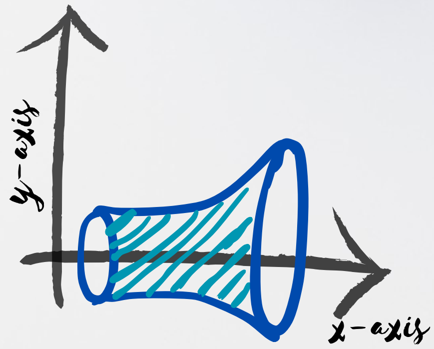
KEMENTERIAN PENGAJIAN TINGGI

Let's Focus

Application of Integration

AREA & VOLUME

MAZIAH BINTI OMAR
MASNIZA BINTI MUSA
NANG SARUNI BINTI NEK ALI



AREA

&

VOLUME

MAZIAH BINTI OMAR
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PREFACE

Area and Volume eBook is designed to assist engineering students in studying sub-topic Area and Volume for Engineering Mathematics 2 course. All contents in this eBook will put students on the track as it is in accordance with the latest syllabus specified by the Polytechnic's syllabus requirement, Ministry of Higher Education.

This eBook consists of comprehensive notes, a number of solved problems for clarifying, solving technique to illustrate the theory and supplementary example questions as the exercise to the subtopic.

Any positive feedback from lecturers and students that would improve the content of this eBook are mostly welcome and appreciated. It is our hope that this eBook is one of the tiny steps that we have made to help students mastery in this course with excellence.

Author's

Masniza binti Musa, Nang Saruni binti Nek Ali
& Maziah binti Omar
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Politeknik Tuanku Sultanah Bahiyah.

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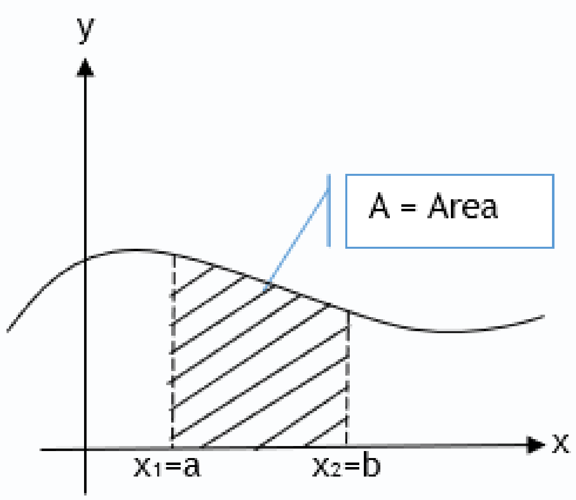
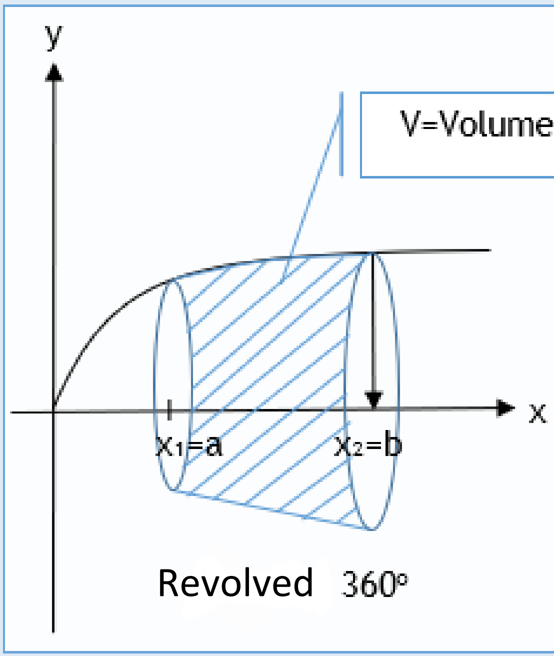
APPLICATION OF INTEGRATION

DBM20023

Application of Integration is applied in various fields like Mathematics, Science, Engineering and others. In Engineering Mathematics 2 (DBM20023), Application of Integration only focus on Area Under a Curve and Volume of Revolution.

Area Under A Curve	Volume of Revolution

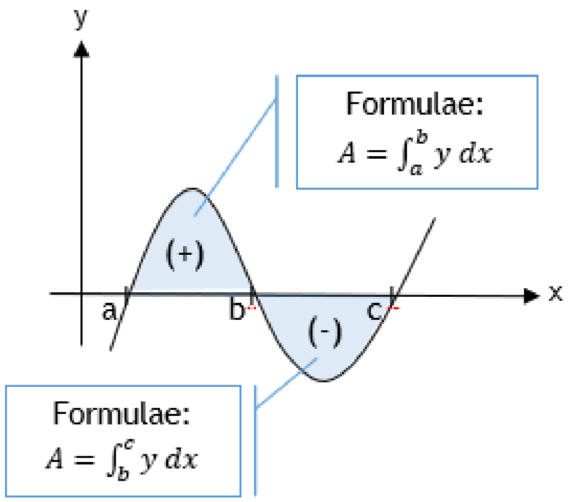




AREA UNDER A CURVE

Area under a curve as the limit of sum of areas

<table>
<tr><th>Area between a curve and the x-axis</th><th>Area between a curve and the y-axis</th></tr>
<tr><td>



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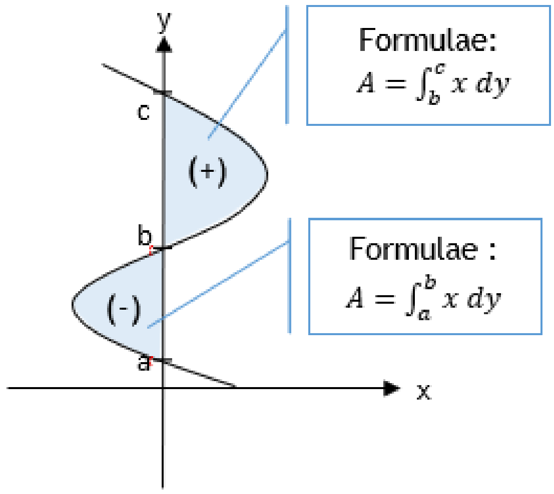


</td></tr>
<tr><td>The area of the region above the x-axis is positive while the area of the region below the x-axis is negative</td><td>The area of the region to the right of the y-axis is positive while the area of the region to the left of the y-axis is negative</td></tr>
</table>

Case 1: Area of the region between a curve and the axis

Example 1:

Find the area bounded by $y = \sqrt{2x-3}$ and x-axis from $x = 4$ and $x = 8$

SOLUTION:

Area of the curve, $A = \int_a^b y\,dx$

$$A = \int_4^8 y\,dx$$

$$A = \int_4^8 \sqrt{2x-3}\,dx$$

$$= \int_4^8 (2x-3)^{\frac{1}{2}}\,dx$$

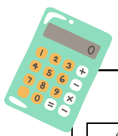
$$= \left[\frac{(2x-3)^{\frac{1}{2}+1}}{\left(\frac{3}{2}\right)(2)}\right]_4^8$$

$$= \left[\frac{(2x-3)^{\frac{3}{2}}}{3}\right]_4^8$$

$$= \frac{[2(8)-3]^{\frac{3}{2}}}{3} - \frac{[2(4)-3]^{\frac{3}{2}}}{3}$$

$$= 15.624 - 3.727$$

$$= \mathbf{11.897\ unit^2}$$

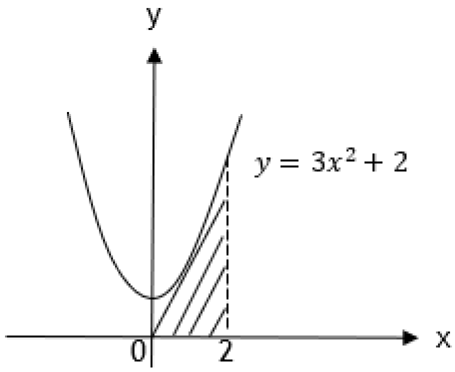


Use scientific calculator:

$\int dx$	$\sqrt{(2x-3)}$	, 4 , 8	)	=

Example 2:

Find the area under the curve of $y = 3x^2 + 2$ in the diagram below:



SOLUTION:

Area of the curve, $A = \int_a^b y\,dx$

$$A = \int_0^2 y\,dx$$

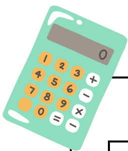
$$A = \int_0^2 (3x^2 + 2)dx$$

$$= \left[\frac{3x^3}{3} + 2x\right]_0^2$$

$$= [x^3 + 2x]_0^2$$

$$= [(2)^3 + 2(2)] - [(0)^3 + 2(0)]$$

$= \mathbf{12\ unit^2}$

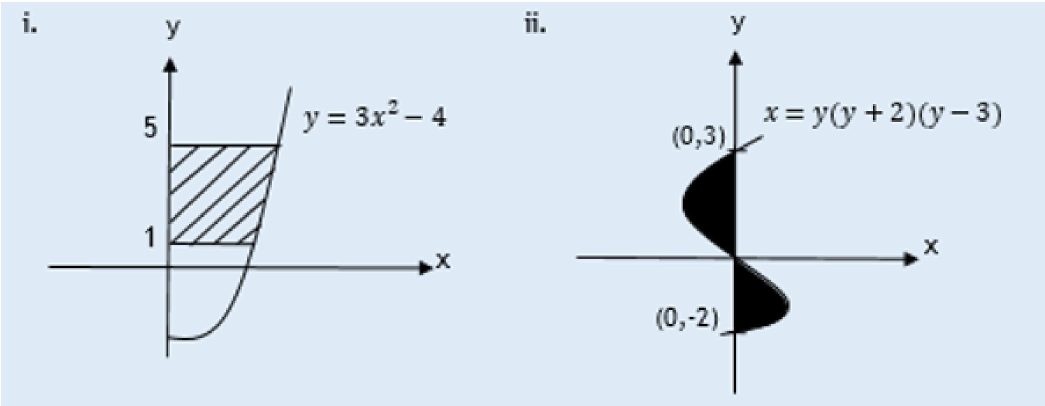


Use scientific calculator:

$\int dx$	$3x^2 + 2$	,0,2	)	=

Example 3:

Calculate the area for the following bounded region below:



SOLUTION:

i. Area of the curve, $A = \int_a^b x\,dy$

$$A = \int_1^5 x\,dy$$

$$A = \int_1^5 \sqrt{\frac{y+4}{3}}\,dy$$

Given $y = 3x^2 - 4$,

then $x = \sqrt{\frac{y+4}{3}}$

$$= \int_1^5 \left(\frac{y+4}{3}\right)^{\frac{1}{2}} dy$$

$$= \left[\frac{\left(\frac{y+4}{3}\right)^{\frac{1}{2}+1}}{\left(\frac{3}{2}\right)\left(\frac{1}{3}\right)}\right]_1^5$$

Use scientific calculator:

$\int dx$	$\sqrt{((x+4) \div 3)}$	, 1 ,5	)	=

$$= \left[2\left(\frac{y+4}{3}\right)^{\frac{3}{2}}\right]_1^5$$

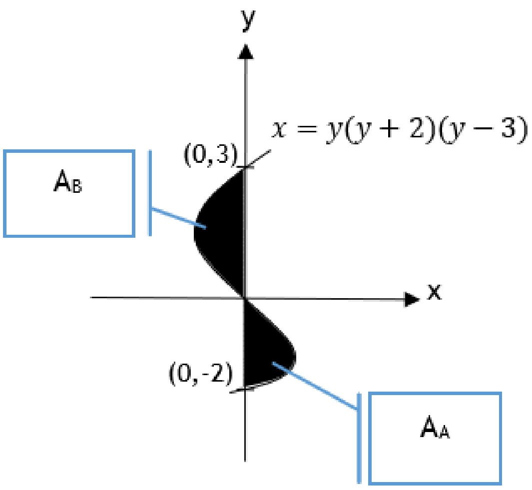
$$= 2\left[\frac{(5)+4}{3}\right]^{\frac{3}{2}} - 2\left[\frac{(1)+4}{3}\right]^{\frac{3}{2}}$$

$$= 10.39 - 4.30$$

$$= \mathbf{6.09\ unit^2}$$

ii. The bounded area, $A = A_A + A_B$

$$A_A = \int_{-2}^{0} y(y+2)(y-3)dy$$
$$= \int_{-2}^{0} y^3 - y^2 - 6y\, dy$$
$$= \left[\frac{y^4}{4} - \frac{y^3}{3} - \frac{6y^2}{2}\right]_{-2}^{0}$$
$$= \left[\frac{(0)^4}{4} - \frac{(0)^3}{3} - 3(0)^2\right] - \left[\frac{(-2)^4}{4} - \frac{(-2)^3}{3} - 3(-2)^2\right]$$
$$= 0 - \left(-\frac{16}{3}\right)$$
$$= \frac{16}{3} \text{ unit}^2$$



$$A_B = \int_{0}^{3} y(y+2)(y-3)dy$$
$$= \int_{0}^{3} y^3 - y^2 - 6y\, dy$$
$$= \left[\frac{y^4}{4} - \frac{y^3}{3} - \frac{6y^2}{2}\right]_{0}^{3}$$
$$= \left[\frac{(3)^4}{4} - \frac{(3)^3}{3} - 3(3)^2\right] - \left[\frac{(0)^4}{4} - \frac{(0)^3}{3} - 3(0)^2\right]$$
$$= -\frac{63}{4} - 0$$
$$= -\frac{63}{4} \text{ unit}^2$$

Negative value is shown the area of the bounded region to the **left** of the y-axis

$$= \frac{63}{4} \text{ unit}^2$$

$\therefore$ The bounded area, $A = A_A + A_B$

$$= \frac{16}{3} + \frac{63}{4}$$

$$= \frac{253}{12} \text{ unit}^2$$

Use scientific calculator:

$\int dx$	$x^3 - x^2 - 6x$	,-2,0	)	=

Use scientific calculator:

$\int dx$	$x^3 - x^2 - 6x$	,0,3	)	=

Example 4:

Given an equation $y = (x-3)^2$. Find the area under the graph bounded by the curve, *x*-axis, the lines $x = 6$ and $x = 1$.

SOLUTION:

Area of the curve, $A = \int_a^b y\, dx$

$$A = \int_1^6 y\, dx$$

$$A = \int_1^6 (x-3)^2 dx$$

$$= \left[\frac{(x-3)^3}{3(1)}\right]_1^6$$

$$= \left[\frac{(6-3)^3}{3}\right] - \left[\frac{(1-3)^3}{3}\right]$$

$$= 9 - (-2.67)$$

$$= \mathbf{11.67} \textbf{ unit}^2$$

Use scientific calculator:

$\int dx$	$(x-3)^2$	,1,6	)	=

Self Assess 1.1

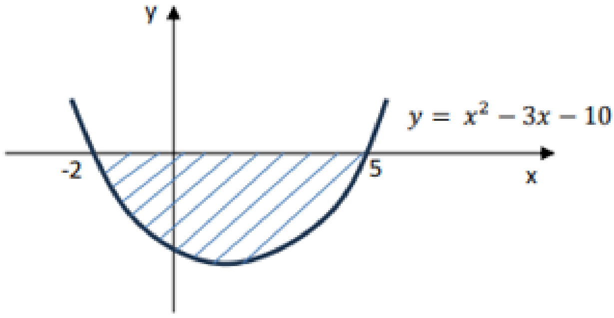
1. Find the area bounded by $y = \sqrt{2x+5}$ and x-axis from $x = -2.5$ and $x = 5$

Answer: 19.365 unit^2

2. Find the area between $x = (y-1)(y-4)$, y = 1 and y = 4.

Answer: $\frac{9}{2}$ unit^2

3. The figure below shows an enclosed region between the curve of $y = x^2 - 3x - 10$ and x-axis between $x = -2$ and $x = 5$. Calculate the area bounded by the curves.



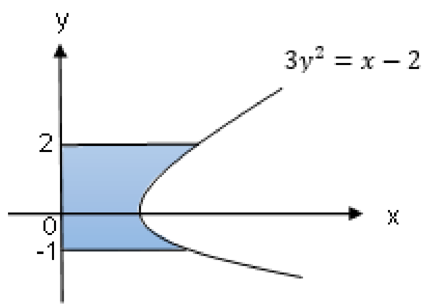
Final Examination II 2022/2023

Answer: 57.16 unit^2

4. Find the area between $y = x^2 - 6x + 5$, x = 1 and x = 3.

Answer: $\frac{16}{3}$ unit^2

5. Find the area for the bounded region below:



Answer: 15 unit^2

Case 2: Area of the region between a curve and the straight line

Example 5:

A straight line $y = x + 8$ meets the curve $y = 10x - x^2$ at points A and B.

i. Find the coordinates of points A and B
ii. Sketch both graph
iii. Calculate the area of the graph bounded by both graph

SOLUTION:

i. Coordinate of point A and B:

Solve using simultaneous equations:

$y = x + 8$ ----------eq. 1
$y = 10x - x^2$ ----------eq. 2

Substitute eq. 1 into eq. 2:

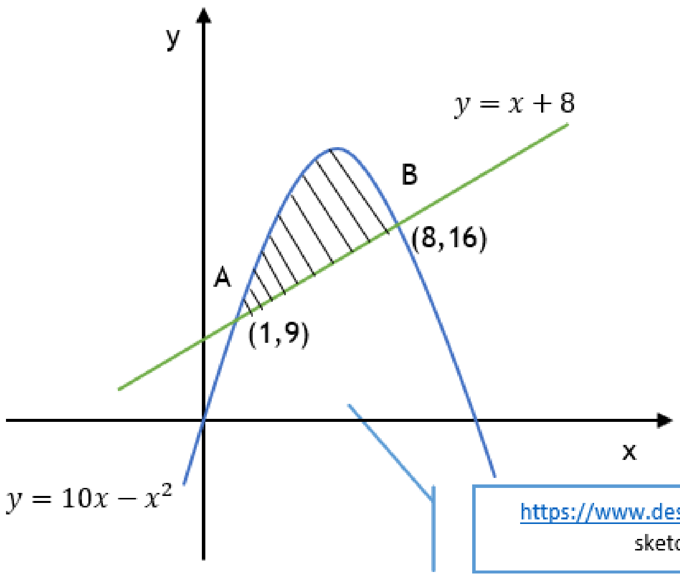
$$x + 8 = 10x - x^2$$
$$x^2 + x - 10x + 8 = 0$$
$$x^2 - 9x + 8 = 0$$
$$(x-1)(x-8) = 0$$
$$x_1 = 1 \qquad x_2 = 8$$
$$y_1 = 9 \qquad y_2 = 16$$

$\therefore$ coordinate of point A and B: (1,9) and (8,16)

Use scientific calculator:

EQN 1	Degree 2	a=1, b= -9, c=8	=

ii. Sketch both graph



https://www.desmos.com/calculator for sketch the graph

iii. Area of the graph bounded by both graph:

$$A = \int_1^8 (10x - x^2)dx - \int_1^8 (x+8)dx$$

$$= \left[\frac{10x^2}{2} - \frac{x^3}{3}\right]_1^8 - \left[\frac{x^2}{2} + 8x\right]_1^8$$

$$= \left[\left(\frac{10(8)^2}{2} - \frac{(8)^3}{3}\right) - \left(\frac{10(1)^2}{2} - \frac{(1)^3}{3}\right)\right] - \left[\left(\frac{(8)^2}{2} + 8(8)\right) - \left(\frac{(1)^2}{2} + 8(1)\right)\right]$$

$$= 144.66 - 87.5$$

$$= \mathbf{57.16\ unit^2}$$

Use scientific calculator:

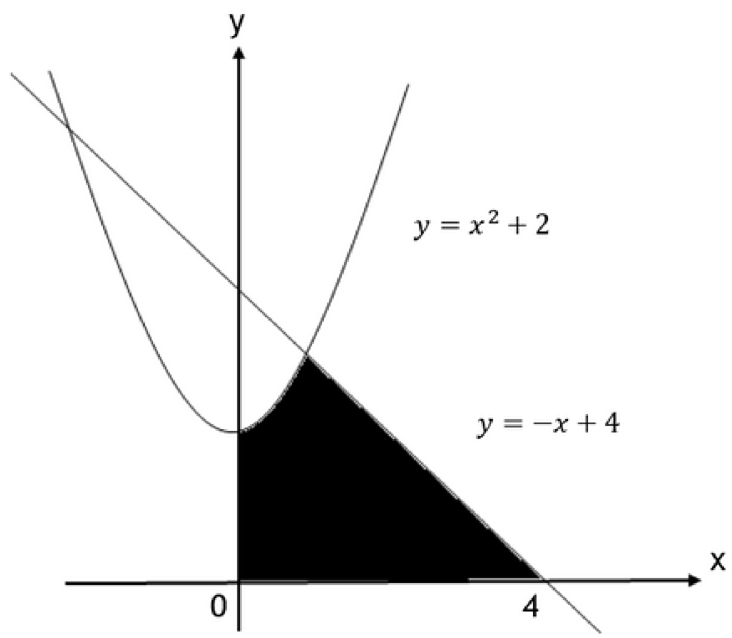
$\int dx$	$10x - x^2$	, 1 , 8	)	=

Use scientific calculator:

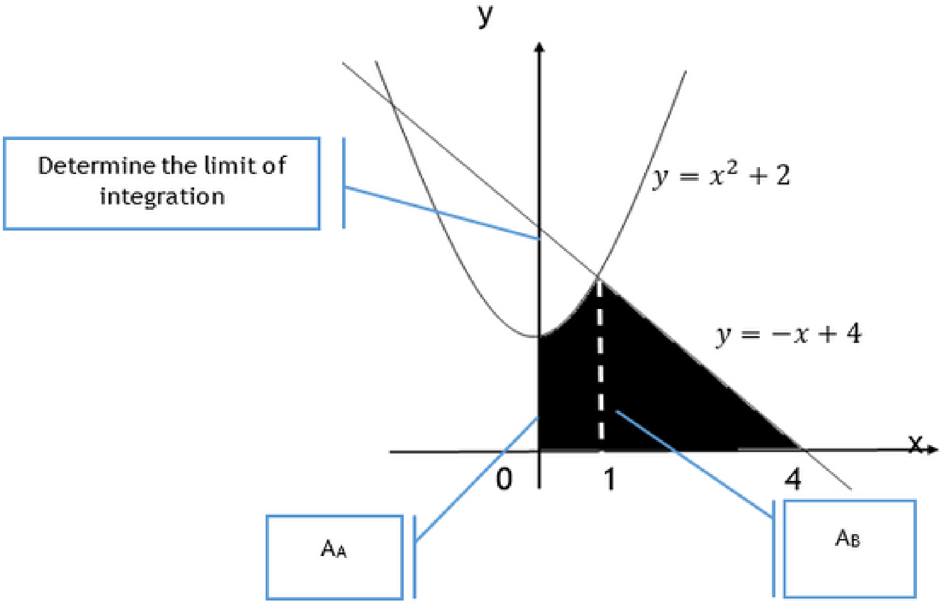
$\int dx$	$x + 8$	, 1 , 8	)	=

Example 6:

Find the area under a curve $y = x^2 + 2$ and a straight line $y = -x + 4$ between x = 0 and x = 4.



SOLUTION:



Area is between x = 0 and x = 4, so the intersection point must be $x = 1$

The bounded area, $A = A_A + A_B$

$$A_A = \int_0^1 y\,dx$$

$$= \int_0^1 x^2 + 2\,dx$$

$$= \left[\frac{x^3}{3} + 2x\right]_0^1$$

$$= \left[\frac{(1)^3}{3} + 2(1)\right] - \left[\frac{(0)^3}{3} + 2(1)\right]$$

$$= \frac{7}{3} - 0$$

$A_A = 2.3$ unit^2

Use scientific calculator:

$\int dx$	$x^2 + 2$	,0 ,1	)	=

$A_B = \int_1^4 y\, dx$

$= \int_1^4 -x + 4\, dx$

$= \left[-\frac{x^2}{2} + 4x\right]_1^4$

$= \left[-\frac{(4)^2}{2} + 4(4)\right] - \left[-\frac{(1)^2}{2} + 4(1)\right]$

$= 8 - 3.5$

$A_B = 4.5 \text{ unit}^2$

$\therefore\ A = A_A + A_B$

$A = 2.3 + 4.5$

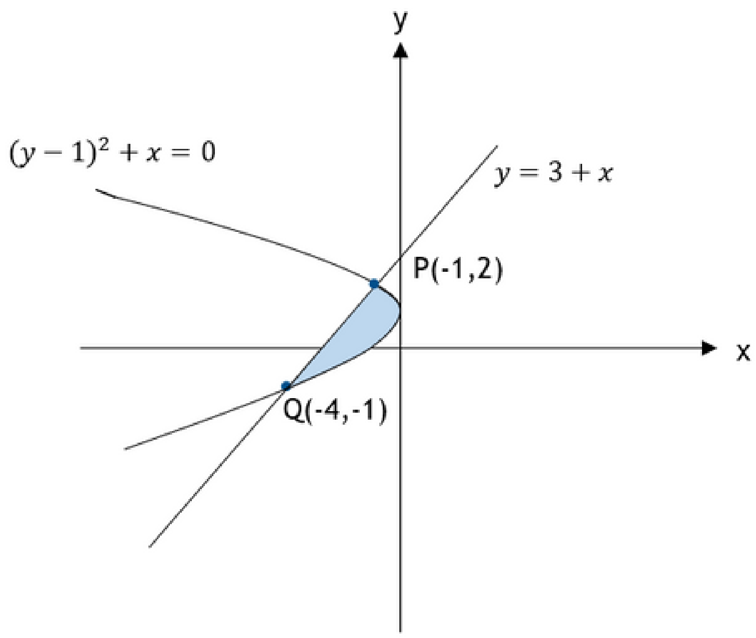
$= \mathbf{6.8\ unit^2}$

Use scientific calculator:

$\int dx$	$-x + 4$	,1 ,4	)	=

Example 7:

The diagram below shows the curve $(y-1)^2 + x = 0$ meets the straight line $y = 3 + x$ at points P and Q. Calculate the area of the shaded region.



SOLUTION:

The area of the shaded region, $A = A_{straight\ line} - A_{curve}$

$$A_{straight\ line} = \int_{-1}^{2} x\, dy$$

$$= \int_{-1}^{2} y - 3\, dy$$

Given $y = 3 + x$,

then $x = y - 3$

$$= \left[\frac{y^2}{2} - 3x\right]_{-1}^{2}$$

$$= \left[\frac{(2)^2}{2} - 3(2)\right] - \left[\frac{(-1)^2}{2} - 3(-1)\right]$$

$$= -4 - 3.5$$

$A_{straight\ line} = -7.5$ unit^2

Negative value is shown the area of the bounded region to the **left** of the y-axis

$\therefore A_{straight\ line} = 7.5$ unit^2

$$A_{curve} = \int_{-1}^{2} x\, dy$$

$$= \int_{-1}^{2} -(y-1)^2\, dy$$

Given $(y-1)^2 + x = 0$,

then $x = -(y-1)^2$

$$= \left[\frac{-(y-1)^3}{3(1)}\right]_{-1}^{2}$$

$$= \left[-\frac{(2-1)^3}{3}\right] - \left[-\frac{(-1-1)^3}{3}\right]$$

$$= -0.33 - 2.67$$

$A_{curve} = -3$ unit^2

Negative value is shown the area of the bounded region to the **left** of the y-axis

$A_{curve} = 3$ unit^2

The area of the shaded region, $A = 7.5 \text{ unit}^2 - 3 \text{ unit}^2$

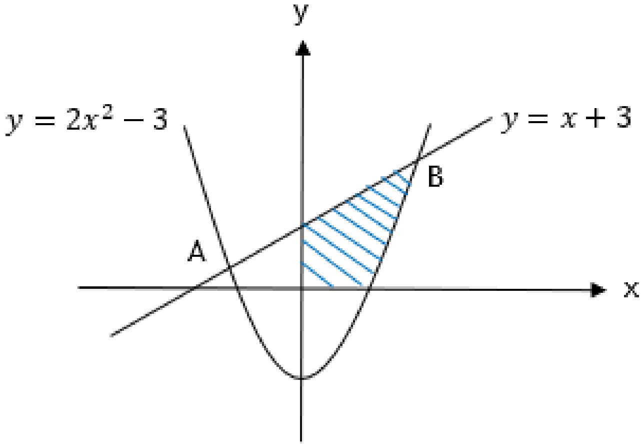
$= 4.5 \text{ unit}^2$

Use scientific calculator:				
$\int dx$	$x - 3$	,-1 ,2	)	=

Use scientific calculator:				
$\int dx$	$-(x-1)^2$	,-1 ,2	)	=

Self Assess 1.2

1. The diagram below shows a shaded region enclosed by $y = 2x^2 - 3$ and straight line $y = x + 3$:



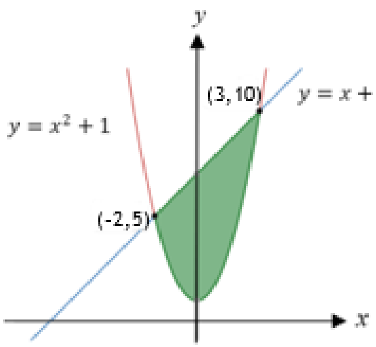
Find the:

i. Coordinate of the points A and B

ii. Area of the shaded region

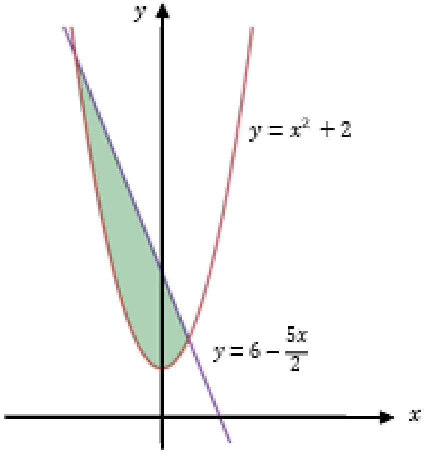
Answer: i. coordinate of $A = (-1.5, 1.5)$ and $B = (2, 5)$ ii. $A = 6.22$ unit²

2. Find the area bounded by the curve $y = x^2 + 1$ and line $y = x + 7$ at x-axis.



Answer: 20.83 unit²

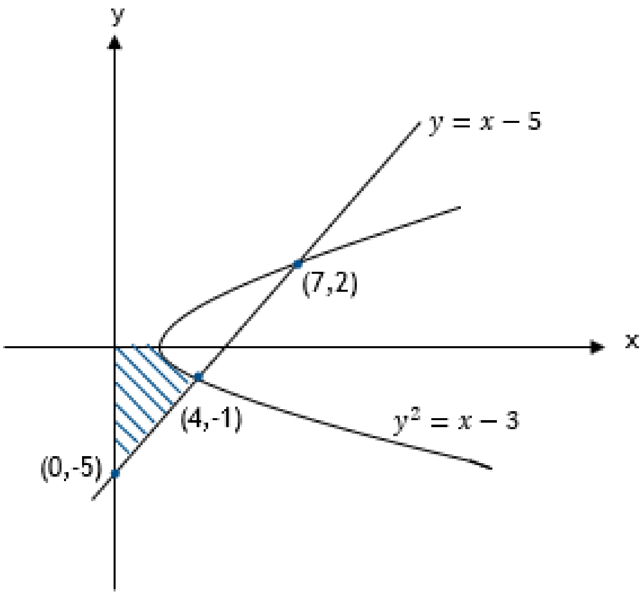
3. The line $y = 6 - \frac{5x}{2}$ intersects the curve $y = x^2 + 2$ at point P and Q.



i. Find the coordinate of P and Q.
ii. Calculate the area bounded by both graphs.

Answer: i. $P(-3.6, 15.0)$ and $Q(1.1, 3.2)$ ii. 17.49 unit^2

4. The diagram shows the region bounded by curve $y^2 = x - 3$ and straight line $y = x - 5$. Calculate the area of the bounded region by both graphs and y-axis.

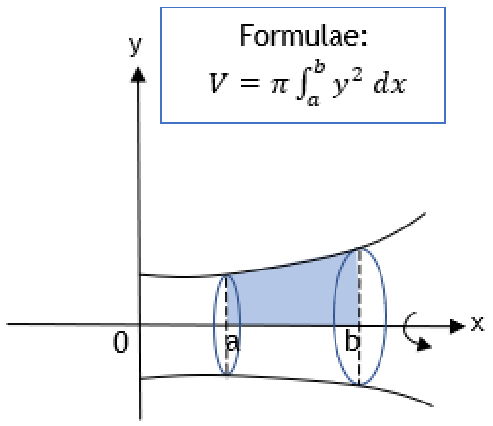


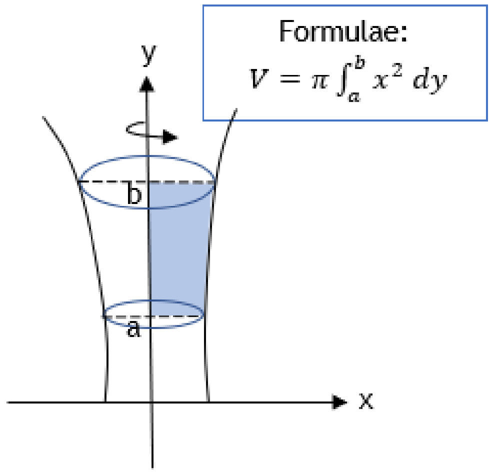
Answer: 11.33 unit^2

VOLUME OF REVOLUTION

Volume of revolution as the limit of a sum of volumes.

The volume of revolution when rotated through 360° about x-axis	The volume of revolution when rotated through 360° about y-axis





Case 1: Volume of relovution about the axis

Example 1:

Calculate the volume that generated by $y = \frac{5}{x}$, $x = 1$ and $x = 6$ when rotated 360° at the x-axis.

SOLUTION:

$$V = \pi \int_a^b y^2\, dx$$

$$= \pi \int_1^6 \left(\frac{5}{x}\right)^2 dx$$

$$= \pi \int_1^6 \frac{5^2}{x^2}\, dx$$

$$= \pi \int_1^6 25x^{-2}\, dx$$

$$= \pi \left[\frac{25x^{-1}}{-1}\right]_1^6$$

$$= \pi \left[-\frac{25}{x}\right]_1^6$$

$$= \pi \left[\left(-\frac{25}{6}\right) - \left(-\frac{25}{1}\right)\right]$$

$$= \frac{125}{6}\pi \text{ unit}^3 \text{ @ } 20.833\pi \text{ unit}^3$$

Use scientific calculator:

$\int dx$	$\left(\frac{5}{x}\right)^2$	,1 ,6	)	=

Example 2:

Find the volume of revolution, when the region bounded by the curve $y = 10 - 2x^2$, the y-axis and the line $y = 10$, is rotated completely about the y-axis.

SOLUTION:

$$V = \pi \int_a^b x^2 \, dy$$
$$= \pi \int_0^{10} 5 - \frac{y}{2} dy$$
$$= \pi \left[5y - \frac{y^2}{2(2)} \right]_0^{10}$$
$$= \pi \left[5y - \frac{y^2}{4} \right]_0^{10}$$
$$= \pi \left[\left(5(10) - \frac{(10)^2}{4} \right) - \left(5(0) - \frac{(0)^2}{4} \right) \right]$$
$= 25\pi$ **unit³**

Given $y = 10 - 2x^2$, make x^2 as a subject,

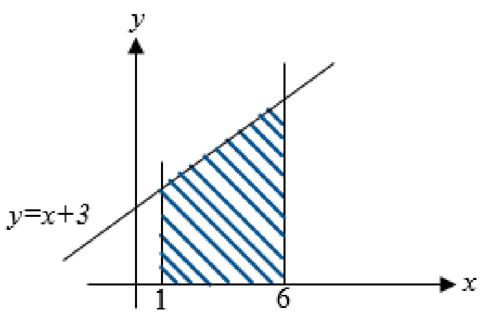
then $x^2 = 5 - \frac{y}{2}$

Use scientific calculator:

$\int dx$	$5 - \left(\frac{x}{2}\right)$	,0 ,10	)	=

Example 3:

The diagram shows the region bounded by the straight lines $y = x + 3$, $x = 1$, $x = 6$ and the x-axis. Find the volume generated in terms of π, when this region is rotated 360° about the x-axis.



SOLUTION:

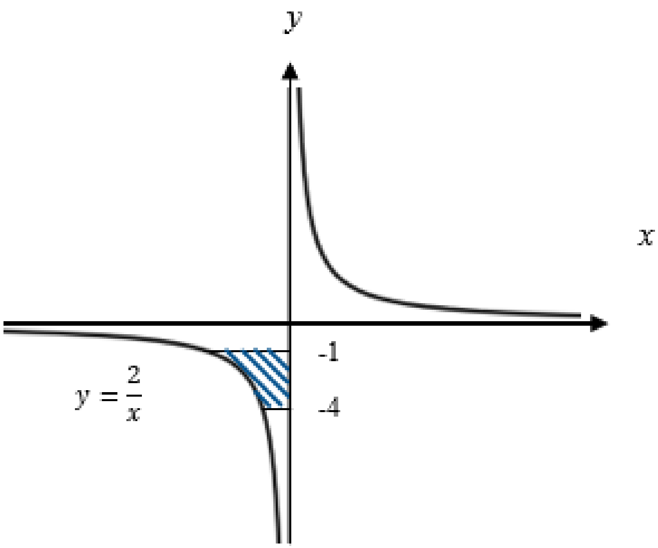
$$V = \pi \int_a^b y^2 \, dx$$
$$= \pi \int_1^6 (x+3)^2 dx$$
$$= \pi \left[\frac{(x+3)^3}{3} \right]_1^6$$
$$= \pi \left[\frac{(6+3)^3}{3} - \frac{(1+3)^3}{3} \right]$$
$= 221.667\pi$ **unit³**

Use scientific calculator:

$\int dx$	$x^2 + 6x + 9$	,1 ,6	)	=

Example 4:

The diagram shows the region bounded by the curve $y = \frac{2}{x}$, the lines $y = -1$, $y = -4$ and the y-axis. Find the volume generated in terms of π, when this region is rotated 360° about the y-axis.



SOLUTION:

$$V = \pi \int_a^b x^2 \, dy$$

$$= \pi \int_{-4}^{-1} \left(\frac{2}{y}\right)^2 dy$$

$$= \pi \int_{-4}^{-1} \frac{4}{y^2} dy$$

$$= \pi \left[-\frac{4}{y}\right]_{-4}^{-1}$$

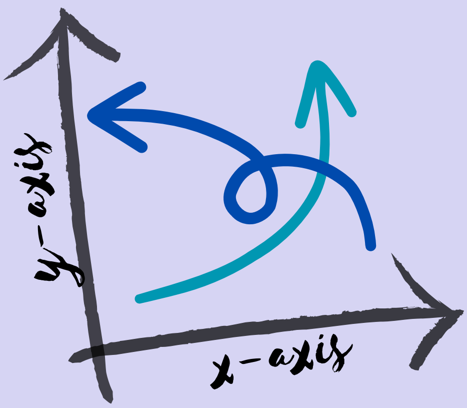
$$= \pi \left[-\frac{4}{-1} - \left(-\frac{4}{-4}\right)\right]$$

$$= \pi[4 - 1]$$

$= 3\pi$ **unit³**

Use scientific calculator:

$\int dx$	$4x^{-2}$	,-4,-1	)	=



Self Assess 2.1

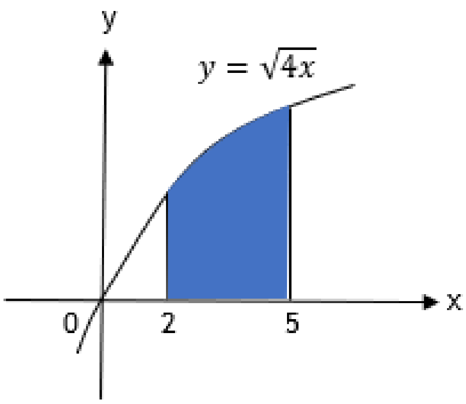
1. Calculate the generated volume between the curve $y = 5x - x^2$, x-axis, x = 3 and x = 5.

Answer: 33.06π unit³

2. Find the volume that generate by $y = 7 - x^2$, y = 0, y = 7 when rotated 360° about the y-axis.

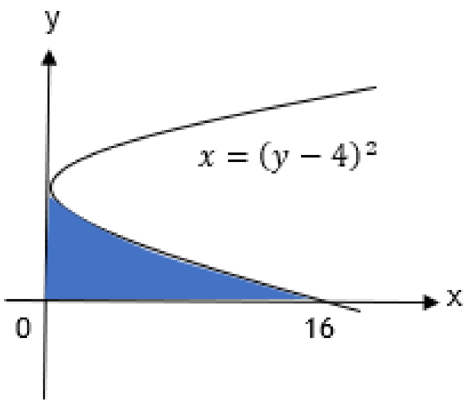
Answer: $24\frac{1}{2}\pi$ unit³

3. Find the volume generated for the diagram below when the shaded region is revolved through 360° about the x-axis.



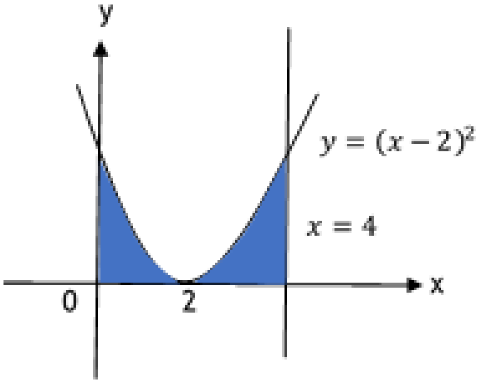
Answer: 42π unit³

4. Find the volume generated for the diagram below when the shaded region is revolved through 360° about the y-axis.



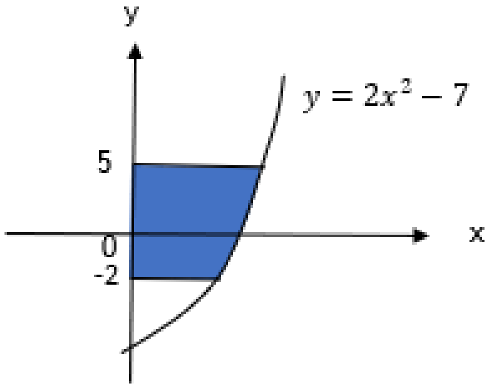
Answer: 204.8π unit³

5. Find the volume generated for the diagram below when the shaded region is revolved through 360° about the x-axis.



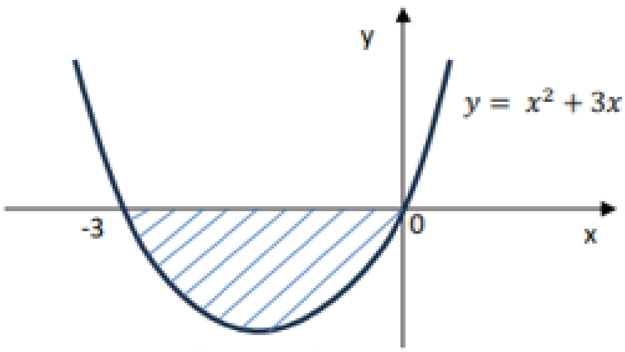
Answer: $\frac{64}{5}\pi$ unit^3

6. Find the volume generated for the diagram below when the shaded region is revolved through 360° about the y-axis.



Answer: 29.75π unit^3

7. The figure below shows the graph $y = x^2 + 3x$ between $x = -3$ and $x = 0$. Calculate the volume when the shaded region is rotated 360° about x- axis.

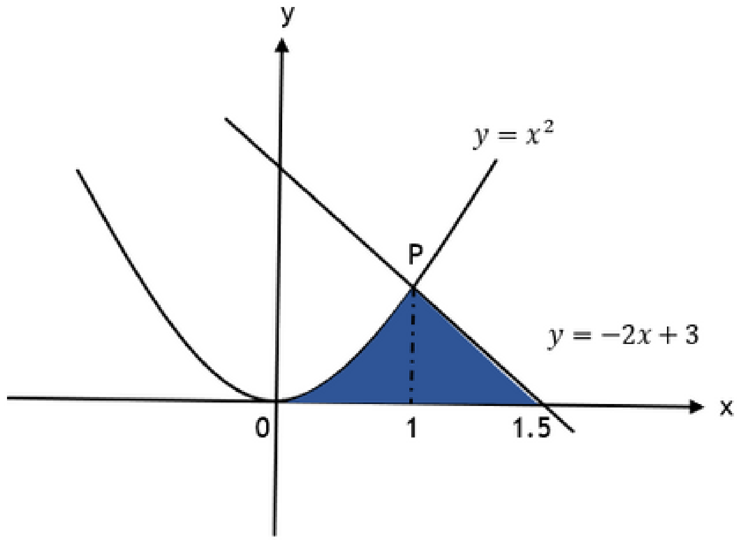


Answer: 8.1π unit^3

Case 2: Volume of revolution of the Region between a Curve and a Straight Line

Example 5:

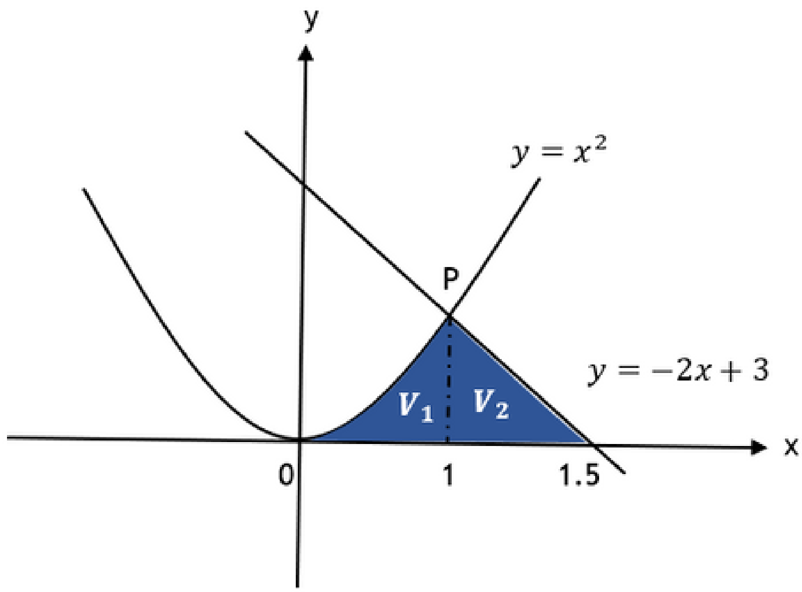
Diagram below shows a region which is enclosed by a curve $y = x^2$ and a line $y = -2x + 3$ meets at point P. Find the value of the volume that is obtained when the shaded region is rotated 360° at x-axis.



SOLUTION:

$$V = V_1 + V_2$$

$$\begin{aligned} V_1 &= \pi \int_a^b y^2 \, dx \\ &= \pi \int_0^1 (x^2)^2 dx \\ &= \pi \int_0^1 x^4 dx \\ &= \pi \left[\frac{x^5}{5}\right]_0^1 \\ &= \pi \left[\left(\frac{1^5}{5}\right) - \left(\frac{0^5}{5}\right)\right] \\ &= \frac{1}{5}\pi \text{ unit}^3 \end{aligned}$$



$$V_2 = \pi \int_a^b y^2 \, dx$$

$$= \pi \int_1^{1.5} (-2x+3)^2 \; dx$$

$$= \pi \int_1^{1.5} 4x^2 - 12\text{x} + 9 \; dx$$

$$= \pi \left[\frac{4x^3}{3} - \frac{12x^2}{2} + 9x \right]_1^{1.5}$$

$$= \pi \left[\left(\frac{4(1.5)^3}{3} - 6(1.5)^2 + 9(1.5) \right) - \left(\frac{4(1)^3}{3} - 6(1)^2 + 9(1) \right) \right]$$

$$= \pi \left(\frac{9}{2} - \frac{13}{3} \right)$$

$$= \frac{1}{6}\pi \text{ unit}^3$$

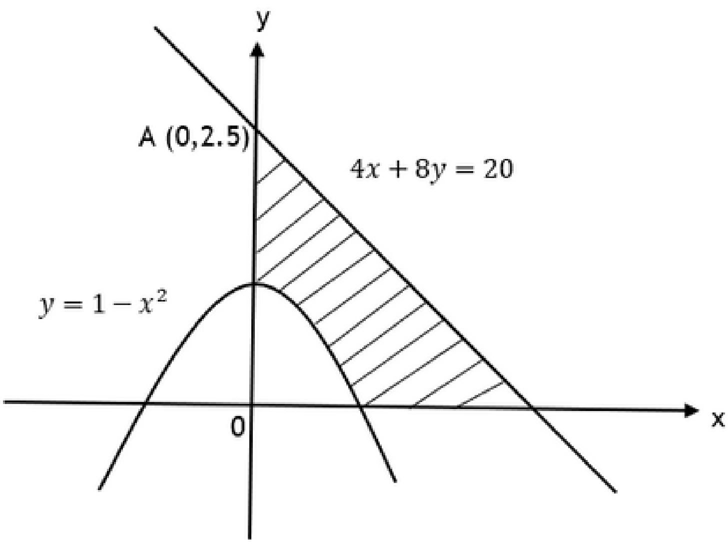
$$\therefore \; V = V_1 + V_2$$

$$= \frac{1}{5}\pi \text{ unit}^3 + \frac{1}{6}\pi \text{ unit}^3$$

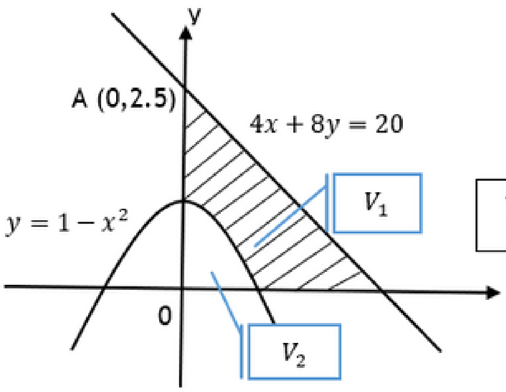
$$= \frac{11}{30}\pi \text{ unit}^3$$

Example 6:

The above diagram shows part of the curve $y = 1 - x^2$ and the straight line $4x + 8y = 20$. The straight line meets y-axis at point A. Find the volume generated when the shaded region is revolved through 360° about the y-axis.

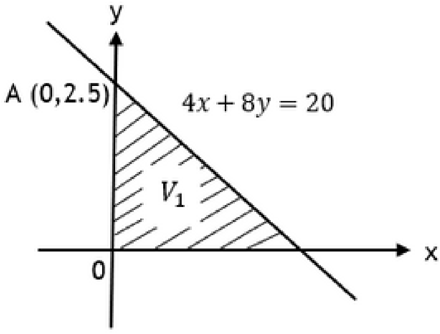


SOLUTION:

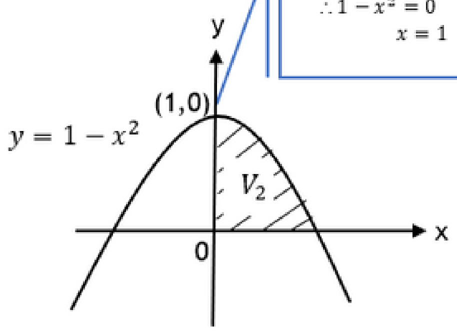


$$Volume, V = V_1 - V_2$$

$$V = V_1 - V_2$$



Find value of x at y-axis, when y = 0
$\therefore 1 - x^2 = 0$
$x = 1$



$$V_1 = \pi \int_a^b x^2 \, dy$$

$$= \pi \int_0^{2.5} (5 - 2y)^2 dy$$

Given $4x + 8y = 20$
make **x** as a subject, then $x = 5 - 2y$

$$= \pi \int_0^{2.5} 4y^2 - 20y + 25 \, dy$$

$$= \pi \left[\frac{4y^3}{3} - \frac{20y^2}{2} + 25x \right]_0^{2.5}$$

$$= \pi \left[\left(\frac{4(2.5)^3}{3} - 10(2.5)^2 + 25(2.5) \right) - \left(\frac{4(0)^3}{3} - 10(0)^2 + 25(0) \right) \right]$$

$$= 20.833\pi \text{ unit}^3$$

$$V_2 = \pi \int_a^b x^2 \, dy$$

$$= \pi \int_0^1 1 - y \, dy$$

Given $y = 1 - x^2$
make x^2 as a subject, then $x^2 = 1 - y$

$$= \pi \left[y - \frac{y^2}{2} \right]_0^1$$

$$= \pi \left[\left(1 - \frac{(1)^2}{2} \right) - \left(0 - \frac{(0)^2}{2} \right) \right]$$

$$= \frac{1}{2}\pi \text{ unit}^3 \text{ @ } 0.5\pi \text{ unit}^3$$

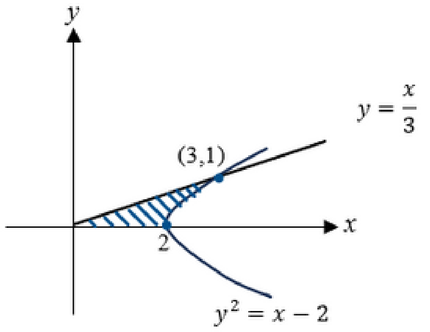
$$\therefore V = V_1 - V_2$$

$$= 20.833\pi \text{ unit}^3 - 0.5\pi \text{ unit}^3$$

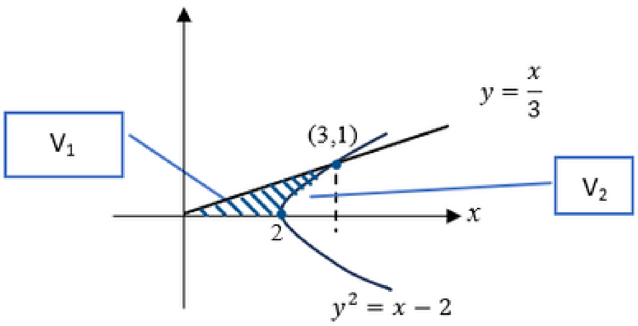
$$= \mathbf{20.333\pi \text{ unit}^3}$$

Example 7:

In the diagram below, the straight line $y = \frac{x}{3}$ is a tangent to the curve $y^2 = x - 2$ at the point (3,1). Find the volume generated when shaded region is rotated completely about the x-axis.



SOLUTION:



$$V = V_1 - V_2$$

$$V_1 = \pi \int_a^b y^2\, dx$$

$$= \pi \int_0^3 \left(\frac{x}{3}\right)^2 dx$$

$$= \pi \int_0^3 \frac{x^2}{9} dx$$

$$= \pi \left[\frac{x^3}{27}\right]_0^3$$

$$= \pi \left[\frac{27}{27} - \frac{0}{27}\right]$$

$$= \pi \text{ unit}^3$$

$$V_2 = \pi \int_a^b y^2\, dx$$

$$= \pi \int_2^3 (x-2) dx$$

$$= \pi \left[\frac{x^2}{2} - 2x\right]_2^3$$

$$= \pi \left[\left(\frac{9}{2} - 6\right) - \left(\frac{4}{2} - 4\right)\right]$$

$$= \pi \left[-\frac{3}{2} - (-2)\right]$$

$$= \frac{\pi}{2} \text{ unit}^3$$

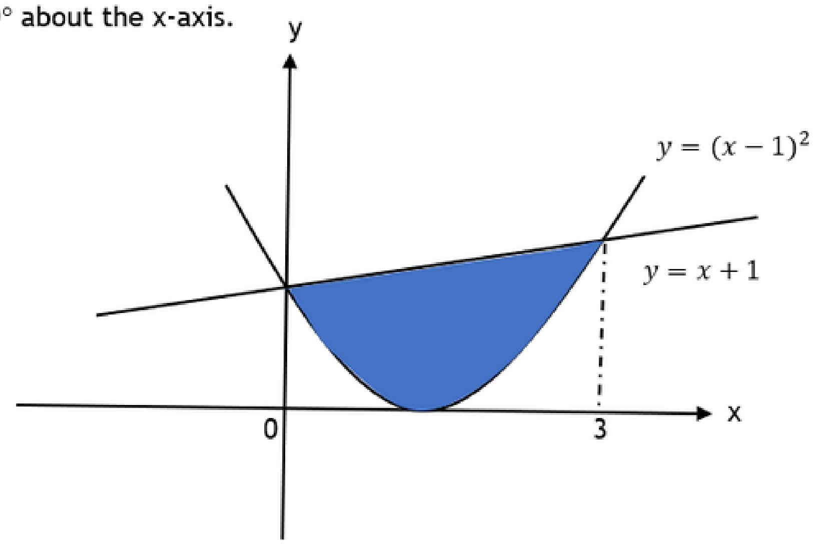
$$\therefore\ V = V_1 - V_2$$

$$= \pi \text{ unit}^3 - \frac{\pi}{2} \text{ unit}^3$$

$$= \frac{\pi}{2} \text{ unit}^3$$

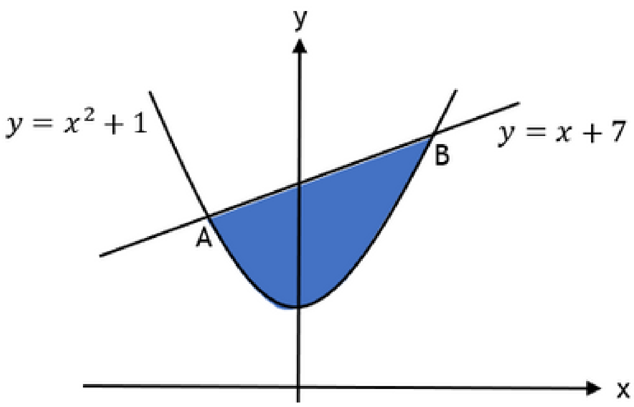
Self Assess 2.2

1. Diagram below shows a region which is enclosed by graph of $y = x + 1$ and $y = (x-1)^2$. Compute the volume of solid revolution formed when the shaded region is rotated 360° about the x-axis.



Answer: $45.24\,\pi$ unit³

2. The diagram below shows a shaded region enclosed by $y = x^2 + 1$ and straight line $y = x + 7$.



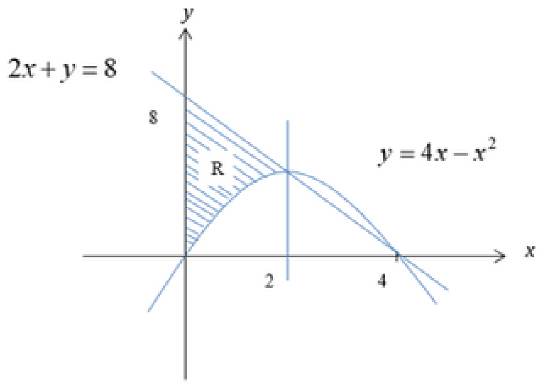
Find the:

i. coordinate of the points A and B

ii. the volume of the solid formed when the shaded region is bounded by the curve $y = x^2 + 1$ and line $y = x + 7$ is rotated 360° about the x-axis.

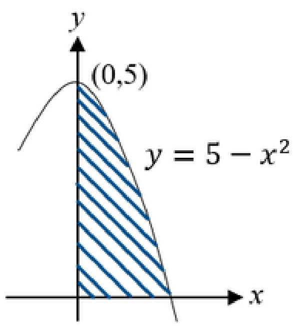
Answer: i. A (-2,5) and B (3,10) ii. $\frac{625}{3}\,\pi$ unit³

3. Figure 6 shows a region bounded by the curve $y = 4x - x^2$, and the line $2x + y = 8$. Determine the volume generated when the region R is rotated through 360° about the x-axis.



Answer: $\frac{288\pi}{5} unit^3$

4. Refer to figure below, calculate the generated volume when this shaded region is rotated 360° around the y-axis.



Answer: $\frac{25}{2}\pi$ unit^3

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