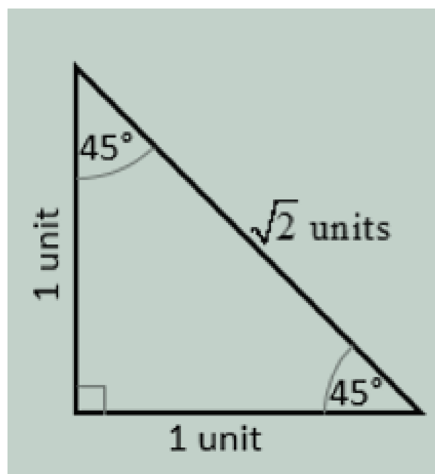
$\therefore \tan 315^o = \tan 45^o$

$= -1$

1.4 Special Triangle

Name of triangle	Shape	Characteristic
Equilateral triangle		- Three equal sides - Three equal angles, always 60°
Isosceles triangle		- Two equal sides - Two equal angles at base
Scalene triangle		- No equal sides - No equal angles





$\boldsymbol{\theta}$	$\mathbf{30^o}$	$\mathbf{45^o}$	$\mathbf{60^o}$
$\sin\theta$	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$
$\cos\theta$	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$
$\tan\theta$	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$

Example 1.4 a

Simplify the following without using calculator.

i) $\dfrac{\sin 60^o}{\cos 60^o}$

$= \dfrac{\left(\frac{\sqrt{3}}{2}\right)}{\left(\frac{1}{2}\right)}$

$= \left(\dfrac{\sqrt{3}}{2}\right) \times \left(\dfrac{2}{1}\right)$

$= \sqrt{3}$

ii) $cos^2 30^o + sin^2 30^o$

$= \left(\dfrac{\sqrt{3}}{2}\right)^2 + \left(\dfrac{1}{2}\right)^2$

$= \dfrac{\left(\sqrt{3}\right)^2}{(2)^2} + \dfrac{(1)^2}{(2)^2}$

$= \dfrac{3}{4} + \dfrac{1}{4}$

$= \dfrac{4}{4} = 1$

iii) $cos^2 45^o - sin^2 45^o$

$= \left(\dfrac{1}{\sqrt{2}}\right)^2 - \left(\dfrac{1}{\sqrt{2}}\right)^2$

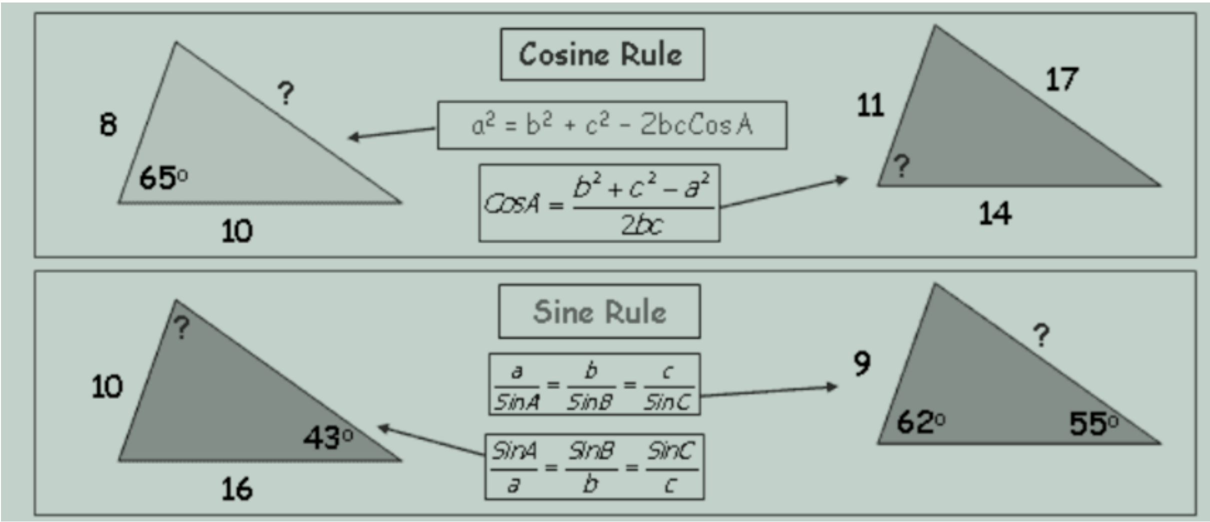
$= \dfrac{(1)^2}{\left(\sqrt{2}\right)^2} + \dfrac{(1)^2}{\left(\sqrt{2}\right)^2}$

$= \dfrac{1}{2} - \dfrac{1}{2}$

$= 0$

1.5 Solving Triangle

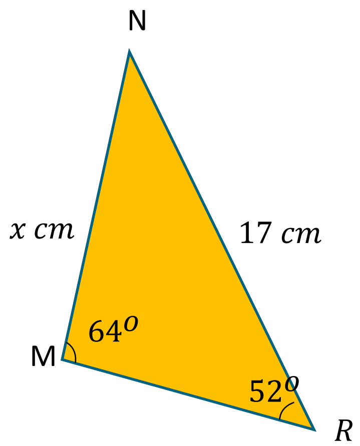
- Pythagoras Theorem can only be use when solving right angle triangles.
- Other than that, we need to use either Sine Rule or Cosine Rule.
- $\Delta\, ABC$ is made up of 6 elements : 3 sides (a,b,c) denoted by the small letters of the opposite vectices and 3 angles (A,B,C) denoted by the capital letters of the vertices.



The Sine Rule is used to solve triangles when	The Cosine Rule is used to solve triangles when
• 2 angles and 1 side are given • 2 sides and non-include angle are given (Non-include angle refers to an angle that is not contained between two sides)	• 3 sides are given • 2 sides and include angle are given (Include angle refers to an angle that is contained between two given sides)

Example 1.5 a

Use the Sine Rule to calculate the size of the side marked x cm.



$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

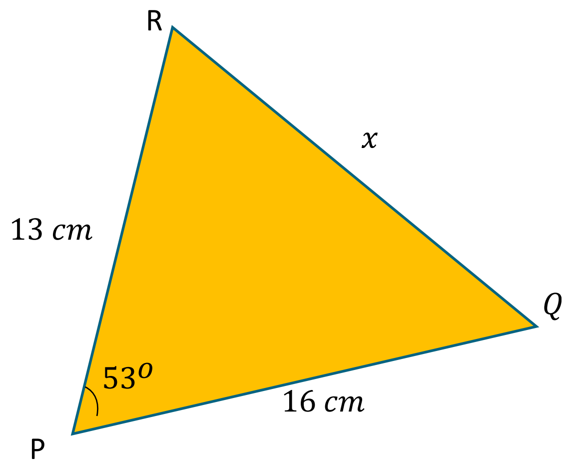
$$\frac{17}{\sin 64} = \frac{x}{\sin 52}$$

$$x = \frac{17}{\sin 64} \times \sin 52$$

$$x = 14.9\ cm$$

Example 1.5 b

Use the Cosine Rule to calculate the size of the side marked x cm.



$$p^2 = r^2 + q^2 - 2rq\ \mathrm{Cos}\ P$$

$$x^2 = 16^2 + 13^2 - 2(16)(13)\ \mathrm{Cos}$$

$$x^2 = 256 + 169 - 416\mathrm{Cos}\ 53$$

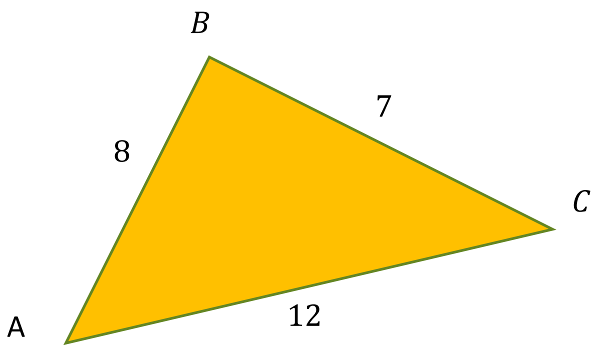
$$x^2 = 174.64$$

$$x = \sqrt{174.64}$$

$$x = 13.22\ cm$$

Example 1.5 c

Find the largest angle for the triangle ABC where AB=8, BC=7 and AC=12



largest angle will be opposite largest side
smallest angle will be opposite smallest side

$$b^2 = a^2 + c^2 - 2ac \text{ Cos } B$$

$$\text{Cos } B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$\text{Cos } B = \frac{7^2 + 8^2 - 12^2}{2(7)(8)}$$

$$\text{Cos } B = \frac{-31}{112}$$

$$B = cos^{-1}(\frac{-31}{112})$$

$$B = 106.1^o$$

Or

$$b^2 = a^2 + c^2 - 2ac \text{ Cos } B$$

$$12^2 = 7^2 + 8^2 - 2(7)(8) \text{ Cos } B$$

$$144 = 49 + 64 - 112\text{Cos } B$$

$$144 = 113 - 112 \text{ Cos } B$$

$$144 - 113 = -112\text{Cos } B$$

$$31 = -112\text{Cos } B$$

$$\frac{31}{-112} = \text{Cos } B$$

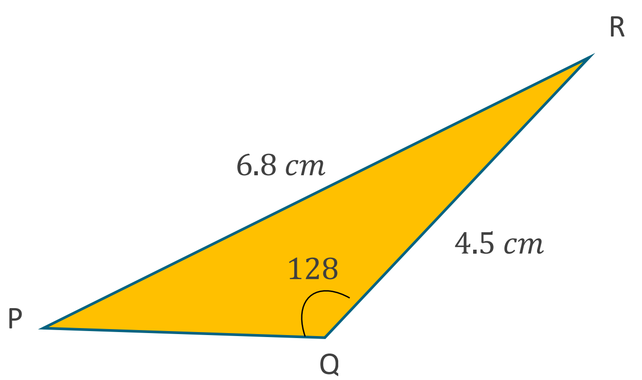
$$\text{Cos } B = \frac{-31}{112}$$

$$B = cos^{-1}(\frac{-31}{112})$$

$$B = 106.1^o$$

Example 1.5 d

The diagram shows ΔPQR. Find $< P$.



$$\frac{q}{\sin Q} = \frac{p}{\sin P}$$

$$\frac{6.8}{\sin 128} = \frac{4.5}{\sin P}$$

$$6.8 \sin P = 4.5 \sin 128$$

$$\sin P = \frac{4.5 \sin 128}{6.8}$$

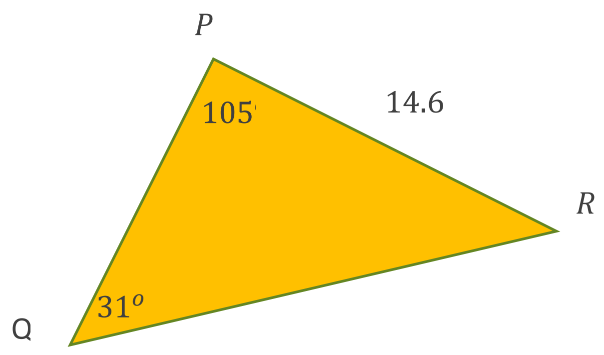
$$\sin P = 0.521$$

$$P = sin^{-1}0.521$$

$$P = 31.4^o = 31^o24'$$

Example 1.5 e

Solve the triangle PQR when $P = 105^o, Q = 31^o$ and $PR = 14.6\ cm$



$$angle\ R = 180^o - 105^o - 31^o$$

$$angle\ R = 44^o$$

$$\frac{q}{\sin Q} = \frac{p}{\sin P}$$

$$\frac{14.6}{\sin 31} = \frac{p}{\sin 105}$$

$$p = \frac{14.6}{\sin 31} \times \sin 105$$

$$p = 27.38\ cm$$

$$\frac{r}{\sin 44} = \frac{q}{\sin Q}$$

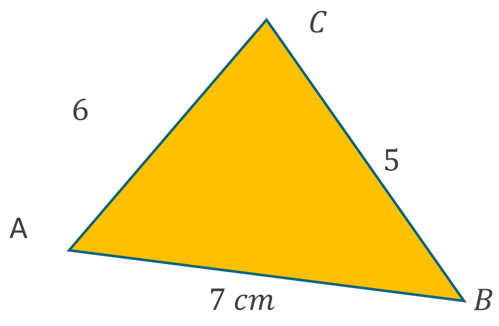
$$\frac{r}{\sin 44} = \frac{14.6}{\sin 31}$$

$$r = \frac{14.6}{\sin 31} \times \sin 44$$

$$r = 19.69\ cm$$

Example 1.5 f

Solve the triangles below by finding the unknown sides and angles.



$$a^2 = b^2 + c^2 - 2bc \text{ Cos } A$$

$$5^2 = 6^2 + 7^2 - 2(6)(7) \text{ Cos } A$$

$$25 = 36 + 49 - 84\text{Cos } A$$

$$\cos A = \frac{25 - 36 - 49}{-84}$$

$$\cos A = 0.714$$

$$A = cos^{-1}0.714$$

$$A = 44^o26'$$

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

$$\frac{5}{\sin 44^o26'} = \frac{7}{\sin C}$$

$$5 \sin C = 7 \sin 44^o26'$$

$$\sin C = \frac{7 \sin 44^o 26'}{5}$$

$$\sin C = 0.980$$

$$C = sin^{-1}0.980$$

$$C = 78^o33'$$

$$angle\ B = 180^o - 44^o26' - 78^o33'$$

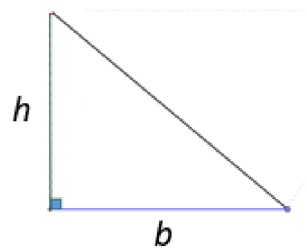
$$angle\ B = 57^o1'$$

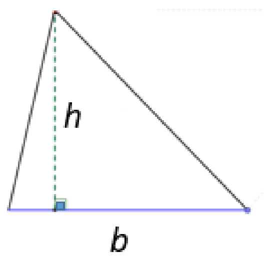
1.6 Area of Triangle

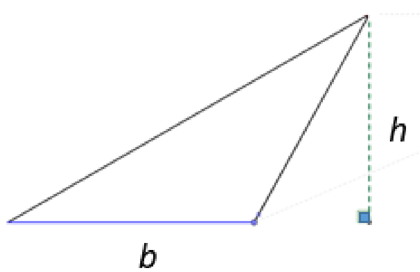
The simplest way to find the area of a triangle is to use this formula.

Area = $\frac{1}{2}$ × base × perpendicular height

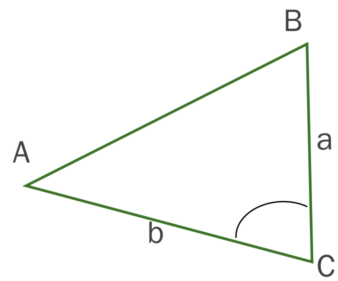
$$A = \frac{1}{2}bh$$



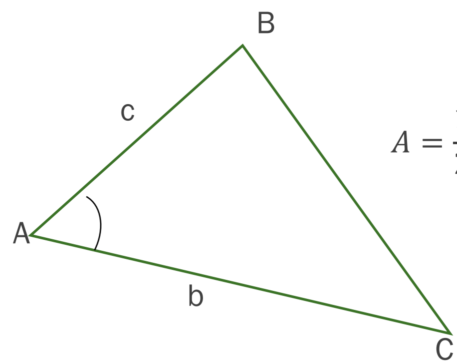




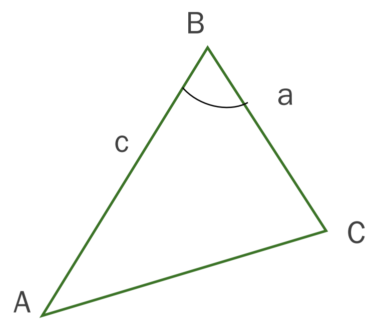
If the perpendicular height is not known, we need to use this formula.



$$A = \frac{1}{2}ab\sin C$$



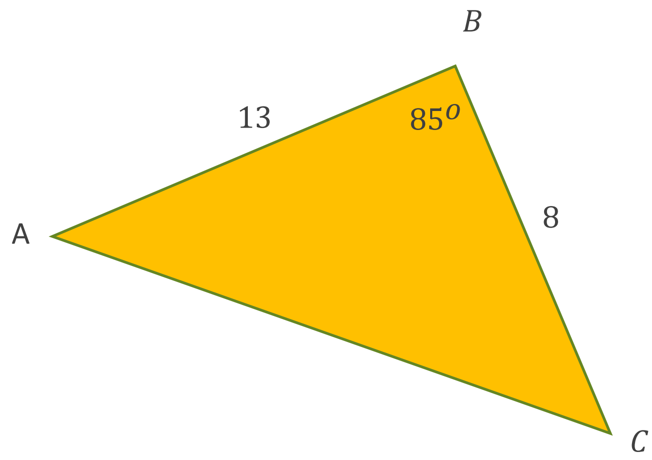
$$A = \frac{1}{2}bc\sin A$$



$$A = \frac{1}{2}ac\sin B$$

Example 1.6 a

Find the area of the triangle ABC



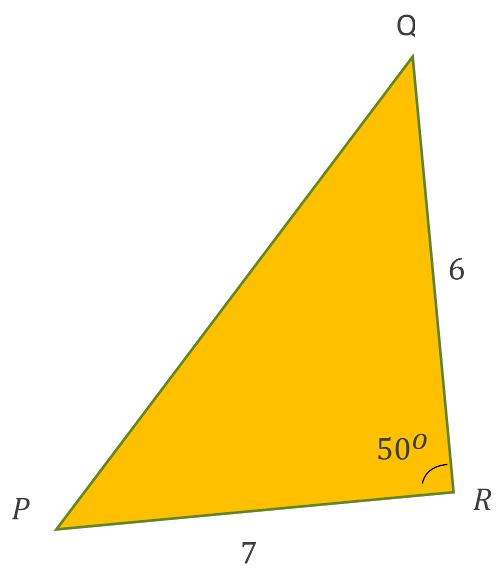
$$Area = \frac{1}{2} ca \sin B$$

$$Area = \frac{1}{2}(13)(8)\sin 85$$

$$Area = 51.8\ unit^2$$

Example 1.6 b

Find the area of the triangle PQR



$$Area = \frac{1}{2} pq \sin R$$

$$Area = \frac{1}{2}(6)(7)\sin 50$$

$$Area = 16.09\ unit^2$$

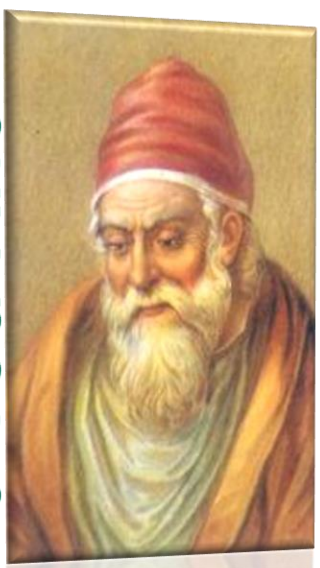
TOPIC 2

CIRCULAR MEASURE

Real life application

Architecture	Astronomy	Design
Astrology	Construction	Physic
Food	Circular object	

Do You Know?



A mathematician named Euclid was the first person to study circle.

Subtopic

2.1 Introduce and define radian and degree.

2.2 Calculate conversion radian to degree and degree to radian.

2.3 Calculate circumference and Arc Length.

2.4 Calculate Area of sector and segment.

2.1 What is radian and degree?

Radian	Degree
• A radian is the standard unit of angular measure • Radian is a SI unit	• A degree is a unit measurement of plane angle • Degree is not a SI unit

2.2 Conversion radian to degree and degree to radian

Conversion of common angles

Units	**Values**												
Radian	0	$\frac{\pi}{12}$	$\frac{\pi}{6}$	$\frac{\pi}{5}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{2\pi}{5}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	$\frac{4\pi}{5}$	π	$\frac{3\pi}{2}$	2π
Degree	0^o	15^o	30^o	36^o	45^o	60^o	72^o	90^o	120^o	144^o	180^o	270^o	360^o

How to convert?

Degree to Radian

$$radian = degree \times \frac{\pi}{180}$$

Radian to Degree

$$degree = radian \times \frac{180}{\pi}$$

Example 2.2 a

Convert degree to radian

i) 55^o

$$= 55 \times \frac{\pi}{180}$$
$$= 0.96\ rad$$

ii) 131^o

$$= 131 \times \frac{\pi}{180}$$
$$= 2.286\ rad$$

iii) 69^o

$$= 69 \times \frac{\pi}{180}$$
$$= 1.204\ rad$$

iv) 25^o

$$= 25 \times \frac{\pi}{180}$$
$$= 0.436\ rad$$

v) 360^o

$$= 360 \times \frac{\pi}{180}$$
$$= 6.283\ rad\ @ = 2\pi\ rad$$

Example 2.2 b

Convert radian to degree

i) $0.568\ rad$

$$= 0.568 \times \frac{180}{\pi}$$

$$= 32.54^o$$

ii) $1.452\ rad$

$$= 1.452 \times \frac{180}{\pi}$$

$$= 83.19^o$$

iii) $5.321\ rad$

$$= 7.321 \times \frac{180}{\pi}$$

$$= 419.46^o$$

iv) $3.796\ rad$

$$= 3.796 \times \frac{180}{\pi}$$

$$= 217.49^o$$

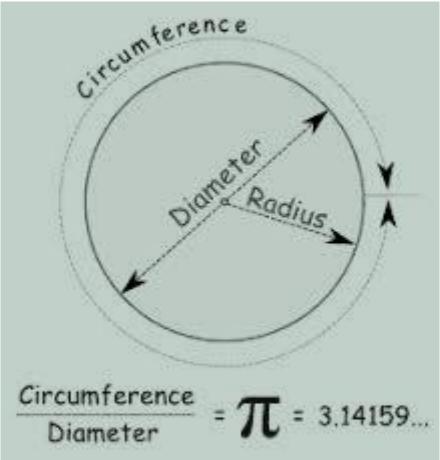
v) $2.058\ rad$

$$= 2.058 \times \frac{180}{\pi}$$

$$= 117.91^o$$

2.3 Circumference and Arc Length

A circle can be defined as the curve traced out by a point that moves so that its distance from a given point is constant.



- The radius is the distance from the center to the edge.
- The diameter starts at one side of the circle, goes through the center and ends on the other side. So, the diameter is twice the radius.

Diameter = 2 x radius

- The circumference is the distance around the edge of the circle. The perimeter of a circle is called the circumference. So, the circumference is perimeter of the whole circle.

Circumference = π x diameter = πd

Circumference =2 x π x radius =2 πr

Example 2.3 a

Find the circumference of each circle with a diameter given (use : $\pi = \frac{22}{7} = 3.142$)

1) 35 cm

$= \pi d$

$= 3.142 \times 35$

$= 109.97\ cm$

2) 26.5 mm

$= \pi d$

$= 3.142 \times 26.5$

$= 83.263\ mm$

3) 7.95 m

$= \pi d$

$= 3.142 \times 7.95$

$= 24.98\ m$

Example 2.3 b

Find the circumference of each circle with a radius given (use : π=22/7=3.142)

1) 10 m

$= 2\pi r$

$= 2 \times 3.142 \times 10$

$= 62.84\ m$

2) 63 km

$= 2\pi r$

$= 2 \times 3.142 \times 63$

$= 395.89\ km$

3) 107 cm

$= 2\pi r$

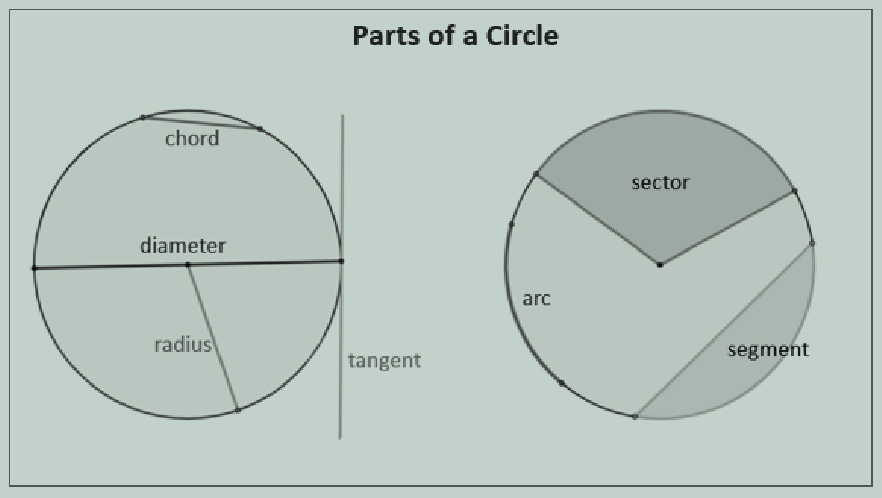
$= 2 \times 3.142 \times 107$

$= 672.39\ m$

2.4 Area of sector and segment (I)

There are two main "slices" of a circle:

1. The "pizza" slice is called a **sector.**
2. The slice made by a chord is called a **segment.**

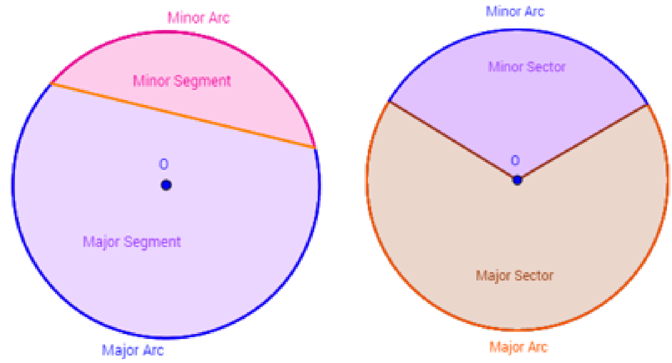


Arc is a part between two points on the circumference of a circle.

The formula of arc length (of a sector or segment) is :

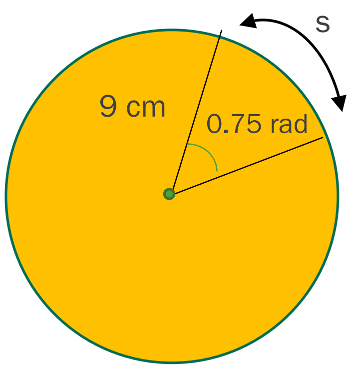
$L = r \times \theta$ (when θ is in radian)

$L = \frac{\theta}{360^o} \times 2\pi r$ (when θ is in degree)



Example 2.4 a

Calculate the length of the arc, s of each of the following circles below:



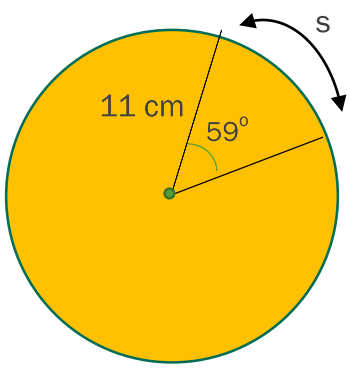
$L = r \times \theta$ (when θ is in radian)

$L = 9 \times 0.75$

$L = 6.75\ cm$

Example 2.4 b

Calculate the length of the arc, s of each of the following circles below:



Method 1

$L = r \times \theta$
(when θ is in radian)

Change degree to radian

$59 \times \frac{\pi}{180} = 1.03\ rad$

$L = 11 \times 1.03$

$L = 11.33\ cm$

Method 2

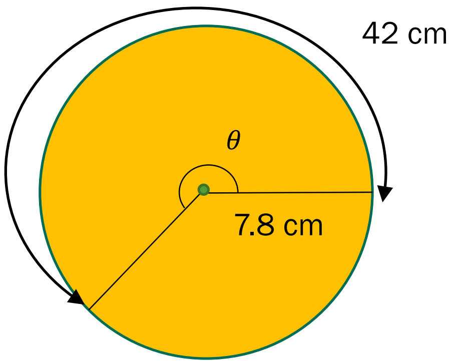
$L = \frac{\theta}{360^o} \times 2\pi r$
(when θ is in degree)

$L = \frac{59}{360^o} \times 2\pi(11)$

$L = 11.33\ cm$

Example 2.4 c

Given length 42 cm and radius is 7.8 cm, find the value of θ in **degree units.**



Method 1

$L = r \times \theta$ (when θ is in radian)

$$42 = 7.8 \times \theta$$

$$\theta = \frac{42}{7.8}$$

$$\theta = 5.385\ rad$$

Change radian to degree

$$5.385 \times \frac{180}{\pi} = 308.5^o$$

Method 2

$L = \frac{\theta}{360^o} \times 2\pi r$ (when θ is in degree)

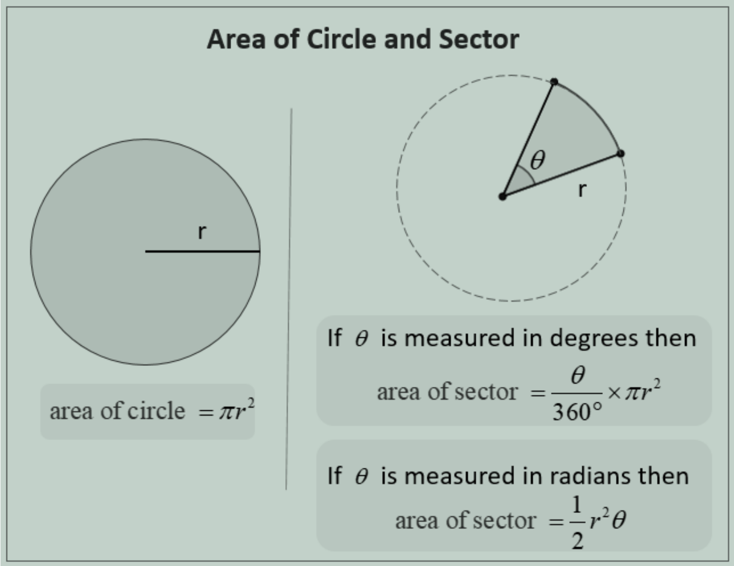
$$42 = \frac{\theta}{360^o} \times 2\pi(7.8)$$

$$42 = \frac{2\pi(7.8)\theta}{360^o}$$

$$\theta = \frac{42 \times 360}{2\pi(7.8)}$$

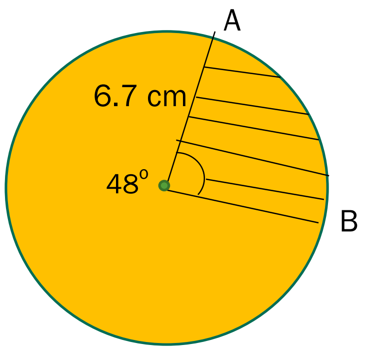
$$\theta = 308.5^o$$

2.5 Area of sector and segment (II)



Example 2.5 a

For each of the following circles, calculate the area of the shaded sector.



Area of sector AOB

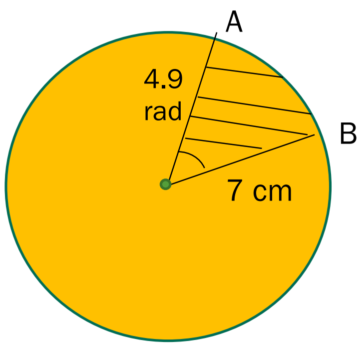
$$= \frac{\theta}{360^o} \times \pi r^2$$

$$= \frac{48}{360} \times \pi (6.7)^2$$

$$= 18.8\ cm^2$$

Example 2.5 b

For each of the following circles, calculate the area of the shaded sector.



Area of sector AOB

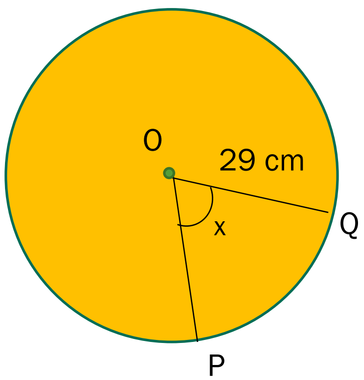
$$= \frac{1}{2} r^2 \theta$$

$$= \frac{1}{2} (7)^2 (4.9)$$

$$= 120.05\ cm^2$$

Example 2.5 c

In the diagram below, O is the centre of the circle. Find the value of x in degrees if the area of a minor sector POQ is 548 cm^2.



Method 1

$$A = \frac{\theta}{360^o} \times \pi r^2$$

$$548 = \frac{\theta}{360^o} \times \pi (29)^2$$

$$\theta = \frac{548 \times 360}{\pi (29)^2}$$

$$\theta = 74.67^o$$

Method 2

$$A = \frac{1}{2} r^2 \theta$$

$$548 = \frac{1}{2} (29)^2 \theta$$

$$\theta = \frac{548 \times 2}{(29)^2}$$

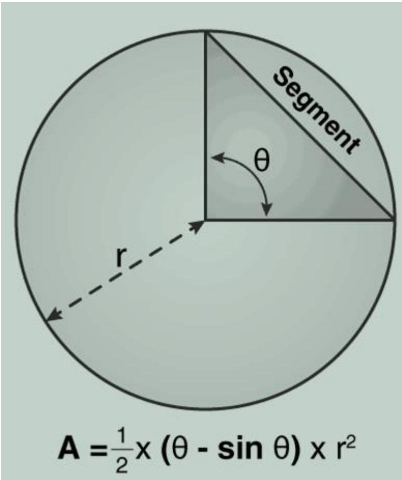
$$\theta = 1.303\ rad$$

Change radian to degree

$$1.303 \times \frac{180}{\pi} = 74.48^o$$

2.6 Area of sector and segment (III)

Area of segment



θ in radian

1. Area of sector

$$A = \frac{1}{2}r^2\theta$$

2. Area of triangle in the circle

$$A = \frac{1}{2}r^2\sin\theta$$

3. Area of segment

= Area of sector-Area of triangle in the circle

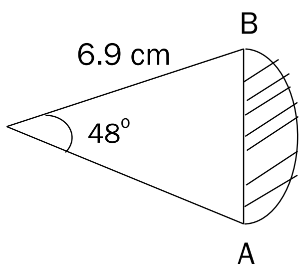
$$A = \frac{1}{2}r^2\theta - \frac{1}{2}r^2\sin\theta$$

$$A = \frac{1}{2}r^2(\theta - \sin\theta)$$

radian   degree

Example 2.6 a

Based on the figure below, calculate the area of the segment.



Area of segment

$$A = \frac{1}{2}r^2(\theta - \sin\theta)$$

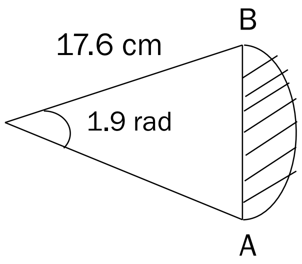
$$A = \frac{1}{2}(6.9)^2\left[\left(48 \times \frac{\pi}{180}\right) - \sin 48\right]$$

radian   degree

$$A = 2.25\ cm^2$$

Example 2.6 b

Based on the figure below, calculate the area of the segment



Area of segment

$$A = \frac{1}{2} r^2 (\theta - \sin\theta)$$

$$A = \frac{1}{2}(17.6)^2 \left[1.9 - \sin\left(1.9 \times \frac{180}{\pi}\right)\right]$$

radian (1.9) degree ($1.9 \times \frac{180}{\pi}$)

$$A = \frac{1}{2}(17.6)^2(1.9 - \sin 108.86)$$

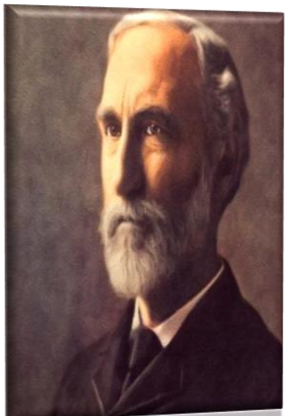
$$A = 147.71\ cm^2$$

TOPIC 3
VECTOR

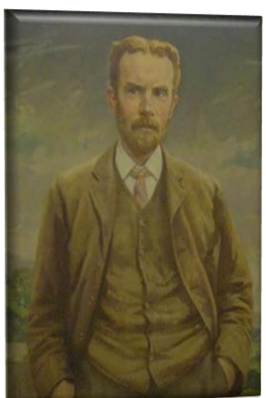
Real life application

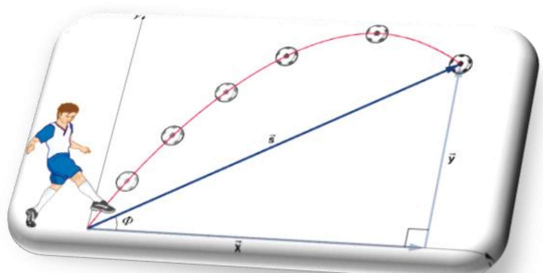
Physic Engineering
(force, torque, velocity, momentum, projectile)
Gaming   Crosswind   Sport
Electromagnetic Field   Weight
3 Dimensional Space

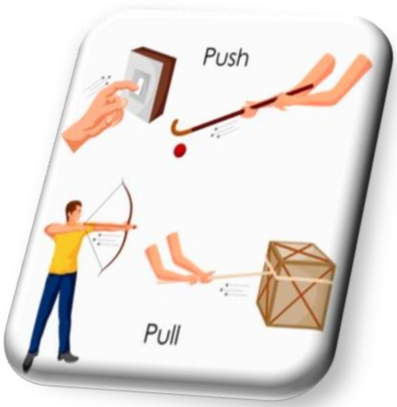
Do You Know?

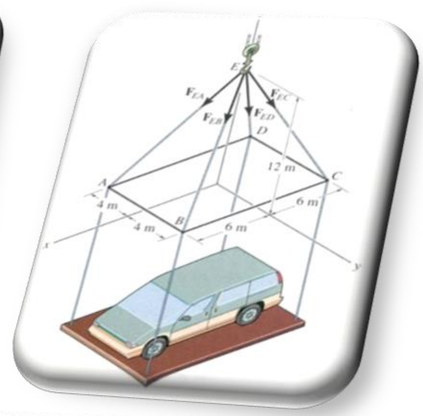


In their modern form, vectors appeared late in the 19th century when Josiah Willard Gibbs and Oliver Heaviside independently developed vector analysis.









Subtopic

3.1 Define and draw a directed line to represent a vector.

3.2 Solve algebraic operation of vector (2 dimensions only).

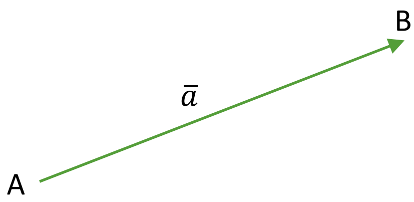
3.3 Demonstrate addition and subtraction of vectors using Parallelogram method.

3.4 Apply the dot product.

3.1 Define and draw a directed line to represent a vector.

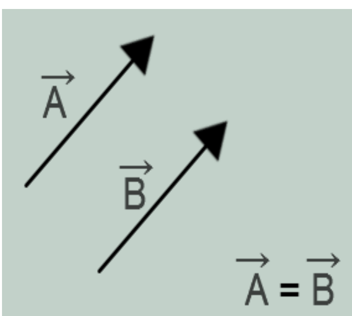
Introduction to vector

- Vector is a quantity that has both magnitude and direction.
- Example of vector quantity : Velocity, Displacement, Force
- Vector can be represented graphically by a directed line segment.
- Notation of a vector : $\overrightarrow{AB}$, $\boldsymbol{a}$ and $\bar{a}$

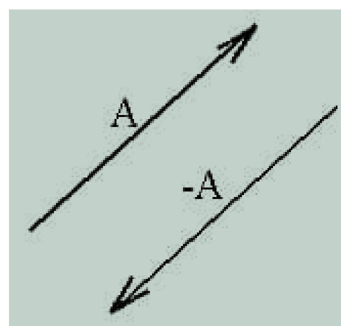


- The arrow shows the direction of the vector from A to B
- The length of line represented the magnitude of the vector, denoted by $|\overrightarrow{AB}|$ or $|a|$

Equal vectors is two vectors that have same magnitude and direction.

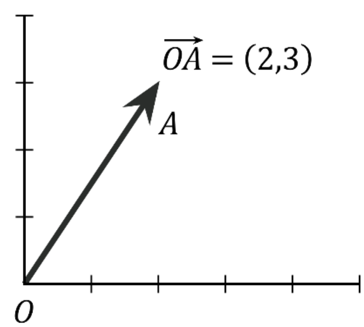


Negative vectors is a vector in the opposite direction of the given vector with equal magnitude.



Position vector

If A is the point P(x,y) and O is the origin, then $\overrightarrow{OA}$ is called position vector of point A.



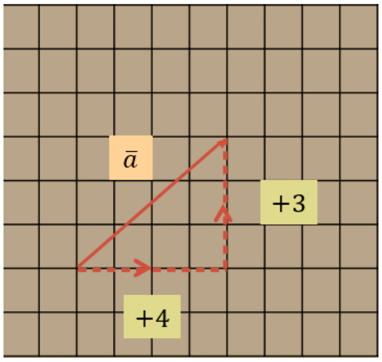
Vector in Cartesian plane

- A vector can be represented by a directed line on Cartesian plane.
- The 2 dimension vector on the Cartesian Plane can be written in the coordinate point form of (x,y) or x**i**+y**j**
- **i** and **j** are unit vector (magnitude of 1 unit), where **i** is a unit vector in the direction of x-axis while **j** in the direction of y-axis. So, x and y represent component for x-axis and y-axis.
- If vector $\overrightarrow{AB} = xi + yj$,then its magnitude, $\left|\overrightarrow{AB}\right| = \sqrt{x^2 + y^2}$

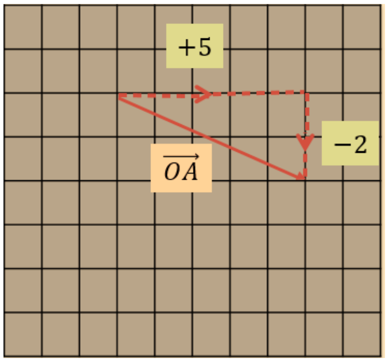
Example 3.1 a

Draw a directed line segment to represent each vector.

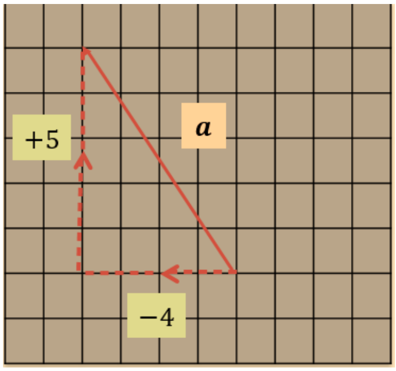
a) $\bar{a} = \begin{pmatrix} 4 \\ 3 \end{pmatrix}$



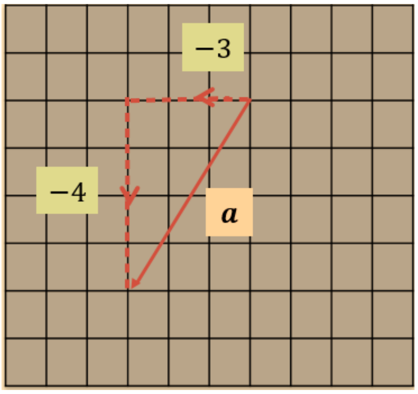
b) $\overrightarrow{OA} = \begin{pmatrix} 5 \\ -2 \end{pmatrix}$



c) $\boldsymbol{a} = \begin{pmatrix} -4 \\ 5 \end{pmatrix}$



d) $\boldsymbol{a} = \begin{pmatrix} -3 \\ -4 \end{pmatrix}$



Example 3.1 b

Calculate the magnitude of the given vector.

a) $\overrightarrow{OA} = 4i + 7j$

$|\overrightarrow{OA}| = \sqrt{4^2 + 7^2}$

$= \sqrt{16 + 49}$

$= \sqrt{65}$

b) $\overrightarrow{OB} = 9i - 4j$

$|\overrightarrow{OB}| = \sqrt{9^2 + (-4)^2}$

$= \sqrt{81 + 16}$

$= \sqrt{97}$

c) $\bar{a} = -2i - 3j$

$|\overrightarrow{OA}| = \sqrt{(-2)^2 + (-3)^2}$

$= \sqrt{4 + 9}$

$= \sqrt{13}$

d) $\boldsymbol{a} = 9j$

$|\overrightarrow{OA}| = \sqrt{0^2 + 9^2}$

$= \sqrt{0 + 81}$

$= \sqrt{81}$=9

3.2 Solve algebraic operation of vector (2 dimensions only).

Addition and subtraction of vector

- To add or subtract two vectors whose components are known, we simply add or subtract the components.
- If vector $a = x_1 i + y_1 j$ and b $= x_2 i + y_2 j$, then

$$a + b = (x_1 + x_2)i + (y_1 + y_2)j$$
$$a - b = (x_1 - x_2)i + (y_1 - y_2)j$$

Scalar Multiplication

- If vector $\boldsymbol{a}$ is multiply with scalar k, then the product is $\mathrm{k}\boldsymbol{a}$.
- The direction $\mathrm{k}\boldsymbol{a}$ is unchanged.

Example 3.2 a

Given vectors $a = 7i + 5j,\ b = i + 9j.$ Find :

i) $a + b$

$a + b = (x_1 + x_2)i + (y_1 + y_2)j$

$= 7i + i + 5j + 9j$

$= 8i + 14j$

ii) $a - b$

$a - b = (x_1 - x_2)i + (y_1 - y_2)j$

$= (7i - i) + (5j - 9j)$

$= 6i - 4j$

Example 3.2 b

Given vectors $a = 4i - 7j,\ b = 11j.$ Find :

i) $5a$

$5a = 5(4i - 7j)$

$= 20i - 35j$

ii) $4a + 7b$

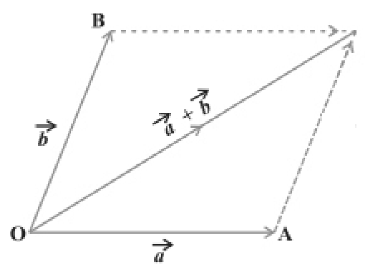
$4a + 7b = 4(4i - 7j) + 7(11j)$

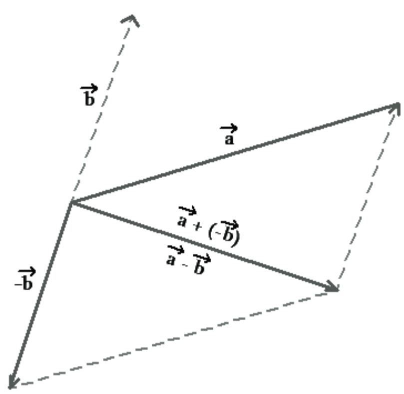
$= 16i - 28j + 77j$

$= 16i + 49j$

3.3 Demonstrate addition and subtraction of vectors using Parallelogram method.

- The addition and subtraction of vectors can be determined by Parallelogram Method
- Subtraction $\boldsymbol{a} - \boldsymbol{b}$ can be taken as the addition of $\boldsymbol{a} + (-\boldsymbol{b})$





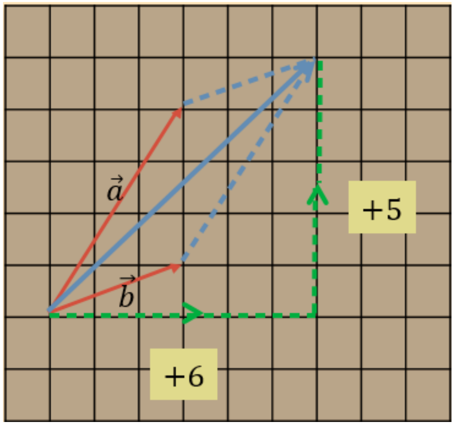
STEPS

1. Joins the initial points of both vectors together.
2. Draw a parallelogram.
3. Draw a diagonal of the parallelogram. The diagonal is vector that presents the addition and subtraction of vectors.

Example 3.3 a

Determine the resultant vector of two vectors by addition operation.

$a = 3i + 4j, \;\; b = 3i + j$



By calculation:

$$a + b = (x_1 + x_2)i + (y_1 + y_2)j$$

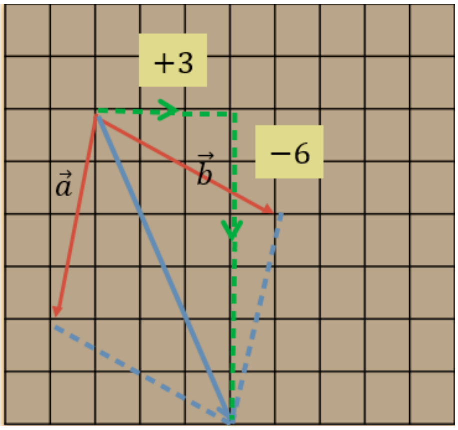
$$= (3 + 3)i + (4 + 1)j$$

$$= 6i + 5j$$

Example 3.3 b

Determine the resultant vector of two vectors by addition operation.

$a = -i - 4j, \;\; b = 4i - 2j$



By calculation:

$$a + b = (x_1 + x_2)i + (y_1 + y_2)j$$

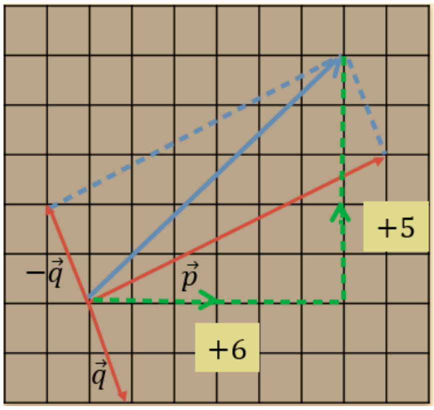
$$= (-1 + 4)i + (-4 + (-2))j$$

$$= 3i - 6j$$

Example 3.3 c

Determine the resultant vector of two vectors by subtraction operation.

$p = 7i + 3j,\ \ q = i - 2j$



By calculation:

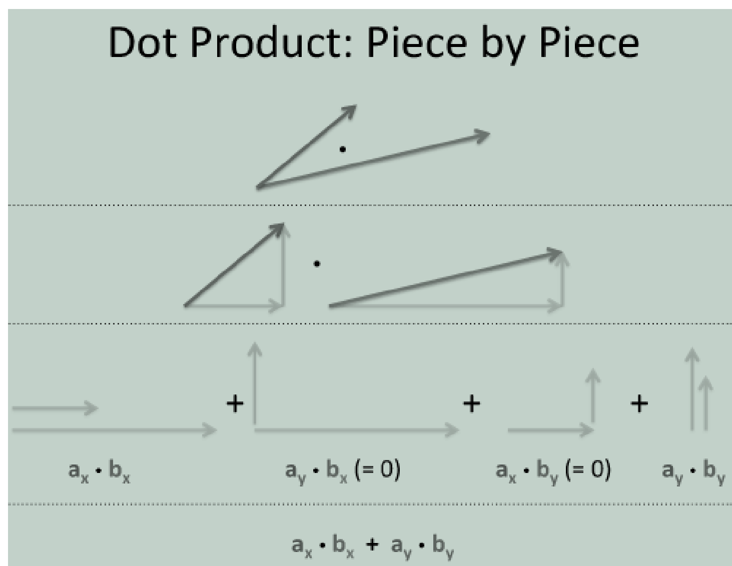
$$p - q = (x_1 + x_2)i + (y_1 + y_2)j$$

$$= (7 - 1)i + (3 - (-2))j$$

$$= 6i + 5j$$

3.4 Apply the dot product.

- Dot product is a scalar product of two vectors a and b given by $a.b = |a||b|\cos\theta$ where θ is the smaller angle between the vectors a and b
- Angle θ between two vectors a and b is given by $\cos\theta = \frac{a.b}{|a||b|}$



Properties of Dot Product

- **Commutative Law**

$$a.b = b.a$$

$$m(a.b) = (ma).b = a.(mb),$$

where m is constant

If vector
$a = x_1 i + y_1 j$ and $= x_2 i + y_2 j$, then

$a.b = x_1 x_2 + y_1 y_2$

- **Distributive Law**

$$a.(b + c) = ab.ac$$
$$(a + b).c = ac.bc$$

- If a is perpendicular to b, then $a.b = 0$
- If a is parallel to b **(a and b are in same direction),** then $a.b = |a||b|$
- If a is parallel to b **(a and b are in opposite direction),** then $a.b = -|a||b|$
- $a.a = |a|^2$

Example 3.4 a

Given vectors $a = 7i + 2j,\ b = 9i - 5j.$ Find :

i) $a.b$

$a.b = (7i + 2j).(9i - 5j)$

$= (7)(9) + (2)(-5)$

$= 63 - 10$

$= 53$

ii) $3a.b$

$3a.b = 3(7i + 2j).(9i - 5j)$

$= (21i + 8j).(9i - 5j)$

$= (21)(9) + (8)(-5)$

$= 189 + (-40)$

$= 149$

Example 3.4 b

Given vectors $p = 7i - 3j,\ q = -i + 12j.$ Find :

i) $p.q$

$p.q = (7i - 3j).(-i + 12j)$

$= (7)(-1) + (-3)(12)$

$= -7 - 36$

$= -43$

ii) Angle, θ between p and q

$$\cos\theta = \frac{p.q}{|p||q|}$$

$$\cos\theta = \frac{-43}{\left(\sqrt{7^2 + (-3)^2}\right)\left(\sqrt{(-1)^2 + 12^2}\right)}$$

$$\cos\theta = \frac{-43}{\left(\sqrt{49 + 9}\right)\left(\sqrt{1 + 144}\right)}$$

$$\cos\theta = \frac{-43}{\left(\sqrt{58}\right)\left(\sqrt{145}\right)}$$

$\cos\theta = -0.469$

⬚ $\theta = cos^{-1} - 0.469$ ⬚ $\theta = 117.97^o$

TOPIC 4
INEQUALITY

Real life application

Comparison of quantities in term of number, price, temperature, size, height, mass, speed. Widely use in business field.

Do You Know?

The symbols “<” and “>” were used for the first time by English mathematician and astronomer Thomas Harriot.

Subtopic

4.1 Understand of inequality notations, range and number line.

4.2 Solve problem related to inequality.

4.1 Introduction of inequalities

- An equality is an algebraic relationship between two unequal quantities
- An equality shows which quantity is greater, than or less than another quantity
- Example:

  Ali run faster than Abu. In equality, we can write a > b where
  a represent "how fast Ali can run" while b represent "how fast Abu can run".
- > Is the sign that represent inequality

Inequalities symbols

Symbols	**Examples**	**Meanings**
$>$	$x > 7$	x is greater than 7
$\geq$	$x \geq M$	x is greater than M or equal to M
$<$	$y < -4$	y is less than -4
$\leq$	$x \leq R$	x is less than R or equal to R

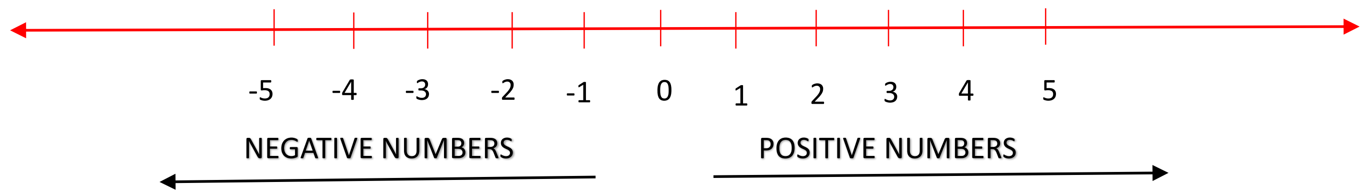
Interval notations

Interval notation is a way to notate the range of values that would make an inequality true.

Notations	**Names**	**Examples**	**Meanings**
(x, y)	Open interval	$(-2,5)$	From -2 to 5 but do not include -2 and 5
$[x, y]$	Closed interval	$[3,12]$	From 3 to 12 and include 3 and 12
$[x, y)$	Half-closed interval	$[-2,9)$	From -2 to 9, include -2 but do not include 9
$(x, y]$	Half-open interval	$(-13,5]$	From -13 to 5, do not include -13 but include 5

Number Line

- An equality also can be represented in number line
- By writing down numbers on number line makes it easy to differentiate which numbers is bigger or smaller

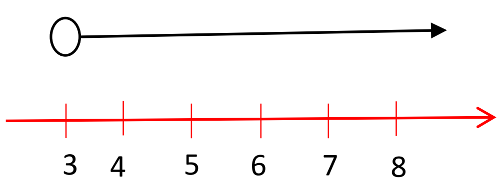


In number line,
numbers on the left are smaller than number on the right.
Example:
-1 is smaller than 1
-4 is smaller than 3

Inequalities in number line

- Use an empty circle, ○ for $<$ and $>$
- Example :

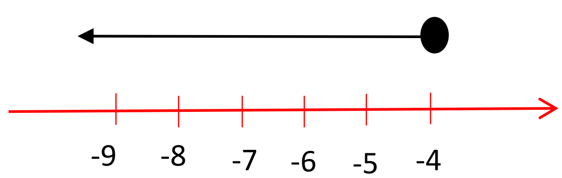
$x > 3$



Interval notation $(3, \infty)$

- Use a solid circle, ● for $\leq$ and $\geq$
- Example :

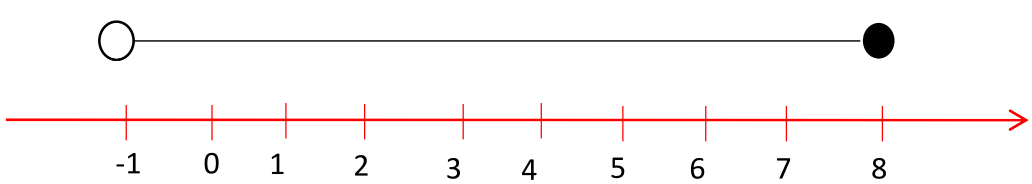
$x \leq -4$



Interval notation $(\infty, -4]$

- Example :

$-1 < x \leq 8$



Interval notation $(-1,8]$

4.2 Solving problems related to inequalities

- Solving inequalities is almost similar as solving algebraic equations
- But in some cases, solving inequalities will change the direction of the inequality.

- An inequality would not change the direction if:
- Add or subtract a number from both side
- Multiply or divide both sides with POSITIVE number

- An inequality would change the direction if:
- Multiply or divide both sides with NEGATIVE number
- Switching left and right hand sides

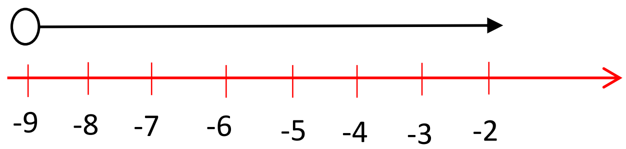
Example 4.2 a

Solve each of the inequalities.

i) $x + 3 > -6$

$x > -6 - 3$

$x > -9$



Interval notation $(-9, \infty)$

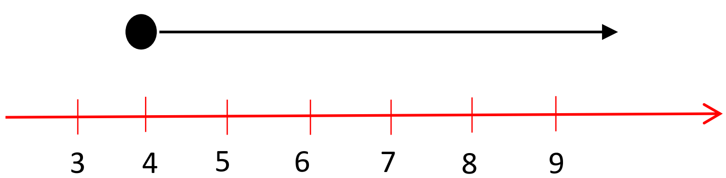
ii) $9 - 4x \leq -7$

$-4x \leq -7 - 9$

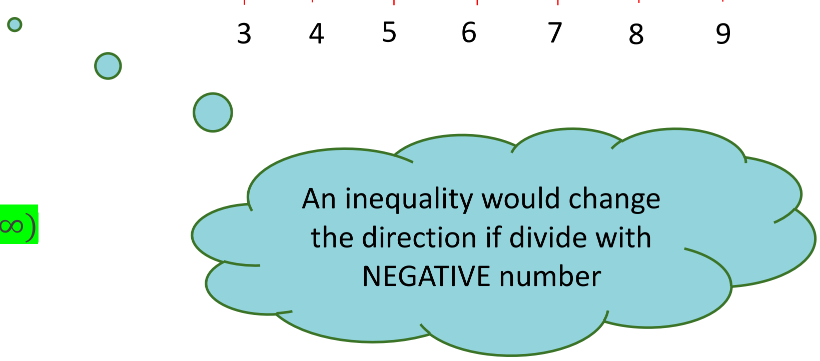
$-4x \leq -16$

$x \geq \frac{-16}{-4}$

$x \geq 4$



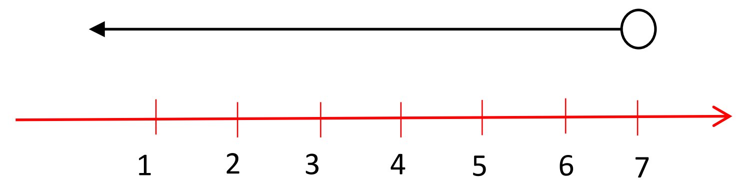
Interval notation $[-4, \infty)$



iii) $3y < 21$

$$y < \frac{21}{3}$$

$$y < 7$$



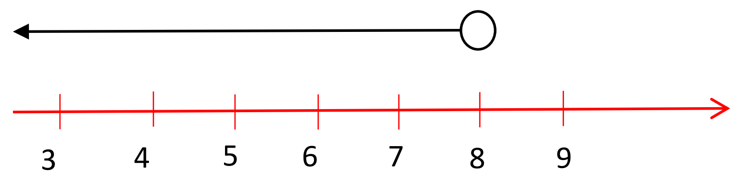
Interval notation $(\infty, 7)$

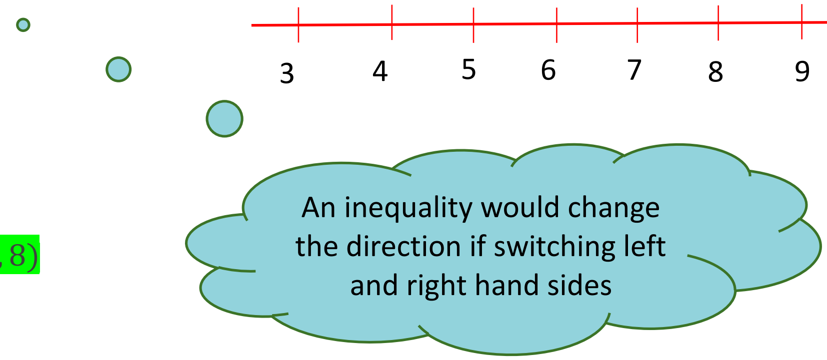
iv) $5 > \ x - 3$

$$x - 3 \ \ < 5$$

$$x \ \ < 5 + 3$$

$$x \ \ < 8$$





Interval notation $(\infty, 8)$

Example 4.2 b

Solve the inequalities.

i) $-2 \leq \frac{4-2x}{5} \leq 4$

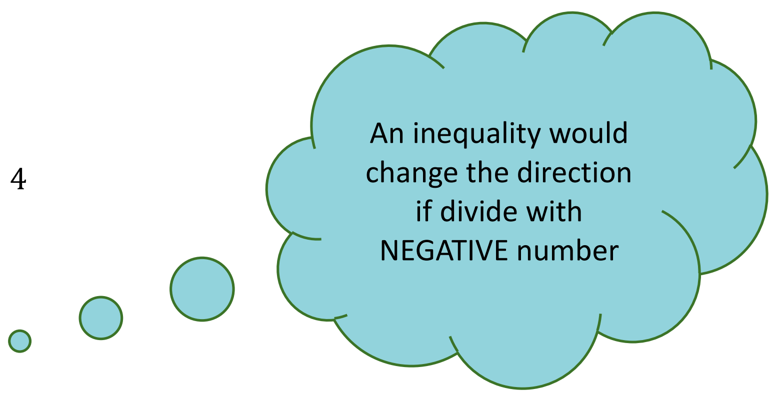
$$-10 \leq 4 - 2x \leq 20$$

$$-10 - 4 \leq -2x \leq 20 - 4$$

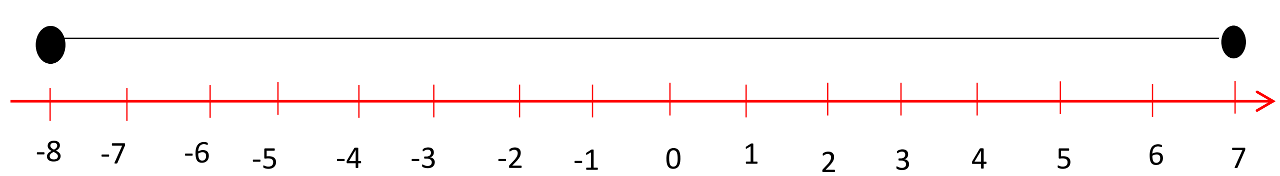
$$-14 \leq -2x \leq 16$$

$$\frac{-14}{-2} \geq x \geq \frac{16}{-2}$$

$$7 \geq x \geq -8$$



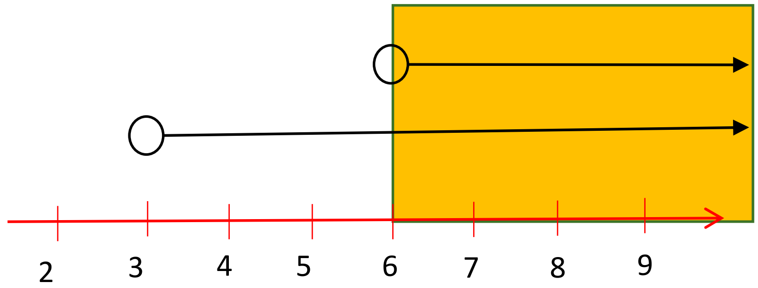
Interval notation $[7, -8]$



Example 4.2 c

Solving two linear inequalities.

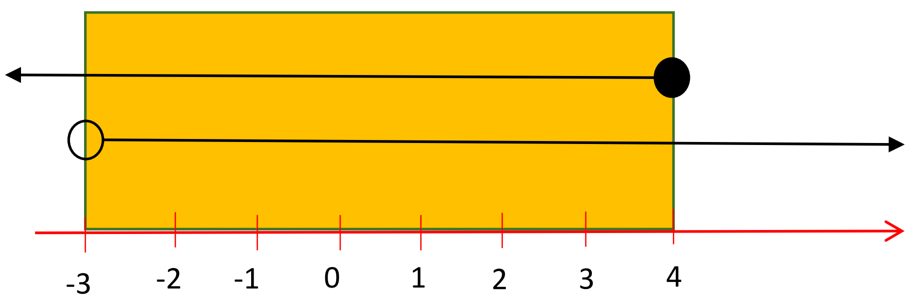
i) $x > 3$ and $x > 6$



The solution is $x > 6$

Interval notation $(6, \infty)$

ii) $x > -3$ and $x \leq 4$



The solution is $-3 < x \leq 4$

Interval notation $[-3,4)$

iii) $-17 \leq 3x - 8 \leq 13$

$-17 + 8 \leq 3x \leq 13 + 8$

$-9 \leq 3x \leq 21$

$\frac{-9}{3} \leq x \leq \frac{21}{3}$

$-3 \leq x \leq 7$

The solution is $-3 \leq x \leq 7$

Interval notation $[-3,7]$

TOPIC 5
MATRICES

Real life application

Solving linear system of equations.

Finance Science Manufacturing

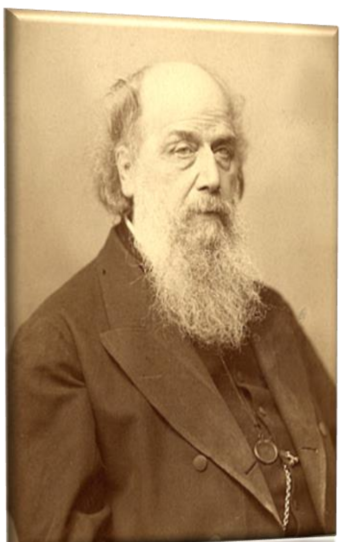
Optimizing Quantum Mechanic

Electrical Circuit Optic

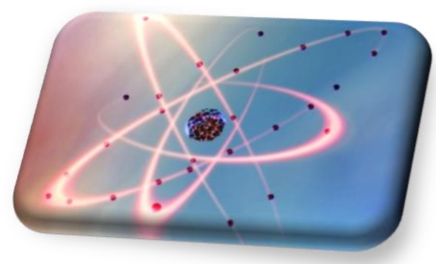
Computer Science

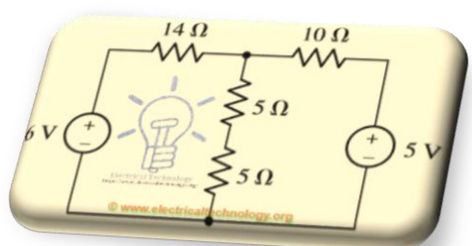
Encryption in wi-fi communication

Do You Know?



James Joseph Sylvester did important work on matrix theory. He discovered the discriminant of a cubic equation and first used the name 'discriminant' for equations of higher order.





Subtopic

5.1 Define and identify of Matrices

5.1.1 Stating the Number of Rows and Columns.

5.1.2 Stating Order of a Matrix (M x N).

5.1.3 Types of matrices.

5.2 Perform basic operations on Matrices.

i. Addition.

ii. Subtraction.

iii. Multiplication.

5.3 Calculate of Inverse Matrix (2 x 2 only).

5.4 Solve Simultaneous Equations By Using Matrix Method (2 Variables Only)

5.1 Understand Matrices

Row And Column

- A m x n matrix is a rectangular array of numbers in m row and n column enclosed in brackets
- The numbers are called the elements of the matrix

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$$

- Capital letters like A, B or Q is used to represent a matrix, and small letters to represent the elements
- The examples of matrices are shown below:

$$A = \begin{pmatrix} 2 \\ 4 \\ -1 \end{pmatrix}, \ B = \begin{pmatrix} 4 & 11 & -5 \\ 0 & 1 & 8 \end{pmatrix}, \ Q = \begin{pmatrix} 6 & 1 \\ 9 & -6 \\ 1 & 3 \end{pmatrix}$$

Notation matrix (m x n)

$A = \begin{pmatrix} -2 & 4 & 7 \\ 8 & -9 & 0 \\ 1 & 2 & 16 \end{pmatrix}$ $= \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$ ▪ Matrix A has 3 rows and 3 columns ▪ Size of Matrix A=3x3 or we can use notation A_{33}	$P = \begin{pmatrix} 10 & -20 & 6 \\ -66 & 30 & 3 \end{pmatrix}$ $= \begin{pmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \end{pmatrix}$ ▪ Matrix P has 2 rows and 3 columns ▪ Size of Matrix P=2x3 or we can use notation P_{23}	$B = \begin{pmatrix} -5 \\ 9 \\ 2 \end{pmatrix} = \begin{pmatrix} b_{11} \\ b_{21} \\ b_{31} \end{pmatrix}$ ▪ Matrix B has 3 rows and 1 columns ▪ Size of Matrix B=3x1 or we can use notation B_{31}

Example 5.1 a

State size of these matrices and identify the elements.

a) $C = \begin{pmatrix} -3 \\ 4 \\ 6 \\ -9 \end{pmatrix}$

Matrix C has 4 rows and 1 columns
Size of Matrix C=4x1

Elements
$c_{11} = -3, \ c_{21} = 4, \ c_{31} = 6, \ c_{41} = -9$

b) $B = \begin{pmatrix} -1 & 3 \\ 2 & 9 \end{pmatrix}$

Matrix B has 2 rows and 2 columns
Size of Matrix B=2x2 or
we can use notation B_{22}

Elements
$b_{11} = -1, b_{12} = 3, \; b_{21} = 2, \quad b_{22} = 9$

c) $A = \begin{pmatrix} -1 & 2 & 4 & 5 \\ 5 & 10 & 3 & 7 \\ 9 & 11 & -10 & 22 \\ 8 & 6 & 3 & 15 \end{pmatrix}$

Matrix A has 4 rows and 4 columns
Size of Matrix A=4x4 or
we can use notation A_{44}

Elements
$a_{11} = -1 \quad a_{12} = 2 \quad a_{13} = 4 \quad a_{14} = 5$
$a_{21} = 5 \quad a_{22} = 10 \quad a_{23} = 3 \quad a_{24} = 7$
$a_{31} = 9 \quad a_{32} = 11 \quad a_{33} = -10 \quad a_{34} = 22$
$a_{41} = 8 \quad a_{42} = 6 \quad a_{43} = 3 \quad a_{44} = 15$

Types of matrices

Row matrix	$(3 \quad 5 \quad -2)$
Column matrix	$\begin{pmatrix} 1 \\ 7 \\ -1 \end{pmatrix}$
Identity matrix	$I = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ $I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ Identity matrix is special because when you multiply a matrix with it or when multiply it with a matrix, the matrix does not change. AI=IA=A BI=IB=B

Null matrix	$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$
Diagonal matrix	$\begin{pmatrix} 0 & 0 & -2 \\ 0 & 8 & 0 \\ 1 & 0 & 0 \end{pmatrix}$
Square matrix	$\begin{pmatrix} 1 & 7 & -2 \\ 0 & 8 & 1 \\ 1 & 1 & 3 \end{pmatrix}$ 3 x 3 $\begin{pmatrix} 1 & -9 \\ 7 & 5 \end{pmatrix}$ 2 x 2
Upper triangular matrix	$\begin{pmatrix} 1 & 7 & -2 \\ 0 & 8 & 1 \\ 0 & 0 & 3 \end{pmatrix}$
Lower triangular matrix	$\begin{pmatrix} 1 & 0 & 0 \\ 5 & -8 & 0 \\ 4 & 3 & 3 \end{pmatrix}$

Transposition matrices

- Transposition is process of interchange the rows of a matrix with its column
- The symbol of transpose of a matrix A is A^T

If $A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix}$ then $A^T = \begin{pmatrix} a_{11} & a_{21} & a_{31} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{pmatrix}$

- The transpose of a transpose is the original matrix.

$$(A^T)^T = A$$

- Some important properties relating to transpose are:

$$(AB)^T = B^T A^T$$

$$(A+B)^T = A^T + B^T$$

Example 5.1 b

i. If $B = \begin{pmatrix} -2 & 4 & 7 \\ 8 & -9 & 0 \\ 1 & 2 & 16 \end{pmatrix}$ ii. If $C = \begin{pmatrix} -2 & 3 & 11 \\ 1 & 9 & 6 \end{pmatrix}$ iii. If $A = \begin{pmatrix} -2 \\ 3 \\ 1 \\ 6 \end{pmatrix}$

then $B^T = \begin{pmatrix} -2 & 8 & 1 \\ 4 & -9 & 2 \\ 7 & 0 & 16 \end{pmatrix}$ then $C^T = \begin{pmatrix} -2 & 1 \\ 3 & 9 \\ 11 & 6 \end{pmatrix}$ then $A^T = \begin{pmatrix} -2 & 3 & 1 & 6 \end{pmatrix}$

5.2 Basic operation on matrices

Addition & subtraction

- Matrix addition and subtraction can only be performed on matrices that have the **same size**.
- The result of a matrix addition/subtraction is a new matrix with the same size.

Example 5.2 a

Given $A = \begin{bmatrix} 3 & -1 & 4 & 9 \\ 1 & 3 & 3 & -2 \\ -5 & 1 & 4 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} -2 & 5 & -8 & 0 \\ -4 & 1 & -3 & 0 \\ -1 & -3 & 7 & -1 \end{bmatrix}$

i.

$$A + B = \begin{bmatrix} 3 & -1 & 4 & 9 \\ 1 & 3 & 3 & -2 \\ -5 & 1 & 4 & 3 \end{bmatrix} + \begin{bmatrix} -2 & 5 & -8 & 0 \\ -4 & 1 & -3 & 0 \\ -1 & -3 & 7 & -1 \end{bmatrix}$$

$$A + B = \begin{bmatrix} 1 & 4 & -4 & 9 \\ -3 & 4 & 0 & -2 \\ -6 & -2 & 11 & 2 \end{bmatrix}$$

ii.

$$A - B = \begin{bmatrix} 3 & -1 & 4 & 9 \\ 1 & 3 & 3 & -2 \\ -5 & 1 & 4 & 3 \end{bmatrix} - \begin{bmatrix} -2 & 5 & -8 & 0 \\ -4 & 1 & -3 & 0 \\ -1 & -3 & 7 & -1 \end{bmatrix}$$

$$A - B = \begin{bmatrix} 5 & -6 & 12 & 9 \\ 5 & 2 & 6 & -2 \\ -4 & 4 & -3 & 4 \end{bmatrix}$$

Example 5.2 b

Given that $A = \begin{bmatrix} 2 & 4 \\ 5 & -6 \end{bmatrix}$ and B= $\begin{bmatrix} 4 & 8 \\ -7 & 3 \end{bmatrix}$. Show that

$$(A + B)^T = A^T + B^T$$

$$(\begin{bmatrix} 2 & 4 \\ 5 & -6 \end{bmatrix} + \begin{bmatrix} 4 & 8 \\ -7 & 3 \end{bmatrix})^T = \begin{bmatrix} 2 & 4 \\ 5 & -6 \end{bmatrix}^T + \begin{bmatrix} 4 & 8 \\ -7 & 3 \end{bmatrix}^T$$

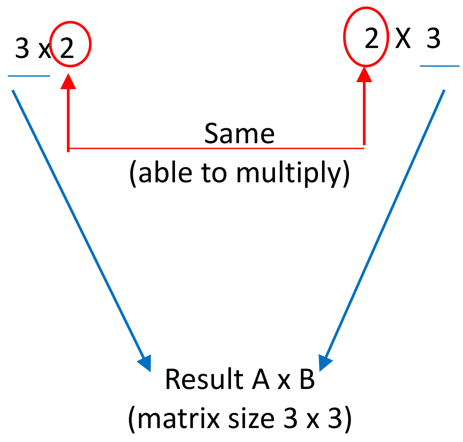
$$\begin{bmatrix} 6 & 12 \\ -2 & -3 \end{bmatrix}^T = \begin{bmatrix} 2 & 5 \\ 4 & -6 \end{bmatrix} + \begin{bmatrix} 4 & -7 \\ 8 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 6 & -2 \\ 12 & -3 \end{bmatrix} = \begin{bmatrix} 6 & -2 \\ 12 & -3 \end{bmatrix}$$

Multiplication

- In order to be able to multiply two matrices AB, we have to ensure that the number of column in matrix A is the same as the number of row in matrix B

If $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}$ and $B = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \end{bmatrix}$



Example 5.2 c

Find the multiplication of A and B if $A = \begin{bmatrix} 4 & 2 \\ 0 & 1 \end{bmatrix}$ and B= $\begin{bmatrix} -3 & 7 \\ 1 & -1 \end{bmatrix}$.

$$AB = \begin{bmatrix} 4 & 2 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} -3 & 7 \\ 5 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} (4 \times -3) + (2 \times 5) & (4 \times 7) + (2 \times -1) \\ (0 \times -3) + (1 \times 5) & (0 \times 7) + (1 \times -1) \end{bmatrix}$$

$$= \begin{bmatrix} -12 + 10 & 28 - 2 \\ 0 + 5 & 0 - 1 \end{bmatrix}$$

$$= \begin{bmatrix} -2 & 26 \\ 5 & -1 \end{bmatrix}$$

Example 5.2 d

If matrix $M = \begin{bmatrix} 1 & 2 & 3 \\ 9 & 3 & 5 \\ 1 & 5 & 12 \end{bmatrix}$, N= $\begin{bmatrix} 2 \\ -1 \\ 5 \end{bmatrix}$ and P= $\begin{bmatrix} -2 & 1 \\ 4 & 1 \end{bmatrix}$.

i. $MN = \begin{bmatrix} 1 & 2 & 3 \\ 9 & 3 & 5 \\ 1 & 5 & 12 \end{bmatrix} \begin{bmatrix} 2 \\ -1 \\ 5 \end{bmatrix}$

$$= \begin{bmatrix} (1 \times 2) + (2 \times -1) + (3 \times 5) \\ (9 \times 2) + (3 \times -1) + (5 \times 5) \\ (1 \times 2) + (5 \times -1) + (12 \times 5) \end{bmatrix}$$

$$= \begin{bmatrix} 2 - 2 + 15 \\ 18 - 3 + 25 \\ 2 - 5 + 60 \end{bmatrix}$$

$$= \begin{bmatrix} 15 \\ 40 \\ 57 \end{bmatrix}$$

ii. $PN = \begin{bmatrix} -2 & 1 \\ 4 & 1 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \\ 5 \end{bmatrix}$

$PN =$ no solution

iii. $NM = \begin{bmatrix} 2 \\ 1 \\ 5 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 1 & 3 & 5 \\ 1 & 5 & 12 \end{bmatrix}$

$NM =$ no solution

Example 5.2 e

Given matrices A , B, C, and D. Calculate matrices AB and CD

i. $$AB = \begin{bmatrix} 1 & 3 & 1 \\ 2 & -1 & 0 \\ 0 & 9 & 2 \end{bmatrix} \begin{bmatrix} -1 & -2 & 5 \\ 4 & 0 & 1 \\ 1 & 3 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} (1 \times -1) + (3 \times 4) + (1 \times 1) & (1 \times -2) + (3 \times 0) + (1 \times 3) & (1 \times 5) + (3 \times 1) + (1 \times 2) \\ (2 \times -1) + (-1 \times 4) + (0 \times 1) & (2 \times -2) + (-1 \times 0) + (0 \times 3) & (2 \times 5) + (-1 \times 1) + (0 \times 2) \\ (0 \times -1) + (9 \times 4) + (2 \times 1) & (0 \times -2) + (9 \times 0) + (2 \times 3) & (0 \times 5) + (9 \times 1) + (2 \times 2) \end{bmatrix}$$

$$= \begin{bmatrix} -1 + 12 + 1 & -2 + 0 + 3 & 5 + 3 + 2 \\ -2 - 4 + 0 & -4 + 0 + 0 & 10 - 1 + 0 \\ 0 + 36 + 2 & 0 + 0 + 6 & 0 + 9 + 4 \end{bmatrix}$$

$$= \begin{bmatrix} 12 & 1 & 10 \\ -6 & -4 & 9 \\ 38 & 6 & 13 \end{bmatrix}$$

ii. $$CD = \begin{bmatrix} 3 & -1 \\ 4 & 2 \end{bmatrix} \begin{bmatrix} -2 & 7 \\ 1 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} (3 \times -2) + (-1 \times 1) & (3 \times 7) + (-1 \times 5) \\ (4 \times -2) + (2 \times 1) & (4 \times 7) + (2 \times 5) \end{bmatrix}$$

$$= \begin{bmatrix} -6 - 1 & 21 - 5 \\ -8 + 2 & 28 + 10 \end{bmatrix}$$

$$= \begin{bmatrix} -7 & 16 \\ -6 & 38 \end{bmatrix}$$

Example 5.2 f

Given $Q = \begin{bmatrix} 2 & -3 & 1 \\ 4 & 0 & 6 \\ 5 & 7 & 8 \end{bmatrix}$. Find 3Q

$$3 \begin{bmatrix} 2 & -3 & 1 \\ 4 & 0 & 6 \\ 5 & 7 & 8 \end{bmatrix} = \begin{bmatrix} 6 & -9 & 3 \\ 12 & 0 & 18 \\ 15 & 21 & 24 \end{bmatrix}$$

5.3 Inverse of 2x2 matrix

- Matrices cannot be divided. However, by defined another matrix called the inverse matrix, it is possible to work with an operation which plays a similar role to division.
- The inverse of a 2x2 matrix A, is another 2x2 matrix denoted by A^{-1} with property that :

$$AA^{-1} = A^{-1}A = I$$

Note that A^{-1} does not mean $\frac{1}{A}$

Given matrix 2x2 $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$, then $A^{-1} = \frac{1}{\text{determinant}}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$

$$A^{-1} = \frac{1}{ad - bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

If the determinant is zero, then it will not have an inverse

Example 5.3 a

i. Find the inverse of matrix $A = \begin{pmatrix} 12 & 1 \\ 4 & 2 \end{pmatrix}$

$$A^{-1} = \frac{1}{ad - bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

$$A^{-1} = \frac{1}{(12)(2) - (1)(4)}\begin{pmatrix} 2 & -1 \\ -4 & 12 \end{pmatrix}$$

$$A^{-1} = \frac{1}{24 - 4}\begin{pmatrix} 2 & -1 \\ -4 & 12 \end{pmatrix}$$

$$A^{-1} = \frac{1}{20}\begin{pmatrix} 2 & -1 \\ -4 & 12 \end{pmatrix}$$

$$A^{-1} = \begin{pmatrix} \frac{2}{20} & \frac{-1}{20} \\ \frac{-4}{20} & \frac{12}{20} \end{pmatrix}$$

$$A^{-1} = \begin{pmatrix} \frac{1}{10} & \frac{-1}{20} \\ \frac{-1}{5} & \frac{3}{5} \end{pmatrix}$$

ii. Find the inverse of matrix $A = \begin{pmatrix} 2 & 9 \\ -3 & 1 \end{pmatrix}$

$$A^{-1} = \frac{1}{ad - bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

$$A^{-1} = \frac{1}{(2)(1) - (9)(-3)}\begin{pmatrix} 1 & -9 \\ 3 & 2 \end{pmatrix}$$

$$A^{-1} = \frac{1}{2 - (-27)}\begin{pmatrix} 1 & -9 \\ 3 & 2 \end{pmatrix}$$

$$A^{-1} = \frac{1}{29}\begin{pmatrix} 1 & -9 \\ 3 & 2 \end{pmatrix}$$

$$A^{-1} = \begin{pmatrix} \frac{1}{29} & \frac{-9}{29} \\ \frac{3}{29} & \frac{2}{29} \end{pmatrix}$$

5.4 Solve simultaneous equation using matrices

To solve simultaneous linear equations

$$ax + by = h$$
$$cx + dy = k$$

- Write the equations in matrix form,

  $$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} h \\ k \end{pmatrix}$$

  Where

  $$A^{-1} = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

- Determine the inverse of A, A^{-1}

- Solve by using formula

  $$\begin{pmatrix} x \\ y \end{pmatrix} = A^{-1}\begin{pmatrix} h \\ k \end{pmatrix}$$

Example 5.4 a

Solve simultaneous linear equations of $3x + 4y = 10$ and $5x + 2y = 14$ by using matrices.

1. Write the equations in matrix form,

$$\begin{pmatrix} 3 & 4 \\ 5 & 2 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 10 \\ 14 \end{pmatrix}$$

Where

$$A = \begin{pmatrix} 3 & 4 \\ 5 & 2 \end{pmatrix}$$

2. Determine the inverse of A, A^{-1}

$$A^{-1} = \frac{1}{ad - bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

$$A^{-1} = \frac{1}{(3)(2) - (4)(5)}\begin{pmatrix} 2 & -4 \\ -5 & 3 \end{pmatrix}$$

$$A^{-1} = \frac{1}{6 - 20}\begin{pmatrix} 2 & -4 \\ -5 & 3 \end{pmatrix}$$

$$A^{-1} = \frac{1}{-14}\begin{pmatrix} 2 & -4 \\ -5 & 3 \end{pmatrix}$$

$$A^{-1} = \begin{pmatrix} \frac{2}{-14} & \frac{-4}{-14} \\ \frac{-5}{-14} & \frac{3}{-14} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{-7} & \frac{2}{7} \\ \frac{5}{14} & \frac{3}{-14} \end{pmatrix}$$

3. Solve by using formula

$$\begin{pmatrix} x \\ y \end{pmatrix} = A^{-1}\begin{pmatrix} h \\ k \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} \frac{1}{-7} & \frac{2}{7} \\ \frac{5}{14} & \frac{3}{-14} \end{pmatrix}\begin{pmatrix} 10 \\ 14 \end{pmatrix}$$

$$= \begin{bmatrix} \left(\frac{1}{-7} \times 10\right) + \left(\frac{2}{7} \times 14\right) \\ \left(\frac{5}{14} \times 10\right) + \left(\frac{3}{-14} \times 14\right) \end{bmatrix}$$

$$= \begin{bmatrix} -\frac{10}{7} + 4 \\ \frac{50}{14} - 3 \end{bmatrix}$$

$$= \begin{bmatrix} 18/7 \\ 4/7 \end{bmatrix}$$

$$x = \frac{18}{7}, y = \frac{4}{7}$$

Example 5.4 b

Solve the following simultaneous linear equations by using matrices.

$$y - 6x - 19 = 0$$

$$2y + 3x + 22 = 0$$

1. Write the equations in matrix form,

$$\begin{pmatrix} 1 & -6 \\ 2 & 3 \end{pmatrix}\begin{pmatrix} y \\ x \end{pmatrix} = \begin{pmatrix} 19 \\ -22 \end{pmatrix}$$

Where

$$A = \begin{pmatrix} 1 & -6 \\ 2 & 3 \end{pmatrix}$$

2. Determine the inverse of A, A^{-1}

$$A^{-1} = \frac{1}{ad - bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

$$A^{-1} = \frac{1}{(1)(3) - (-6)(2)}\begin{pmatrix} 3 & 6 \\ -2 & 1 \end{pmatrix}$$

$$A^{-1} = \frac{1}{3 - (-12)}\begin{pmatrix} 3 & 6 \\ -2 & 1 \end{pmatrix}$$

$$A^{-1} = \frac{1}{15}\begin{pmatrix} 3 & 6 \\ -2 & 1 \end{pmatrix}$$

$$A^{-1} = \begin{pmatrix} \frac{3}{15} & \frac{6}{15} \\ \frac{-2}{15} & \frac{1}{15} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{5} & \frac{2}{5} \\ \frac{-2}{15} & \frac{1}{15} \end{pmatrix}$$

3. Solve by using formula

$$\begin{pmatrix} y \\ x \end{pmatrix} = A^{-1}\begin{pmatrix} h \\ k \end{pmatrix}$$

$$\begin{pmatrix} y \\ x \end{pmatrix} = \begin{pmatrix} \frac{1}{5} & \frac{2}{5} \\ \frac{-2}{15} & \frac{1}{15} \end{pmatrix}\begin{pmatrix} 19 \\ -22 \end{pmatrix}$$

$$= \begin{bmatrix} \left(\frac{1}{5} \times 19\right) + \left(\frac{2}{5} \times -22\right) \\ \left(\frac{-2}{15} \times 19\right) + \left(\frac{1}{15} \times -22\right) \end{bmatrix}$$

$$= \begin{bmatrix} \frac{19}{5} - \frac{44}{5} \\ \frac{-38}{15} - \frac{22}{15} \end{bmatrix}$$

$$= \begin{bmatrix} -5 \\ -4 \end{bmatrix}$$

$$y = -5, x = -4$$


PRACTICE
MAKES
PERFECT


TUTORIAL

TUTORIAL : QUESTION

TOPIC 1 : TRIGONOMETRY

1. In Diagram 1a below, PQR is a right-angled triangle. Find the value of each of the following trigonometric functions.

   a. tan ɵ

   b. sin ɵ

   c. cos ɵ

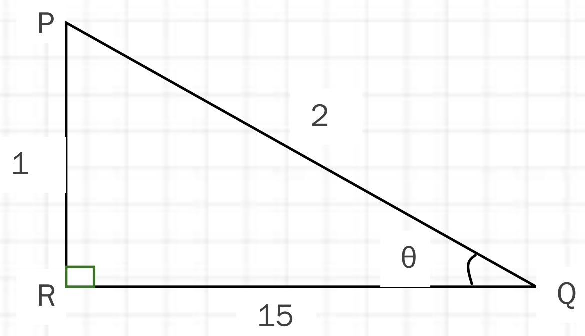


Diagram 1a

2. In Diagram 1b below, ABC is a right-angled triangle and ADC is a straight line. Calculate the length of BC and round off the answer into two decimal places.

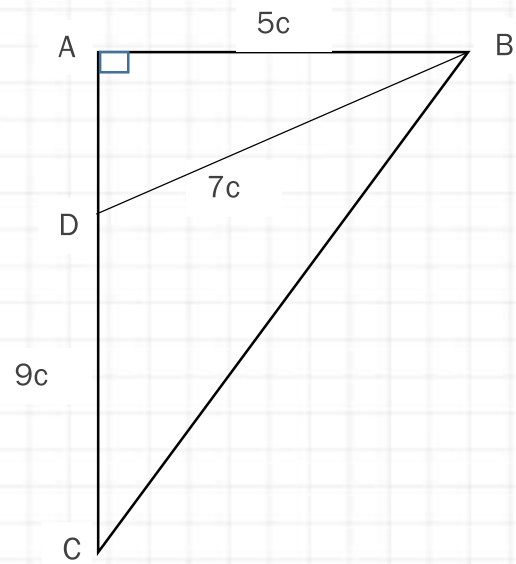


Diagram 1b

3. Refer to Diagram 1c and calculate the angle of PMN.

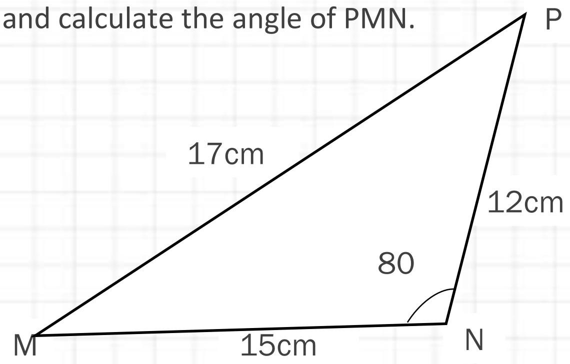


Diagram 1c

4. Diagram 1d below is a right-angled triangle. Determine:

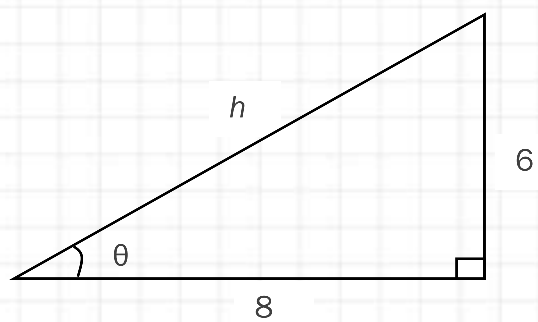


Diagram 1d

a. the value of *h*

b. cos ϴ

c. tan ϴ

d. cosec ϴ

5. Given a triangle EFG below (Diagram 1e). Using an appropriate method, calculate:

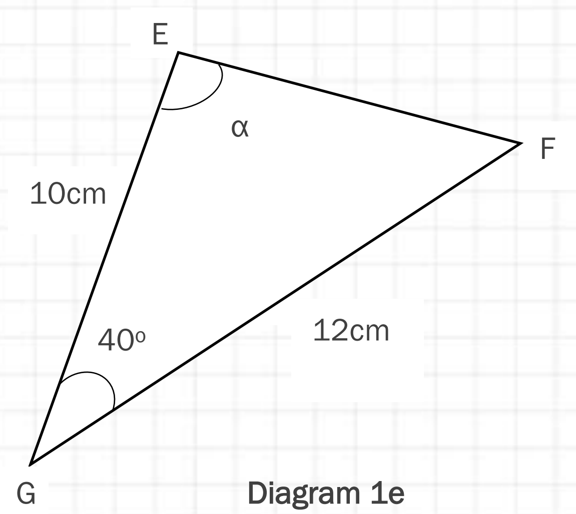


Diagram 1e

a. the length of EF

b. angle of α

c. the area of triangle EFG

6. Determine angle α, ϴ and β in Diagram 1f below:

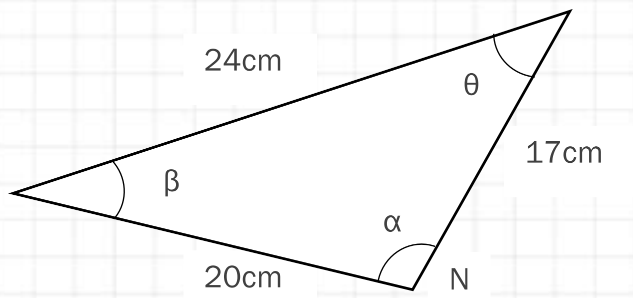


Diagram 1f

7. Based on Diagram 1g below, calculate:

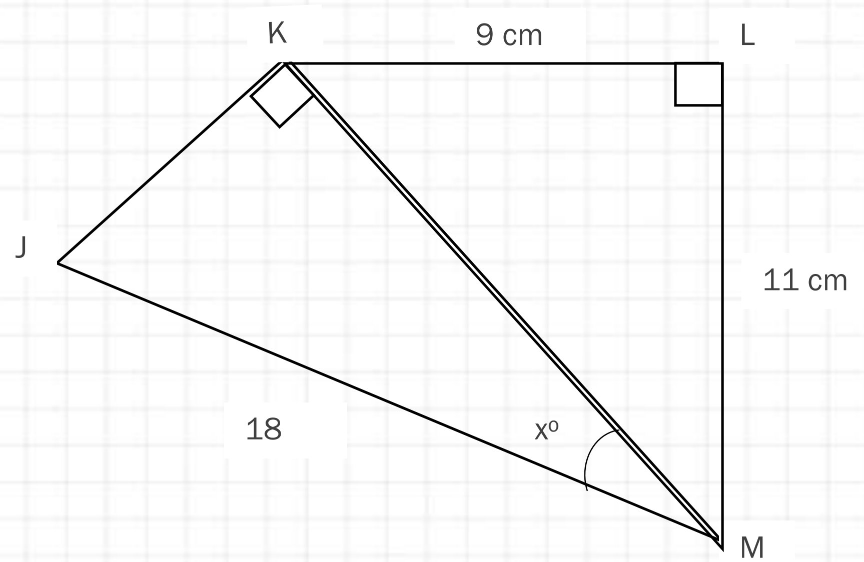


Diagram 1g

a. length of KM

b. angle xº

8. Diagram 1h below shows a triangle of PRS. Find the length of PR

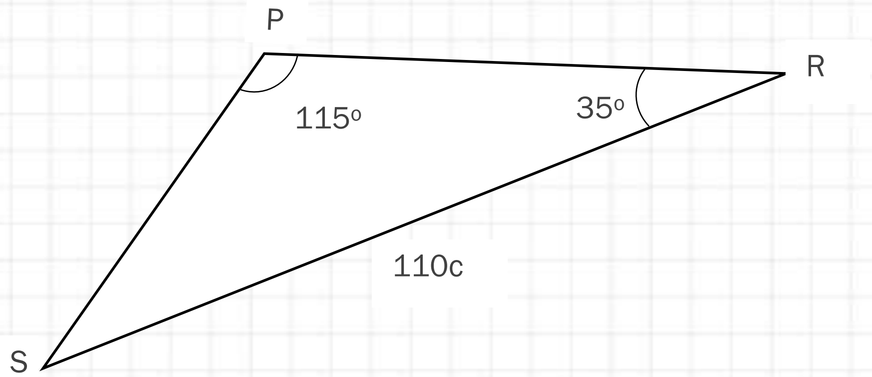


Diagram 1h

9. Express each of the following trigonometric functions in term the of trigonometric ratio of acute angle:

   a. tan 225°

   b. sin(-172°)

   c. cos(-240°)

10. Evaluate and sketch the diagram to show the angle of the quadrant lies on the following trigonometric functions:

   a. tan 145°

   b. cot 220°

   c. $sin\frac{4}{3}\pi$

TOPIC 2 : CIRCULAR MEASURE

1. Convert each of the following angle in degree to radian and in radian to degree:

   a. 78°

   b. 153 °

   c. 3.43 rad

   d. 0.62π rad

   e. $\frac{3}{5}\pi\ rad$

2. Diagram 2a below shows a circle with center O. Determine:

    a. the value of ɵ in degree unit

    b. circumference of the circle

    c. area of circle

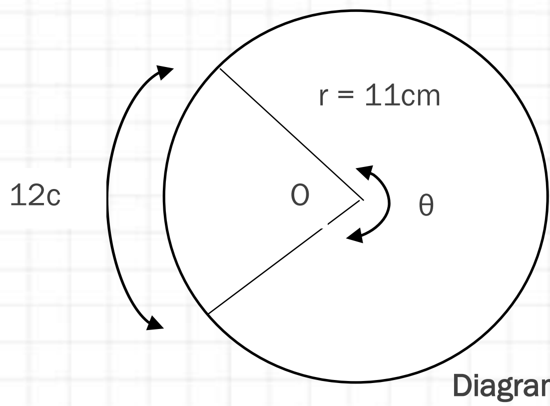


Diagram 2a

3. Diagram 2b below shows a sector of KOL. Find:

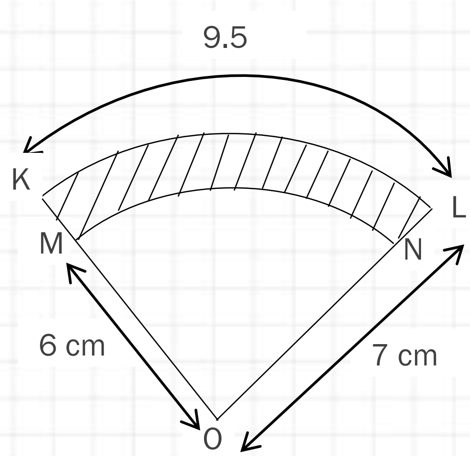


Diagram 2b

a. angle of KOL
b. arc length of MN
c. length of NL
d. perimeter of KLNM
e. area of sector MON

4. A sector have angle 3.578rad. Find ;

   a. the radius for the sector if its length of arc 34 m

   b. the radius for the sector if its area $161.46\ m^2$

5. If the diameter of the circle is 8.5 cm. Find the area of the shaded sector as shown in Diagram 2c.

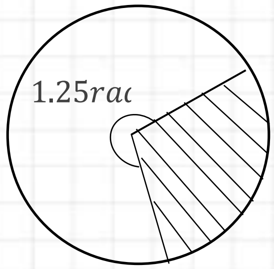


Diagram 2c

6. Diagram 2d shows a circle with center O with radius 10 cm. If the area of shaded region is 255 cm^2, find the value of x.

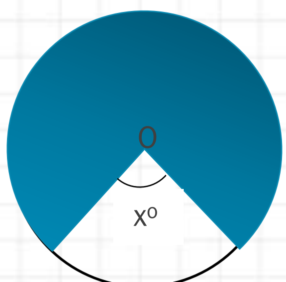


Diagram 2d

7. In Diagram 2e, ABC is a sector with center A. The area of this sector is 87.54 cm^2. Calculate:

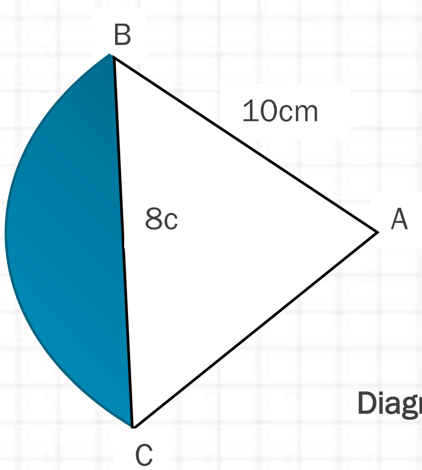


Diagram 2e

a. angle BAC in radian

b. the length of arc BC

c. the area of shaded region

8. Diagram 2f below shows a sector DEF with a radius of 12cm and an angle of 1.78 radian. Calculate the area of triangle DEF.

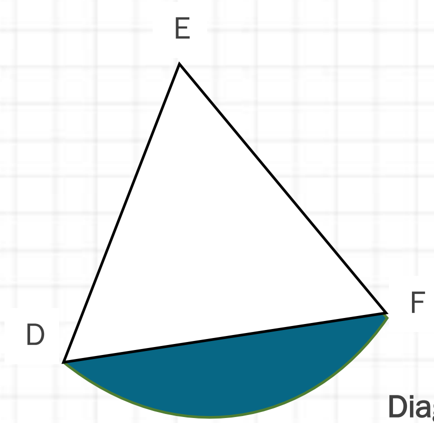


Diagram 2f

9. Find the area of **UNSHADED** sector in Diagram 2g below if the diameter of the circle is 7 cm.

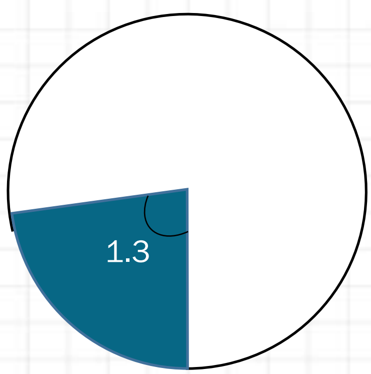


Diagram 2g

10. The arc length of a sector is 55 cm with radius is 11 cm. Find:

a. angle in degree and area of the sector

b. area and circumference of the circle

TOPIC 3 : VECTOR

1. Given $\tilde{a}$ = 2i + 3j and $\tilde{b}$ = 4i – 5j. Find:

   a. $\tilde{a} - 2\tilde{b}$

   b. $\tilde{a}.\tilde{b}$

   c. magnitude for vector $5\tilde{a}$

2. If $\overrightarrow{OA}$ = 4i + 6j and and $\overrightarrow{OB}$ = – 4i + 6j, calculate angle ɵ between $\overrightarrow{OA}$ and $\overrightarrow{OB}$.

3. Given that $\tilde{p} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$, $\tilde{q} = \begin{pmatrix} 1 \\ -5 \end{pmatrix}$ and $\tilde{r} = \begin{pmatrix} 4 \\ 7 \end{pmatrix}$. Find:

   a. $2\tilde{p}$

   b. $\tilde{p} - \tilde{q} + \tilde{r}$

   c. $2\tilde{q} + \tilde{r}$

   d. magnitude of vector $3\tilde{r}$

4. If $\tilde{m} = 3i - j$ and $\tilde{n} = 2i + 4j$, find:

   a. $\tilde{m} \cdot \tilde{n}$

   b. $|\tilde{m} - 2\tilde{n}|$

   c. $\tilde{m} \cdot (\tilde{m} + \tilde{n})$

   d. $3|\tilde{m}| + 2|\tilde{n}|$

5. Calculate the magnitude of the following vectors:

   a. $\overrightarrow{OA} = 5i - 6j$

   b. $\tilde{b} = -i + 7j$

   c. $\overrightarrow{OB} = \begin{pmatrix} 8 \\ 3 \end{pmatrix}$

6. Given vector $\overrightarrow{OP} = (-5\,,7)$ and $\overrightarrow{OR} = (4\ \,,6)$. Determine angle between $\overrightarrow{OP}$ and $\overrightarrow{OR}$.

7. Sketch a directed line segment to represent each of the following vectors.

a. $\tilde{w} = -6i + 7j$

b. $\tilde{s} = (5\,,-3)$

c. $\tilde{z} = \begin{pmatrix} 0 \\ -4 \end{pmatrix}$

8. Diagram 3a below shows Cartesian plane, state the named vector in term of x*i* +y*j* for vector

a. $\overrightarrow{OA}$

b. $\overrightarrow{OB}$

c. $\overrightarrow{OC}$

d. $\overrightarrow{OD}$

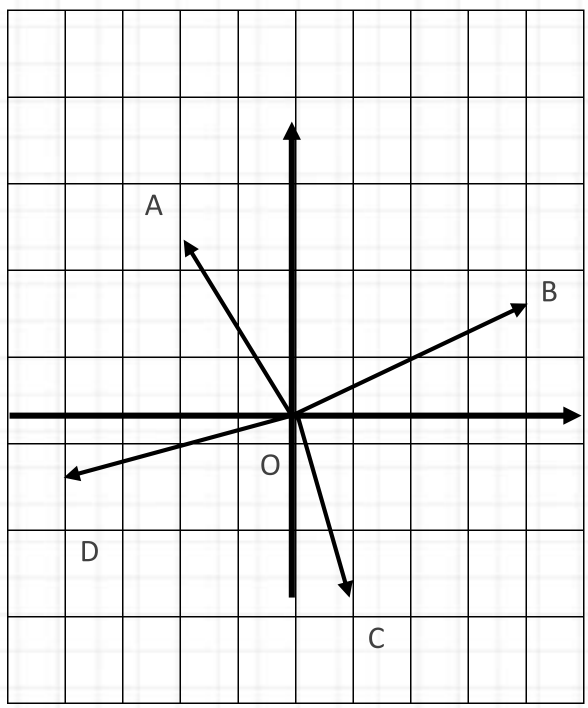


Diagram 3a

9. Given vector $\overrightarrow{OS} = (5\,,2)$ and $\overrightarrow{OS} \cdot \overrightarrow{OR} = 9$. Determine angle between $\overrightarrow{OS}$ and $\overrightarrow{OR}$.

10. If $\tilde{r} = 3i + 2j$ and $= 2i$, find:

a. $-\tilde{r} \cdot 4\tilde{s}$

b. $\tilde{s} - \frac{1}{2}\tilde{r}$

c. $|\tilde{r} + 5\tilde{s}|$

TOPIC 4 : INEQUALITY

1. Solve the following inequalities:
   a. $4x > 36$
   b. $-10 < 2x$
   c. $5x > 12 + 4x$
   d. $9x - 30 < -3$
   e. $7m < 21$
   f. $5y > 15 - 10y$
   g. $r + 5 > -5$

2. Show the interval notation, sketch the number liner and state the value of x for each of the following.
   a. $\{x : x < 6\}$
   b. $\{x : 2 < x \leq 7\}$

3. If x is an integer, find the values of x that satisfy the following simultaneous inequalities and draw a number line.
   a. $15 - 10x > 5$
   b. $4x + 10 \geq 26$
   c. $x - 1 \leq 3$ and $5 - 3x < 2$
   d. $1 \leq 4x - 3 \leq 17$

4. Calculate each of the following inequality and express your answer using Interval Notation.
   a. $4x - 1 > 11$
   b. $2x + 2 \geq -8 + x$
   c. $-2(x - 1) < 6$
   d. $2(6 - y) > 10$

5. Show the value of s in the form of number line if $5 + 4s \leq 60 - 7s$
   a. $5 + 4s \leq 60 - 7s$
   b. $9 + 6s \leq 53 - 5s$

6. Solve the following inequality and express your answer using Interval Notation.

$$\frac{8-6x}{6} \leq 8$$

TOPIC 5 : MATRICES

1. State the order of the matrix

   a. $\begin{pmatrix} 1 & 2 \\ 4 & 3 \\ 5 & 6 \end{pmatrix}$

   b. $\begin{pmatrix} 1 & 0 & 2 \end{pmatrix}$

   c. $\begin{pmatrix} 1 & 4 & 9 \\ 2 & 8 & 6 \end{pmatrix}$

   d. $\begin{pmatrix} 2 & -2 & 4 \end{pmatrix}$

   e. $\begin{pmatrix} 9 \\ 8 \\ 7 \end{pmatrix}$

   f. $\begin{pmatrix} 3 & 1 \\ 0 & -8 \end{pmatrix}$

   g. $\begin{pmatrix} 0 \\ 9 \end{pmatrix}$

   h. $\begin{pmatrix} 0 & 10 \\ -1 & 7 \\ 7 & 5 \end{pmatrix}$

2. List FOUR types of matrices

3. Given that $P=\begin{pmatrix}2 & 4\\ -3 & 5\end{pmatrix}$ and $Q=\begin{pmatrix}6 & 4\\ 0 & -4\end{pmatrix}$. Determine:

   a. $P+Q$

   b. P^T-Q

   c. $3Q$

   d. PQ

4. Given that $M=\begin{pmatrix}5 & 4\\ -2 & 1\end{pmatrix}$ and $N=\begin{pmatrix}0 & 7\\ -2 & 6\end{pmatrix}$, find:

   a. $M+N$

   b. $2M-N$

5. Given $S=\begin{pmatrix}x+2 & 7\\ 0 & 4\end{pmatrix}$ and $T=\begin{pmatrix}4 & y-1\\ 11 & 2\end{pmatrix}$. If $3S+T=\begin{pmatrix}25 & 24\\ 11 & 14\end{pmatrix}$, find:

   a. the value of x and y

   b. inverse matrix T

6. Solve $\begin{pmatrix}1 & 0\\ 2 & -1\\ 3 & 2\end{pmatrix}\begin{pmatrix}1\\ 2\end{pmatrix}$

7. Solve the simultaneous linear equations below by using matrix method:

   a. $2x+2y=10$
      $4x-4y=8$

   b. $7x+7y=35$
      $5x-3y=-7$

   c. $5x+y=17$
      $-2x+3y=17$

8. Find the inverse matrix for the following matrix.

$$\begin{pmatrix}8 & -1\\ 0 & 3\end{pmatrix}$$

TUTORIAL : SOLUTION

TOPIC 1 : TRIGONOMETRY

Solution Q1:

a. tan ɵ = $\frac{12}{15}$

b. sin ɵ = $\frac{12}{20}$ @ $\frac{3}{5}$

c. cos ɵ = $\frac{15}{20}$ @ $\frac{3}{4}$

Solution Q2:

$AB^2 + AD^2 = DB^2$ (using Pythagoras Theorem to find AD)

$5^2 + AD^2 = 7^2$

$25 + AD^2 = 49$

$AD^2 = 49 - 25$

$AD^2 = 49 - 25$

$AD^2 = 24$

$\mathrm{AD} = \sqrt{24}$

$\mathrm{AD} = 4.90\ cm$

therefore, AC = 4.90 + 9 = 13.90 cm

$BC^2 = AB^2 + AC^2$ (using Pythagoras Theorem to find BC)

$BC^2 = AB^2 + AC^2$

$BC^2 = 5^2 + 13.90^2$

$BC^2 = 5^2 + 13.90^2$

$BC^2 = 218.21$

$BC = \sqrt{218.21}$

Solution Q3:

$\frac{12}{sin\measuredangle PMN} = \frac{17}{sin80^o}$ (using Sine Rule)

$17\ sin\measuredangle PMN = 12sin80^o$

$sin\measuredangle PMN = \frac{12sin80^o}{17}$

$sin\measuredangle PMN = 0.6952$

$\measuredangle PMN = sin^{-1}0.6952$

$\boldsymbol{\measuredangle PMN = 44.04^o}$

Solution Q4:

a. the value of *h*

$h^2 = 6^2 + 8^2$ (using Pythagoras Theorem)

$h^2 = 100$

$h = \sqrt{100}$

$\boldsymbol{h = 10}$

b. cos Ө = $\frac{8}{10}$ @ $\frac{4}{5}$

c. tan Ө= $\frac{6}{8}$ **@** $\frac{3}{4}$

d. cosec Ө = $\frac{10}{6}$ **@** $\frac{5}{3}$

Solution Q5:

a. the length of EF

$EF^2 = 10^2 + 12^2 - 2(10)(12)\cos 40^o$ (using Cosine Rule)

$EF^2 = 100 + 144 - 240\cos 40^o$

$EF^2 = 60.15$

$EF = \sqrt{60.15}$

$\boldsymbol{EF = 7.76cm}$

b. angle of α

$\frac{12}{sin\alpha} = \frac{7.76}{sin40^o}$ (using Sine Rule)

$7.76\ sin\alpha = 12sin40^o$

$sin\alpha = \frac{12sin40^o}{7.76}$

$sin\alpha = 0.9940$

$\alpha = sin^{-1}0.9940$

$\boldsymbol{\alpha = 83.72^o}$

c. the area of triangle EFG

$= \frac{1}{2} \times 10 \times 12\ sin40°$

<u>= 38.</u>57cm^2

Solution Q6:

$24^2 = 20^2 + 17^2 - 2(20)(17)\cos\alpha$

(using Cosine Rule)

$576 = 400 + 289 - 680\cos\alpha$

$576 = 689 - 680\cos\alpha$

$\cos\alpha = \frac{576 - 689}{-680}$

$\cos\alpha = 0.1662$

$\alpha = cos^{-1}0.1662$

$\boldsymbol{\alpha = 80.43°}$

$\frac{17}{sin\beta} = \frac{24}{sin80.43°}$ (using Sine Rule)

$24sin\beta = 17sin80.43°$

$sin\beta = \frac{17sin80.43°}{24}$

$sin\beta = 0.6985$

$\beta = sin^{-1}0.6985$

$\boldsymbol{\beta = 44.31°}$

$\theta = 180° - 44.31° - 80.43°$

$\boldsymbol{\theta = 55.26°}$

Solution Q7:

a. length of KM

$BC^2 = AB^2 + AC^2$

(using Pythagoras Theorem to find BC)

$KM^2 = KL^2 + LM^2$

$KM^2 = 9^2 + 11^2$

$KM^2 = 202$

$KM = \sqrt{202}$

$\boldsymbol{KM = 14.21cm}$

b. angle x°

$cosx° = \frac{14.21}{18}$

$cosx° = 0.7894$

$x° = cos^{-1}0.7894$

$\boldsymbol{x° = 37.87°}$

Solution Q8:

$\measuredangle PSR = 180° - 115° - 35$

$\measuredangle PSR = 30°$

Therefore,

$\frac{PR}{sin30°} = \frac{110}{sin115°}$ (using Sine Rule)

$PRsin115° = 110sin30°$

$PR = \frac{110sin30°}{sin115°}$

$\boldsymbol{PR}$ **=60.69cm**

Solution Q9:

a. tan 225°

Acute angle = 225° – 180°

(3^{rd} quadrant, anti-clockwise)

= 45°

at 3^{rd} quadrant tan is positive,

therefore

tan 225° = tan 45°

b. sin(-172°)

Acute angle = 180° – 172°

(3^{rd} quadrant, clockwise)

= 8°

at 3^{rd} quadrant sin is negative,

therefore

sin(-172°) = -sin (8°)

c. cos(-240°)

Acute angle = 240° – 180°

(2^{nd} quadrant, clockwise)

= 60°

at 2^{rd} quadrant cos is negative,

therefore

cos(-240°) = -cos (60°)

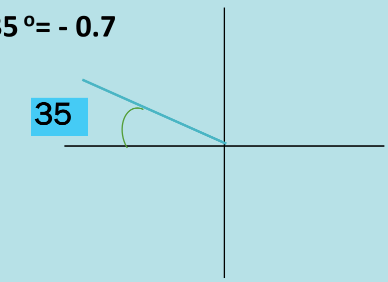
Solution Q10:

a. tan 145°

acute angle, α = 180 ° - 145 °

(2^{nd} quadrant) = 35 °

-tan 35 °= - 0.7



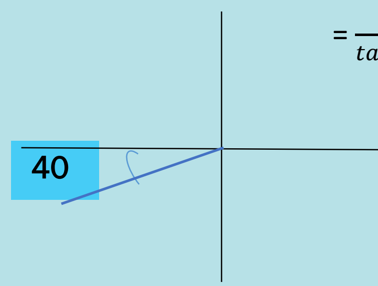
b. cot 220°

acute angle, α = 220 ° - 180 °

(3^{rd} quadrant) = 40 °

cot 220 °= $\frac{1}{tan220°}$

= $\frac{1}{tan40°}$ **= 1.1918**

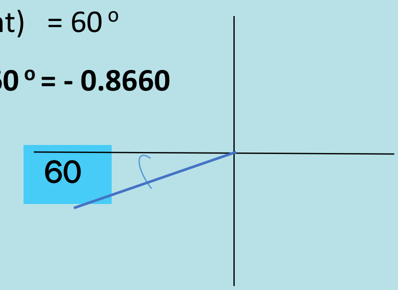


c. $sin\frac{4}{3}\pi$

Convert $\frac{4}{3}\pi$ $into\ degree\ unit, \frac{4}{3}\pi \times \frac{180}{\pi} = 240°$

acute angle, α = 240 ° - 180 ° (3^{rd} quadrant) = 60 °

-sin 60 ° = - 0.8660



TOPIC 2 : CIRCULAR MEASURE

Solution Q1:

a. $78^\circ = 78^\circ \times \frac{\pi}{180^\circ}$

= **1.36 rad**

b. $153^\circ = 153^\circ \times \frac{\pi}{180^\circ}$

= **2.67 rad**

c. $3.43 \text{ rad} = 3.43 \times \frac{180^\circ}{\pi}$

= **196.52°**

d. $0.62\pi \text{ rad} = 0.62\pi \times \frac{180^\circ}{\pi}$

= **111.6°**

e. $\frac{3}{5}\pi\, rad = \frac{3}{5}\pi \times \frac{180^\circ}{\pi}$

= **108°**

Solution Q2:

a. circumference of the circle

$= 2\pi r$

$= 2\pi(11)$

= 69.12 cm

b. the value of ɵ in degree unit

major arc length = 69.12 - 12

= 57.12cm

$57.12 = \frac{\theta}{360}(69.12)$

$\theta = \frac{57.12(360)}{69.12}$

$\boldsymbol{\theta = 297.5^\circ}$

c. area of circle

$\mathrm{A} = \pi r^2$

$\mathrm{A} = \pi(11)^2$

$\mathbf{A = 380.13cm^2}$

Solution Q3:

a. angle of KOL ,Ɵ

$s = r\theta$

$9.5 = 7\theta$

$\theta = \frac{9.5}{7}$

$\theta = 1.357\, rad$

@

$1.357 \times \frac{180^\circ}{\pi} = \mathbf{77.75}^\circ$

Solution Q3:

b. arc length of MN

$s = r\theta$

$s = 6 \times 1.357$

$s = \mathbf{8.14\ cm}$

Solution Q3:

c. length of NL

= 7 – 6

= **1 cm**

d. perimeter of KLNM

= 9.5 + 1 + 8.14 + 1

= **19.64 cm**

e. area of sector MON

$$A = \frac{1}{2}r^2\theta$$

$$A = \frac{1}{2} \times 6^2 \times 1.357$$

$$\boldsymbol{A = 24.426\ cm^2}$$

Solution Q4:

a. the radius for the sector if its length of arc 34 m

$$s = r\theta$$

$$34 = r \times 3.578$$

$$r = \frac{34}{3.578}$$

$$\boldsymbol{r = 9.50m}$$

b. the radius for the sector if its area 161.46m²

$$A = \frac{1}{2}r^2\theta$$

$$161.46 = \frac{1}{2}r^2 \times 3.578$$

$$r^2 = \frac{161.46 \times 2}{3.578}$$

$$r^2 = 90.25$$

$$r = \sqrt{90.25}$$

$$\boldsymbol{r = 9.5\ m}$$

Solution Q5:

Radius, r = $\frac{8.5}{2}$ = 4.25 cm

Angle of shaded sector = 2π – 1.25

= 5.033 rad

Area of shaded sector, $A = \frac{1}{2}r^2\theta$

$$A = \frac{1}{2} \times 4.25^2 \times 5.033$$

$$\boldsymbol{A = 45.45\ cm^2}$$

Solution Q6:

$$\text{A} = \frac{1}{2}r^2\theta$$

$$255 = \frac{1}{2} \times 10^2 \times \theta$$

$$\theta = \frac{255 \times 2}{10^2}$$

$$\theta = 5.1 \text{ rad}$$

$$x^\circ = (2\pi - 5.1) \times \frac{180^\circ}{\pi}$$

$$\boldsymbol{x^\circ = 67.79}$$

Solution Q7:

a. angle BAC in radian

$$A = \frac{1}{2}r^2\theta$$

$$87.54 = \frac{1}{2} \times 10^2 \times \theta$$

$$\theta = \frac{87.54 \times 2}{10^2}$$

$$\boldsymbol{\theta = 1.75\ \text{rad}}$$

b. the length of arc BC

$$s = r\theta$$

$$s = 10 \times 1.75$$

$$\boldsymbol{s = 17.5\ cm}$$

c. the area of shaded region

$$A = \frac{1}{2}r^2\theta - \frac{1}{2}r^2 \sin\theta$$

$$A = 87.54 - \left(\frac{1}{2} \times 10^2 \times \sin 1.75\right)$$

........(calculator in radian mode)

$$A = 87.54 - 49.2$$

$$\boldsymbol{A = 38.34\ cm^2}$$

b. area and circumference of the circle

$$A = \pi r^2$$

$$A = \pi \times 11^2$$

A = 380.13 cm^2

Circumference = 2πr

= 2 x π x 11

= 69.12 cm

Solution Q8:

$$A = \frac{1}{2}r^2 \sin\theta$$

$$A = \frac{1}{2} \times 12^2 \times \sin 1.78$$

........(calculator in radian mode)

A = 70.43 cm^2

Solution Q9:

ϴ = 2π – 1.3

= 4.983 rad

$$A = \frac{1}{2}r^2\theta$$

$$A = \frac{1}{2} \times 7^2 \times 4.983$$

A = 122.08 cm^2

Solution Q10:

a. angle in degree and area of the sector

$$s = r\theta$$

$$55 = 11 \times \theta$$

$$\theta = \frac{55}{11}$$

$$\theta = 5\ rad$$

$$= 5 \times \frac{180°}{\pi}$$

= 286.48°

$$A = \frac{1}{2}r^2\theta$$

$$A = \frac{1}{2} \times 11^2 \times 5$$

= 302.5 cm^2

TOPIC 3 : VECTOR

Solution Q1:

a. $\tilde{a} - 2\tilde{b}$

= (2i + 3j) – 2(4i – 5j)

= (2i + 3j) – (8i – 10j)

= 2i – 8i + 3j + 10j

= **- 6i + 13 j**

b. $\tilde{a}.\tilde{b}$

= (2i + 3j). (4i – 5j)

= (2i.4i) + (3j.-5j)

= 8 + (-15)

= **-7**

c. magnitude for vector $5\tilde{a}$

= $|5\tilde{a}|$

= $|5(2\text{i} + 3\text{j})|$

= $|10\text{i} + 15\text{j})|$

$|10\text{i} + 15\text{j})|$ = $\sqrt{10^2 + 15^2}$

= $\sqrt{325}$

= **18.03**

Solution Q2:

$\overrightarrow{OA} \cdot \overrightarrow{OB}$ = (4i + 6j).(– 4i + 6j)

= (4i . – 4i) + (6j.6j)

= -16 + 36

= 20

$|\overrightarrow{OA}|$=$\sqrt{4^2 + 6^2}$

=$\sqrt{52}$

$|\overrightarrow{OB}| = \sqrt{(-4)^2 + 6^2}$

=$\sqrt{52}$

$\overrightarrow{OA} \cdot \overrightarrow{OB} = |\overrightarrow{OA}||\overrightarrow{OB}|cos\theta$

$20 = \sqrt{52}\sqrt{52}cos\theta$

$20 = 52cos\theta$

$cos\theta = \frac{20}{52}$

$cos\theta = 0.3846$

$\theta = cos^{-1}0.3846$

$\boldsymbol{\theta = 67.38°}$

Solution Q3:

a. $2\tilde{p} = 2\binom{3}{2}$

$= \binom{\mathbf{6}}{\mathbf{4}}$

b. $\tilde{p} - \tilde{q} + \tilde{r}$

$= \binom{3}{2} - \binom{1}{-5} + \binom{4}{7}$

$= \binom{\mathbf{6}}{\mathbf{14}}$

c. $2\tilde{q} + \tilde{r} = 2\binom{1}{-5} + \binom{4}{7}$

$= \binom{2}{-10} + \binom{4}{7}$

$= \binom{\mathbf{6}}{\mathbf{-3}}$

d. magnitude of vector $3\tilde{r}$

$3\tilde{r} = 3\binom{4}{7}$

$= \binom{12}{21}$

$|3\tilde{r}| = \sqrt{12^2 + 21^2}$

$= \sqrt{585}$

$= \mathbf{24.19}$

Solution Q4:

a. $\tilde{m} \cdot \tilde{n} = (3i - j).(2i + 4j)$

$= (3i.2i) + (-j.4j)$

$= 6 + (-4)$

$= \mathbf{2}$

b. $|\tilde{m} - 2\tilde{n}|$

$\tilde{m} - 2\tilde{n} = (3i - j) - 2(2i + 4j)$

$= (3i - j) - (4i + 8j)$

$= 3i - 4i - j - 8j$

$= -i - 9j$

$|\tilde{m} - 2\tilde{n}| = \sqrt{(-1)^2 + (-9)^2}$

$= \sqrt{82}$

$= \mathbf{9.06}$

c. $\tilde{m} \cdot (\tilde{m} + \tilde{n})$

$= (3i - j).[(3i - j) + (2i + 4j)]$

$= (3i - j).(5i - 3j)$

$= (3i.5i) + (-j.-3j)$

$= 15 + 3$

$= \mathbf{18}$

d. $3|\tilde{m}| + 2|\tilde{n}| = 3\sqrt{3^2 + (-1)^2} + 2\sqrt{2^2 + 4^2}$

$= 3\sqrt{10} + 2\sqrt{20}$

$= 3(3.16) + 2(4.47)$

$= \mathbf{18.42}$

Solution Q5:

a. $\overrightarrow{OA} = 5i - 6j$

$|\overrightarrow{OA}| = \sqrt{5^2 + (-6)^2}$

$= \sqrt{25 + 36}$

$= \sqrt{61}$

$= \mathbf{7.81}$

b. $\tilde{b} = -i + 7j$

$|\tilde{b}| = \sqrt{(-1)^2 + 7^2}$

$= \sqrt{1 + 49}$

$= \sqrt{50}$

$= \mathbf{7.07}$

c. $\overrightarrow{OB} = \binom{8}{3}$

$|\overrightarrow{OB}| = \sqrt{8^2 + 3^2}$

$= \sqrt{64 + 9}$

$= \sqrt{73}$

$= \mathbf{8.54}$

Solution Q6:

$\overrightarrow{OP} \cdot \overrightarrow{OR} = (-5 \times 4) + (7 \times 6)$

$= -20 + 42$

$= 22$

$|\overrightarrow{OP}| = \sqrt{(-5)^2 + 7^2}$

$= \sqrt{74}$

$|\overrightarrow{OR}| = \sqrt{4^2 + 6^2}$

$= \sqrt{52}$

$\overrightarrow{OP} \cdot \overrightarrow{OR} = |\overrightarrow{OP}||\overrightarrow{OR}| cos\theta$

$22 = \sqrt{74}\sqrt{52} cos\theta$

$22 = 62.03 cos\theta$

$cos\theta = \frac{22}{62.03}$

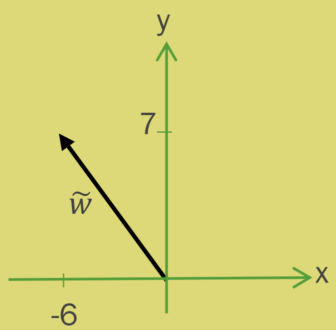
$cos\theta = 0.3547$

$\theta = cos^{-1} 0.3547$

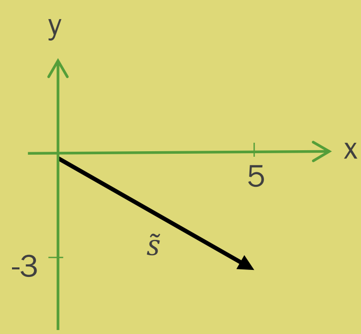
$\boldsymbol{\theta = 69.22}°$

SolutionQ7:

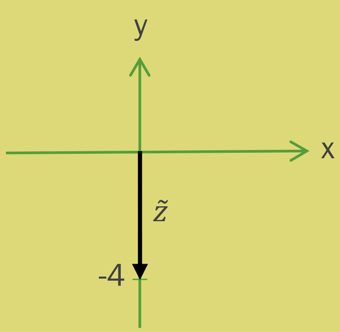
a. $\widetilde{w} = -6i + 7j$



b. $\tilde{s} = (5\,,-3)$



c. $\tilde{z} = \binom{0}{-4}$



Solution Q8:

a. $\overrightarrow{\boldsymbol{OA}} = -\mathbf{2}\boldsymbol{i} + \mathbf{3}\boldsymbol{j}$

b. $\overrightarrow{\boldsymbol{OB}} = \mathbf{4}\boldsymbol{i} + \mathbf{2}\boldsymbol{j}$

c. $\overrightarrow{\boldsymbol{OC}} = \boldsymbol{i} - \mathbf{3}\boldsymbol{j}$

iv. $\overrightarrow{\boldsymbol{OD}} = -\mathbf{4}\boldsymbol{i} - \boldsymbol{j}$

Solution Q9:

$\overrightarrow{OS} \cdot \overrightarrow{OR} = 9$

$(5,2) \cdot (x, y) = 9$

$(5x) + (2y) = 9$

$5(1) + 2(2) = 9$

$x = 1, y = 2$

$\overrightarrow{OR} = (1,2)$

$|\overrightarrow{OR}| = \sqrt{1^2 + 2^2}$

$= \sqrt{5}$

$|\overrightarrow{OS}| = \sqrt{5^2 + 2^2}$

$= \sqrt{29}$

$\overrightarrow{OS} \cdot \overrightarrow{OR} = |\overrightarrow{OR}||\overrightarrow{OS}|cos\theta$

$9 = \sqrt{5}\sqrt{29}cos\theta$

$\cos\theta = \frac{9}{\sqrt{5}\sqrt{29}}$

$\cos\theta = 0.7474$

$\theta = \cos^{-1} 0.7474$

$\boldsymbol{\theta} = \mathbf{41.63}°$

Solution Q10:

a. $-\tilde{r} \cdot 4\tilde{s}$

$= -(3i + 2j) \cdot 4(2i)$

$= (-3i - 2j) \cdot 8i$

$= (-3i.8i) + (2j.0j)$

$= -24 + 0$

$= \mathbf{-24}$

b. $\tilde{s} - \frac{1}{2}\tilde{r}$

$= 2i - \frac{1}{2}(3i + 2j)$

$= 2i - (\frac{3}{2}i + j)$

$= 2i - \frac{3}{2}i - j$

$= \mathbf{\frac{1}{2}i - j}$

Solution Q10:

c. $|\tilde{r} + 5\tilde{s}|$

$\tilde{r} + 5\tilde{s} = (3i + 2j) + 5(2i)$

$= 3i + 2j + 10i$

$= 13i + 2j$

$|\tilde{r} + 5\tilde{s}| = \sqrt{13^2 + 2^2}$

$= \sqrt{169 + 4}$

$= \sqrt{173}$

$= \mathbf{13.15}$

TOPIC 4 : INEQUALITY

Solution Q1:

a. $4x > 36$

$x > \frac{36}{4}$

$x > 9$

b. $-10 < 2x$

$\frac{-10}{2} < x$

$-5 < x$

$x > -5$

Solution Q1:

c. $5x > 12 + 4x$

$5x - 4x > 12$

$x > 12$

d. $9x - 30 < -3$

$9x < -3 + 30$

$9x < 27$

$x < \frac{27}{9}$

$x < 3$

Solution Q1:

e. $7m < 21$

$m < \frac{21}{7}$

$m < 3$

f. $5y > 15 - 10y$

$5y + 10y > 15$

$15y > 15$

$y > \frac{15}{15}$

$y > 1$

g. $r + 5 > -5$

$r > -5 - 5$

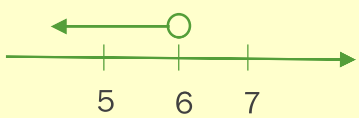
$r > -10$

Solution Q2:

a. $\{x : x < 6\}$

Interval notation $(\infty, 6)$

Number Line

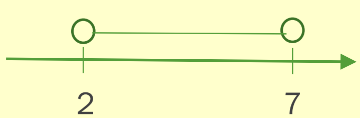


$x = 5,4,3,2,1,\ldots.\infty$

b. $\{x : 2 < x \leq 7\}$

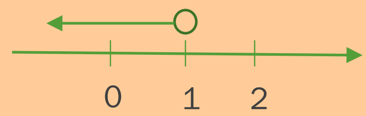
Interval Notation $(2,7]$

Number Line

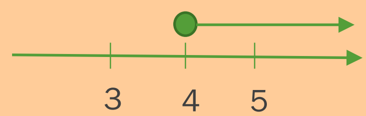


Solution Q3:

a.
$$15 - 10x > 5$$
$$-10x > 5 - 15$$
$$-10x > -10$$
$$x < \frac{-10}{-10}$$
$$x < 1$$



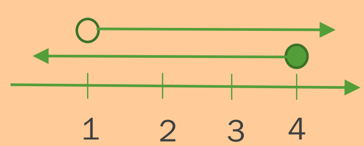
b.
$$4x + 10 \geq 26$$
$$4x \geq 26 - 10$$
$$4x \geq 16$$
$$x \geq \frac{16}{4}$$
$$x \geq 4$$



c. $x - 1 \leq 3$ and $5 - 3x < 2$

$$x - 1 \leq 3$$
$$x \leq 3 + 1$$
$$x \leq 4$$

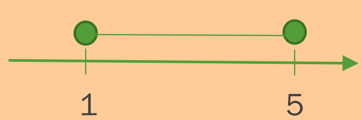
$$5 - 3x < 2$$
$$-3x < 2 - 5$$
$$-3x < -3$$
$$x < \frac{-3}{-3}$$
$$x > 1$$



d. $1 \leq 4x - 3 \leq 17$

$$1 \leq 4x - 3$$
$$4x \leq -3 - 1$$
$$4x \leq -4$$
$$x \geq 1$$

$$4x - 3 \leq 17$$
$$4x \leq 17 + 3$$
$$4x \leq 20$$
$$x \leq 5$$



Solution Q4:

a.
$$4x-1>11$$
$$4x>11+1$$
$$4x>12$$
$$x>\frac{12}{4}$$
$$x>3$$

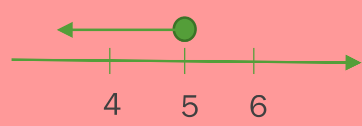
b.
$$2x+2\geq -8+x$$
$$2x-x\geq -8-2$$
$$x\geq -10$$

c.
$$-2(x-1)<6$$
$$-2x+2<6$$
$$-2x<6-2$$
$$-2x<4$$
$$x>\frac{4}{-2}$$
$$x>-2$$

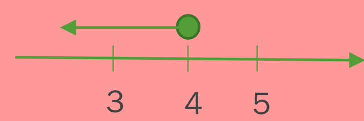
d.
$$2(6-y)>10$$
$$12-2y>10$$
$$-2y>10-12$$
$$-2y>-2$$
$$y<\frac{-2}{-2}$$
$$y<1$$

Solution Q5:

a.
$$5+4s\leq 60-7s$$
$$4s+7s\leq 60-5$$
$$11s\leq 55$$
$$s\leq\frac{55}{11}$$
$$s\leq 5$$



b.
$$9+6k\leq 53-5k$$
$$6s+5s\leq 53-9$$
$$11s\leq 44$$
$$s\leq 4$$



Solution Q6:

$$\frac{8-6x}{6}\leq 8$$

$$8-6x\leq 48$$
$$-6x\leq 48-8$$
$$-6x\leq 40$$
$$x\geq\frac{40}{-6}$$
$$x\geq\frac{20}{-3}$$

Interval Notation

$$[\frac{20}{-3},\infty)$$

TOPIC 5 : MATRICES

Solution Q1:

a. Order of matrix $= 3\times 2$

b. Order of matrix $= 1\times 3$

c. Order of matrix $= 2\times 3$

d. Order of matrix $= 1\times 3$

e. Order of matrix $= 3\times 1$

f. Order of matrix $= 2\times 2$

g. Order of matrix $= 2\times 1$

h. Order of matrix $= 3\times 2$

Solution Q2:

-Row matrix
-Column matrix/Vector matrix
-Zero matrix / Null matrix
-Diagonal matrix
-Scalar matrix
-Unit matrix
-Upper triangular matrix
-Lower triangular matrix

Solution Q3:

a. $P+Q$

$$\begin{pmatrix} 2 & 4 \\ -3 & 5 \end{pmatrix} + \begin{pmatrix} 6 & 4 \\ 0 & -4 \end{pmatrix}$$

$$\begin{pmatrix} 2+6 & 4+4 \\ -3+0 & 5-4 \end{pmatrix}$$

$$\begin{pmatrix} 8 & 8 \\ -3 & 1 \end{pmatrix}$$

b. $P^T - Q$

$$P^T = \begin{pmatrix} 2 & -3 \\ 4 & 5 \end{pmatrix}$$

$$\begin{pmatrix} 2 & -3 \\ 4 & 5 \end{pmatrix} - \begin{pmatrix} 6 & 4 \\ 0 & -4 \end{pmatrix}$$

$$\begin{pmatrix} 2-6 & -3-4 \\ 4-0 & 5-(-4) \end{pmatrix}$$

$$\begin{pmatrix} -4 & -7 \\ 4 & 9 \end{pmatrix}$$

Solution Q3:

c. $3Q$

$$3\times \begin{pmatrix} 6 & 4 \\ 0 & -4 \end{pmatrix}$$

$$\begin{pmatrix} 3\times 6 & 3\times 4 \\ 3\times 0 & 3\times -4 \end{pmatrix}$$

$$\begin{pmatrix} 18 & 12 \\ 0 & -12 \end{pmatrix}$$

d. PQ

$$\begin{pmatrix} 2 & 4 \\ -3 & 5 \end{pmatrix} \times \begin{pmatrix} 6 & 4 \\ 0 & -4 \end{pmatrix}$$

$$\begin{pmatrix} (2\times 6)+(4\times 0) & (2\times 4)+(4\times -4) \\ (-3\times 6)+(5\times 0) & (-3\times 4)+(5\times -4) \end{pmatrix}$$

$$\begin{pmatrix} 12+0 & 8+(-16) \\ -18+0 & (-12)+(-20) \end{pmatrix}$$

$$\begin{pmatrix} 12 & -8 \\ -18 & -32 \end{pmatrix}$$

Solution Q4:

a. $M + N$

$$\begin{pmatrix} 5 & 4 \\ -2 & 1 \end{pmatrix} + \begin{pmatrix} 0 & 7 \\ -2 & 6 \end{pmatrix}$$

$$\begin{pmatrix} 5+0 & 4+7 \\ -2+-2 & 1+6 \end{pmatrix}$$

$$\begin{pmatrix} 5 & 11 \\ -4 & 7 \end{pmatrix}$$

b. $2M - N$

$$2\begin{pmatrix} 5 & 4 \\ -2 & 1 \end{pmatrix} - \begin{pmatrix} 0 & 7 \\ -2 & 6 \end{pmatrix}$$

$$\begin{pmatrix} 10 & 8 \\ -4 & 2 \end{pmatrix} - \begin{pmatrix} 0 & 7 \\ -2 & 6 \end{pmatrix}$$

$$\begin{pmatrix} 10-0 & 8-7 \\ -4--2 & 2-6 \end{pmatrix}$$

$$\begin{pmatrix} 10 & 1 \\ -2 & -4 \end{pmatrix}$$

Solution Q5:

a. $3S = 3\begin{pmatrix} x+2 & 7 \\ 0 & 4 \end{pmatrix}$

$$3S + T = \begin{pmatrix} 3x+6 & 21 \\ 0 & 12 \end{pmatrix} + \begin{pmatrix} 4 & y-1 \\ 11 & 2 \end{pmatrix}$$

$$\begin{pmatrix} 25 & 24 \\ 11 & 14 \end{pmatrix} = \begin{pmatrix} 3x+6 & 21 \\ 0 & 12 \end{pmatrix} + \begin{pmatrix} 4 & y-1 \\ 11 & 2 \end{pmatrix}$$

$$\begin{pmatrix} 3x+6+4 & 21+y-1 \\ 0+11 & 12+2 \end{pmatrix} = \begin{pmatrix} 25 & 24 \\ 11 & 14 \end{pmatrix}$$

$$3x+6+4 = 25$$

$$x = 5$$

$$21+y-1 = 24$$

$$y = 4$$

b. $T = \begin{pmatrix} 4 & 3 \\ 11 & 2 \end{pmatrix}$

$$T^{-1} = \frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

$$= \frac{1}{8-33}\begin{pmatrix} 2 & -3 \\ -11 & 4 \end{pmatrix}$$

$$= \frac{1}{-25}\begin{pmatrix} 2 & -3 \\ -11 & 4 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{-2}{25} & \frac{3}{25} \\ \frac{11}{25} & \frac{-4}{25} \end{pmatrix}$$

Solution Q6:

a) $\begin{pmatrix} 1 & 0 \\ 2 & -1 \\ 3 & 2 \end{pmatrix}\begin{pmatrix} 1 \\ 2 \end{pmatrix}$

$$= \begin{bmatrix} (1\times 1)+(0\times 2) \\ (2\times 1)+(-1\times 2) \\ (3\times 1)+(2\times 2) \end{bmatrix}$$

$$= \begin{pmatrix} 1 \\ 0 \\ 7 \end{pmatrix}$$

Solution Q7:

a. $A = \begin{pmatrix} 2 & 2 \\ 4 & -4 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 10 \\ 8 \end{pmatrix}$

$$A^{-1} = \frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

$$= \frac{1}{-2-2}\begin{pmatrix} -4 & -2 \\ -4 & 2 \end{pmatrix}$$

$$= \frac{1}{-4}\begin{pmatrix} -4 & -2 \\ -4 & 2 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{-4}{-4} & \frac{-2}{-4} \\ \frac{-4}{-4} & \frac{2}{-4} \end{pmatrix}$$

$$= \begin{pmatrix} 1 & \frac{1}{2} \\ 1 & \frac{-1}{2} \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = A^{-1}\begin{pmatrix} h \\ k \end{pmatrix}$$

$$= \begin{pmatrix} 1 & \frac{1}{2} \\ 1 & \frac{-1}{2} \end{pmatrix}\begin{pmatrix} 10 \\ 8 \end{pmatrix}$$

$$= \begin{bmatrix} (1\times 10)+\left(\frac{1}{2}\times 8\right) \\ (1\times 10)+\left(\frac{-1}{2}\times 8\right) \end{bmatrix}$$

$$= \begin{bmatrix} 10+4 \\ 10-4 \end{bmatrix}$$

$$= \begin{bmatrix} 14 \\ 6 \end{bmatrix}$$

$x = 14, y = 6$

Solution Q7:

b. $A = \begin{pmatrix} 7 & 7 \\ 5 & -3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 35 \\ -7 \end{pmatrix}$

$$A^{-1} = \frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

$$= \frac{1}{-21-35}\begin{pmatrix} -3 & -7 \\ -5 & 7 \end{pmatrix}$$

$$= \frac{1}{-56}\begin{pmatrix} -3 & -7 \\ -5 & 7 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{3}{56} & \frac{1}{8} \\ \frac{5}{56} & \frac{-1}{8} \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = A^{-1}\begin{pmatrix} h \\ k \end{pmatrix}$$

$$= \begin{pmatrix} \frac{3}{56} & \frac{1}{8} \\ \frac{5}{56} & \frac{-1}{8} \end{pmatrix}\begin{pmatrix} 35 \\ -7 \end{pmatrix}$$

$$= \begin{bmatrix} \left(\frac{3}{56}\times 35\right)+\left(\frac{1}{8}\times -7\right) \\ \left(\frac{5}{56}\times 35\right)+\left(\frac{-1}{8}\times -7\right) \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 4 \end{bmatrix}$$

$x = 1, y = 4$

Solution Q7:

c. $A = \begin{pmatrix} 5 & 1 \\ -2 & 3 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 17 \\ 17 \end{pmatrix}$

$$A^{-1} = \frac{1}{ad-bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

$$= \frac{1}{15+2}\begin{pmatrix} 3 & -1 \\ 2 & 5 \end{pmatrix}$$

$$= \frac{1}{17}\begin{pmatrix} 3 & -1 \\ 2 & 5 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{3}{17} & \frac{-1}{17} \\ \frac{2}{17} & \frac{5}{17} \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = A^{-1}\begin{pmatrix} h \\ k \end{pmatrix}$$

$$= \begin{pmatrix} \frac{3}{17} & \frac{-1}{17} \\ \frac{2}{17} & \frac{5}{17} \end{pmatrix}\begin{pmatrix} 17 \\ 17 \end{pmatrix}$$

$$= \begin{bmatrix} \left(\frac{3}{17}\times 17\right)+\left(\frac{-1}{17}\times 17\right) \\ \left(\frac{2}{17}\times 17\right)+\left(\frac{5}{17}\times 17\right) \end{bmatrix}$$

$$= \begin{bmatrix} 2 \\ 7 \end{bmatrix}$$

$x = 2, y = 7$

Solution Q8:

$$\begin{pmatrix} 8 & -1 \\ 0 & 3 \end{pmatrix}$$

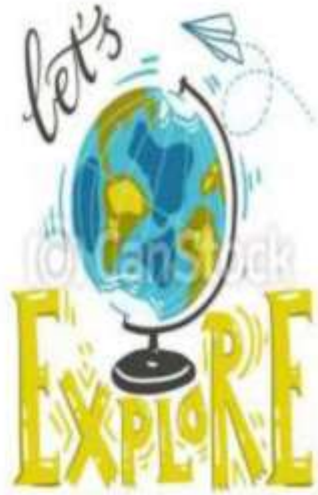
$$A^{-1} = \frac{1}{ad-bc}\begin{pmatrix} 3 & 1 \\ 0 & 8 \end{pmatrix}$$

$$= \frac{1}{24-0}\begin{pmatrix} 3 & 1 \\ 0 & 8 \end{pmatrix}$$

$$= \frac{1}{24}\begin{pmatrix} 3 & 1 \\ 0 & 8 \end{pmatrix}$$

$$= \begin{pmatrix} \frac{3}{24} & \frac{1}{24} \\ 0 & \frac{8}{24} \end{pmatrix}$$

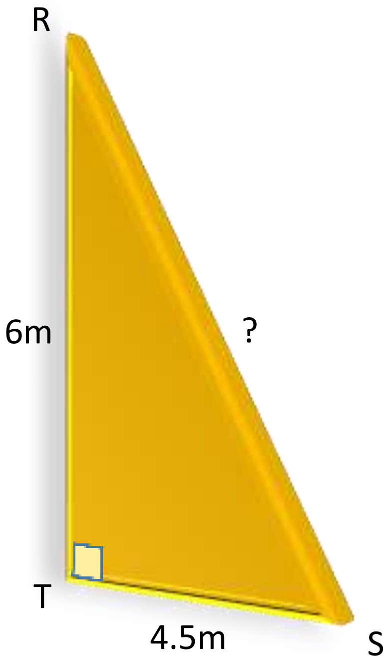
$$= \begin{pmatrix} \frac{1}{8} & \frac{1}{24} \\ 0 & \frac{1}{3} \end{pmatrix}$$


let's
EXPLORE


APPLICATION

Application of Theorem Pythagoras

A fireman climbs up a ladder to save a child who is trapped on the third floor as shown in the diagram. The third floor is 8m high from the horizontal ground. The base of the ladder is 3m away from the wall of the building. How long is the ladder?



$$c^2 = a^2 + b^2$$

$$c^2 = 3^2 + 8^2$$

$$c^2 = 73$$

$$c = \sqrt{73}$$

$$c = 8.54\text{m}$$

Thus, the ladder is 8.54 m long.

Application of Circular Measure

A landscape designer intends to build a rectangular recreational park with a length of 70m and a width of 55m. At every corner of the park, a quadrant with radius 8m will be plated with flowers. A circular shaped fish pond with a diameter of 15m will be built in the middle of the park. The remaining areas will be planted with grass. Calculate the area covered with the grass.

Area rectangular recreational park	Area of circular pond	Area quadrant flower	Area grass
$A_R = l \times w$ $A_R = 70 \times 55$ $A_R = 3850$	$A_p = \pi r^2$ $A_p = \frac{22}{7}(7.5)^2$ $A_p = 176.8$	$A_f = 4 \times \frac{1}{4}\pi r^2$ $A_f = \pi r^2$ $A_f = \frac{22}{7}(8)^2$ $A_f = 201.1$	$A_G = A_R - A_p - A_f$ $A_G = 3850 - 176.8 - 201.1$ $A_G = 3472.1 \text{ m}^2$

Thus, the area needed to cover with grass is 3472.1 m^2.

Application of Inequalities

Amina buys fruits in a night market near her house. She has RM175 and plans to buy *m* kg of mangoes and *n* kg of rambutans. The weight of the mangoes is at most three times the weight of the rambutans. The total weight of the fruits is not less than 12kg. The price of 1kg of mangoes is RM6 and the price of 1kg of rambutans is RM4. If Amina's brother sells 1kg of mangoes for RM 10 and 1kg of rambutans for RM 7, find the maximum profit that can be gained by Dania's brother.

$$m = \text{mango} \quad n = \text{rambutan}$$

$$m + n \geq 12$$

$$6m + 4n \leq 175$$

$$m \leq 3n$$

The first step in solving this kind of situation is by using inequalities.

Application of Inequalities

A candy factory produces two products A and B. The profits from the sales of products A and B are RM12 and RM29 respectively. The factory can produce *x* units of product A and *y* units of product B in a day. Find the maximum profit that can be obtained if the production of products A and B must satisfy the following conditions:

- The total number of units of products A and B produced in a day is not more than 500 units.
- The number of units of product B produced in a day is not more than two times the number of units of product A produced.
- The number of units of product B produced in a day is at least 300 units.

$x = \text{product A} \quad y = \text{product B}$

$$x + y \geq 500$$

$$y \leq 2x$$

$$y \geq 300$$

The first step in solving this kind of situation is by using inequalities.

Application of Inverse Matrices

A group took a trip on a bus, at RM3 per child and RM3.20 per adult for a total of RM118.40. They took the train back at RM3.50 per child and RM3.60 per adult for a total of RM135.20. How many children, and how many adults?

$$x = \text{children} \qquad y = \text{adult}$$

1. Write the equations in matrix form,

$$\begin{pmatrix} 3 & 3.20 \\ 3.50 & 3.60 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 118.40 \\ 135.20 \end{pmatrix}$$

Where

$$A = \begin{pmatrix} 3 & 3.20 \\ 3.50 & 3.60 \end{pmatrix}$$

2. Determine the inverse of A, A^{-1}

$$A^{-1} = \frac{1}{ad - bc}\begin{pmatrix} d & -b \\ -c & a \end{pmatrix}$$

$$A^{-1} = \frac{1}{(3)(3.60) - (3.50)(3.20)}\begin{pmatrix} 3.60 & -3.20 \\ -3.50 & 3 \end{pmatrix}$$

$$A^{-1} = \frac{1}{10.80 - 11.20}\begin{pmatrix} 3.60 & -3.20 \\ -3.50 & 3 \end{pmatrix}$$

$$A^{-1} = \frac{1}{-0.40}\begin{pmatrix} 3.60 & -3.20 \\ -3.50 & 3 \end{pmatrix}$$

$$A^{-1} = \begin{pmatrix} \frac{3.60}{-0.40} & \frac{-3.20}{-0.40} \\ \frac{-3.50}{-0.40} & \frac{3}{-0.40} \end{pmatrix}$$

$$= \begin{pmatrix} -9 & 8 \\ 8.75 & -7.5 \end{pmatrix}$$

3. Solve by using formula

$$\begin{pmatrix} x \\ y \end{pmatrix} = A^{-1}\begin{pmatrix} h \\ k \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -9 & 8 \\ 8.75 & -7.5 \end{pmatrix}\begin{pmatrix} 118.40 \\ 135.20 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{bmatrix} (-9 \times 118.40) + (8 \times 135.20) \\ (8.75 \times 118.40) + (-7.5 \times 135.20) \end{bmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{bmatrix} 16 \\ 22 \end{bmatrix}$$

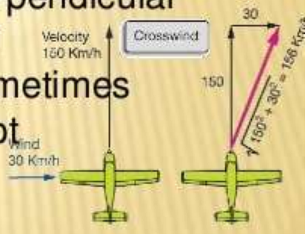
$$x = 16, y = 22$$

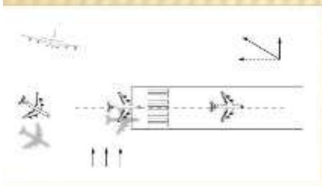
Thus, there were 16 children and 22 adults.

Application of Vector

CROSSWIND

We are familiar to the term of crosswind. A crosswind is any wind That has a perpendicular component to the line or direction of travel. When a plan come to land sometimes It face difficulties for crosswind. A pilot can find out the resultant velocity and direction by with help of vector.





VECTOR IN GAMING

In Games, vectors are used to store positions directions and velocities. The position vectors indicates how far the object is the velocity vector indicates how much time it will take or how much force we should give and the direction vector indicates in which way we should apply the force.

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