TUANKU SULTANAH BAHIYAH

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KOLEJ KOMUNITI MALAYSIA

Easy Technique

RLC Series & Parallel AC Circuits

vol.1

Complex Number Method

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Easy Technique RLC Series and Parallel AC Circuits: Complex Number Method

Synopsis

This book contains two main topics that focus on the method of calculating series and parallel circuits using the complex number. The content is based on the latest syllabus for the DET20033 course, where the survey found that students are less proficient in using calculators to calculate impedance, current and voltage values using this method. Findings reveal that an approach to continuous training on how to use calculators can help improve student’s skills to master this topic. This book will benefit students especially those taking the DET20033 course. This book is also suitable for general readers who interested in circuit calculations using the complex number method.

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TOPIC: 1

INTRODUCTION OF BASIC AC CIRCUITS

INTRODUCTION OF BASIC AC CIRCUITS

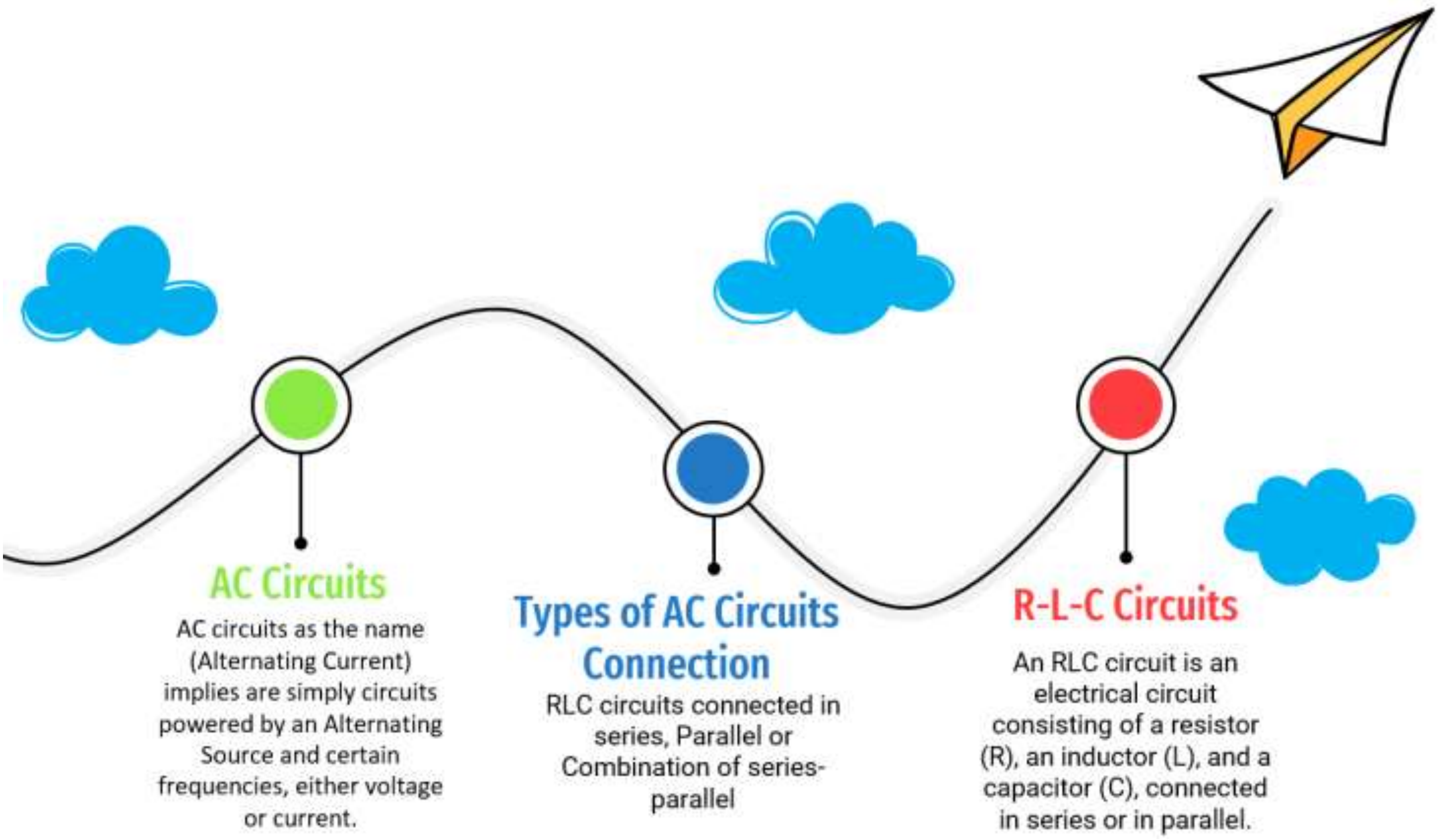


Figure 1.1: Basic AC Circuit Introduction line map

TOPIC: 2

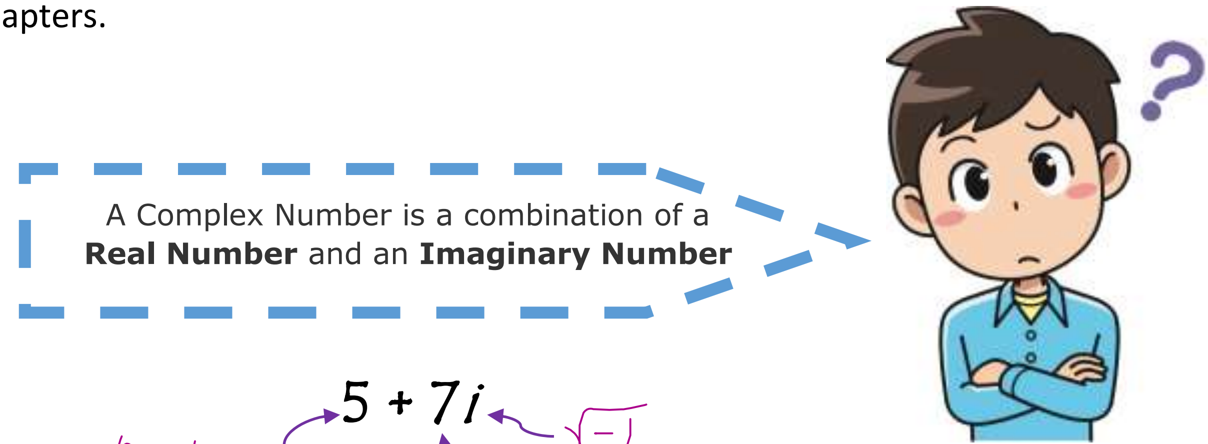
COMPLEX NUMBERS

COMPLEX NUMBERS

Complex numbers are used in many scientific fields, including engineering, electromagnetism, quantum physics and applied mathematics, such as chaos theory. Any physical motion which is periodic, such as an oscillating beam, string, wire, pendulum, electronic signal or electromagnetic wave can be represented by a complex number function.

This can make calculations with the various components simpler than with real numbers and sines and cosines. In control theory, systems are often transformed from the time domain to the frequency domain using the Laplace transform. In fluid dynamics, complex functions are used to describe potential flow in two dimensions. In electrical engineering, the Fourier transform is used to analyse varying voltages and currents. Complex numbers are used in signal analysis and other fields for a convenient description for periodically varying signals.

This use is also extended into digital signal processing and digital image processing, which utilize digital versions of Fourier analysis (and wavelet analysis) to transmit, compress, restore and otherwise process digital audio signals, still images and video signals. Knowledge of complex numbers is clearly absolutely essential for further studies in so many engineering disciplines and is used extensively in many of the ensuing chapters.



$$5 + 7i$$

Real Number

Imaginary Number

$\sqrt{-1}$

Imaginary Numbers when **squared** give a **negative** result.

Normally this doesn't happen, because:

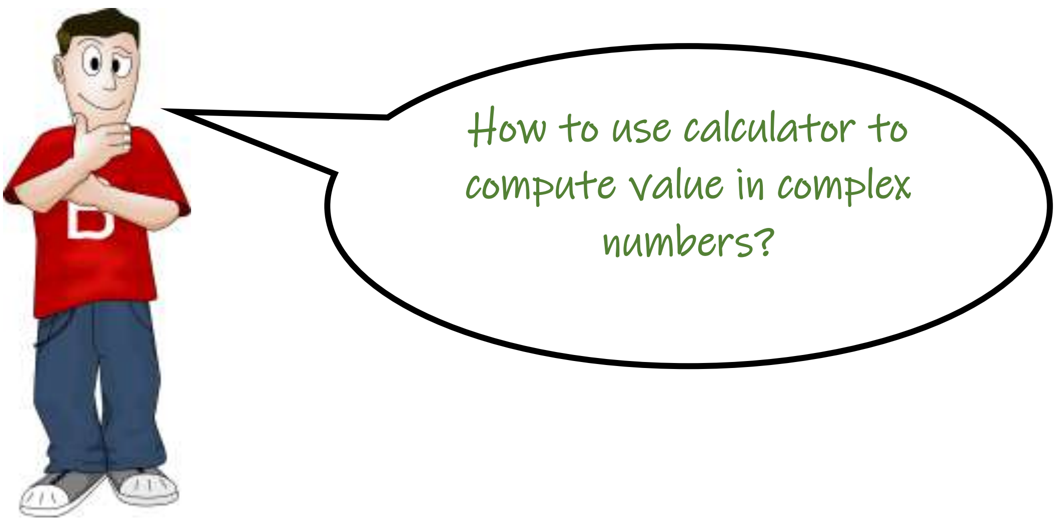
- ✓ when we square a positive number we get a positive result, and
- ✓ when we square a negative number we also get a positive result (because a negative time a negative gives a positive), for example **−2 × −2 = +4**

So, a Complex Number has a real part and an imaginary part.

But either part can be **0**, so all Real Numbers and Imaginary Numbers are also Complex Numbers.

Table 2.1: Complex Number Table

Complex Number	**Real Part**	**Imaginary Part**	
3 + 2i	3	2	
5	5	0	Purely Real
−6i	0	−6	Purely Imaginary



TOPIC: 3

BASIC SERIES R-L, R-C AND R-L-C CIRCUITS

BASIC SERIES R-L, R-C AND RLC CIRCUITS

The whole of electronics components can be slip into two broad categories, one being the Active components and the other as Passive components. The Passive components include the Resistor (R), Capacitor (C) and the Inductor (L). These are the three most used components in electronics circuit and you will find them in almost every application circuit.

These three components together in different combinations will form the RC, RL and RLC circuits and they have many applications like from filtering circuits, Tube light chokes, multi vibrators etc. So, in this tutorial we will learn the basic of these circuits, the theory behind them and how to use them in our circuits. Before we jump into the main topics lets understand what an R, L and C does in a circuit.

Resistor: Resistors are denoted by the letter “R”. A resistor is an element that dissipates energy mostly in form of heat. It will have a Voltage drop across it which remains fixed for a fixed value of current flowing through it.

Capacitor: Capacitors are denoted by the letter “C”. A capacitor is an element which stores energy (temporarily) in form of electric field. Capacitor resists changes in voltage. There are many types of capacitors, out of which the ceramic capacitor and the electrolytic capacitors are mostly used. They charge in one direction and discharge in opposite direction

Inductor: Inductors are denoted by the letter “L”. A Inductor is also similar to capacitor, it also stores energy but is stored in form of magnetic field. Inductors resist changes current. Inductors are normally a coil wound wire and is rarely used compared to the former two components.

When these Resistor, Capacitor and Inductors are put together we can form circuits like RC, RL and RLC circuit which exhibits time and frequency dependent responses that will e useful in many AC applications as mentioned already. A RC/RL/RLC circuit can be used as a filter, oscillator and much more.

Basic Principle of RC/RL and RLC circuits

Before we start with each topic let us understand how a Resistor, Capacitor and an Inductor behave in an electronic circuit. For the purpose of understanding let us consider a simple circuit consisting of a capacitor and resistor in series with a power supply (5V).

In this case when the power supply is connected to the RC pair, the voltage across the Resistor (Vr) increase to its maximum value while the voltage across the capacitor (Vc) stays at zero, then slowly the capacitor starts to build charge and thus the voltage across the resistor will decrease and the voltage across the capacitor will increase until the resistor voltage (Vr) has reached Zero and Capacitor voltage (Vc) has reached its maximum value.

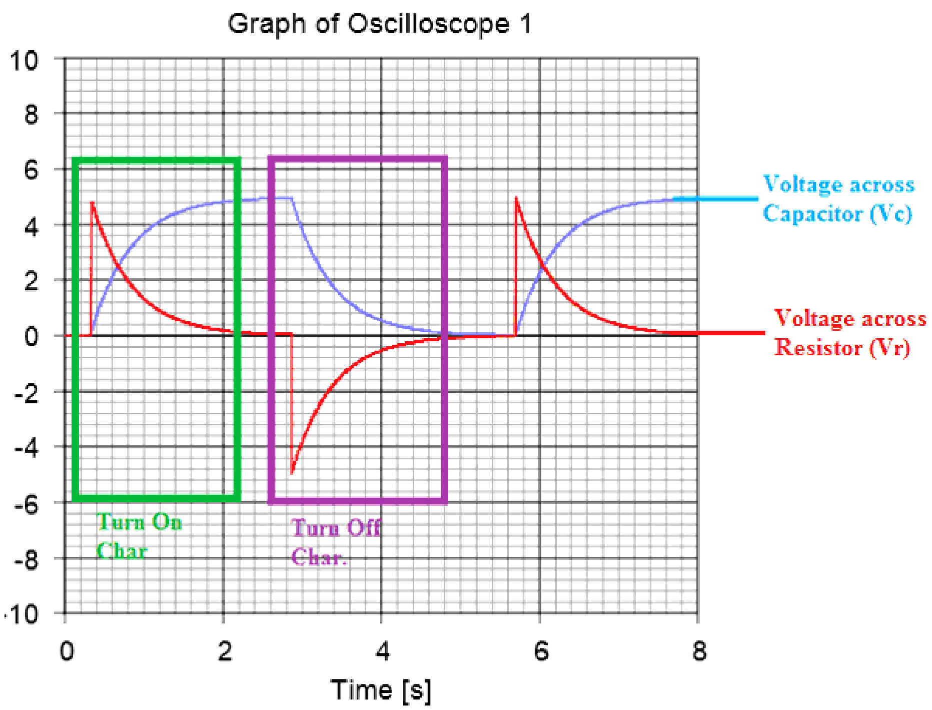


Figure 3.1: Characteristic of Voltage across capacitor and resistor

When the switch is turned on the voltage across the resistor (red wave) reaches its maximum and the voltage across capacitor (blue wave) remains at zero. Then the capacitor charges up and Vr becomes zero and Vc becomes maximum. Similarly, when the switch is turned off capacitor discharges and hence the negative voltage appears across the Resistor and as the capacitor discharges both the capacitor and resistor voltage become zero as shown above.

The same can be visualized for inductors as well. Replace the capacitor with an inductor and the waveform will just be mirrored, that is the voltage across the resistor (Vr) will be zero when the switch is turned on since the whole voltage will appear across the Inductor (Vl). As the inductor charges up the voltage across (Vl) it will reach zero and the voltage across the resistor (Vr) will reach the maximum voltage.

RC series circuit

The RC circuit (Resistor Capacitor Circuit) will consist of a Capacitor and a Resistor connected either in series or parallel to a voltage or current source. These types of circuits are also called as RC filters or RC networks since they are most commonly used in filtering applications.

An RC circuit can be used to make some crude filters like low-pass, high-pass and Band-Pass filters. A first order RC circuit will consist of only one Resistor and one Capacitor.

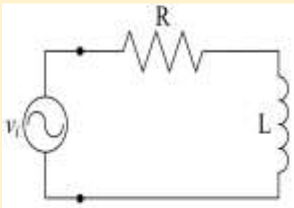
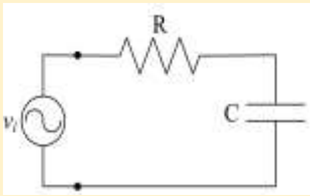
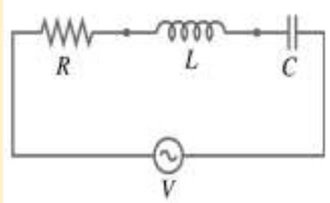
RL series circuit

The RL Circuit (Resistor Inductor Circuit) will consist of an Inductor and a Resistor again connected either in series or parallel. A series RL circuit will be driven by voltage source and a parallel RL circuit will be driven by a current source. RL circuit are commonly used in as passive filters, a first order RL circuit with only one inductor and one capacitor.

RLC series Circuit

A RLC circuit as the name implies will consist of a Resistor, Capacitor and Inductor connected in series or parallel. The circuit forms an Oscillator circuit which is very commonly used in Radio receivers and televisions. It is also very commonly used as damper circuits in analogue applications.

Table 3.2: R-L-C Diagram and Characteristic Table

Circuits Types	**R-L**	**R-C**	**R-L-C**
Circuit Diagram			
Impedance , Z Formula	**Z = R+jXL** Where, *j* = *i* (imaginary)	**Z = R-jXC** Where, *j* = *i* (imaginary)	**Z = R+jXL-jXC** Where, *j* = *i* (imaginary)
Current, I Formula	**I = V∟Ø°/Z∟Ø°**		
Voltage Drop Formula	**VR** = I∟Ø° x R∟0° **VL** = I∟Ø° x XL∟90°	**VR** = I∟Ø° x R∟0° **VC** = I∟Ø° x XC∟-90°	**VR** = I∟Ø° x R∟0° **VL** = I∟Ø° x XL∟90° **VC** = I∟Ø° x XC∟-90°

TOPIC: 4

BASIC PARALLEL R-L, R-C AND R-L-C CIRCUITS

BASIC PARALLEL R-L, R-C AND RLC CIRCUITS

The **Parallel RLC Circuit** is the exact opposite to the series circuit we looked at in the previous tutorial although some of the previous concepts and equations still apply. However, the analysis of a *parallel RLC circuits* can be a little more mathematically difficult than series RLC circuits so in this tutorial about parallel RLC circuits only pure components are assumed to keep things simple.

This time instead of the current being common to the circuit components, the applied voltage is now common to all so we need to find the individual branch currents through each element. The total impedance, Z of a parallel RLC circuit is calculated using the current of the circuit similar to that for a DC parallel circuit, the difference this time is that admittance is used instead of impedance. Consider the parallel RLC circuit below.

Parallel RLC Circuit

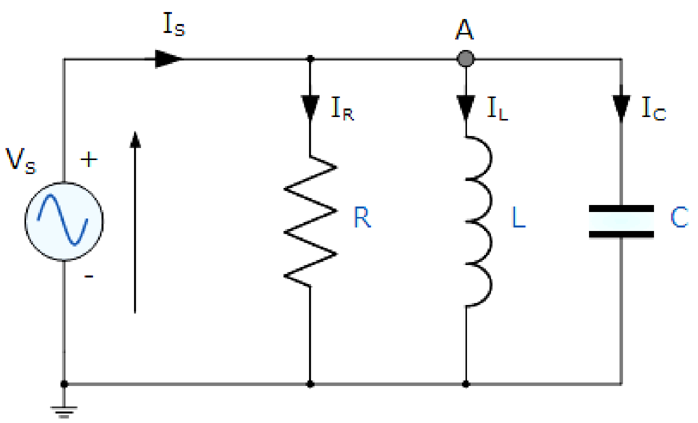


Figure 4.1: RLC Parallel Circuit Diagram

In the above parallel RLC circuit, we can see that the supply voltage, V_S is common to all three components whilst the supply current I_S consisting of three parts. The current flowing through the resistor, I_R, the current flowing through the inductor, I_L and the current through the capacitor, I_C.

But the current flowing through each branch and therefore each component will be different to each other and also to the supply current, I_S.

The total current drawn from the supply will not be the mathematical sum of the three individual branch currents but their vector sum.

Like the series RLC circuit, we can solve this circuit using the phasor or vector method but this time the vector diagram will have the voltage as its reference with the three current vectors plotted with respect to the voltage. The phasor diagram for a parallel RLC circuit is produced by combining together the three individual phasors for each component and adding the currents vectorially.

Since the voltage across the circuit is common to all three circuit elements we can use this as the reference vector with the three current vectors drawn relative to this at their corresponding angles. The resulting vector current I_S is obtained by adding together two of the vectors, I_L and I_C and then adding this sum to the remaining vector I_R. The resulting angle obtained between V and I_S will be the circuits phase angle as shown below.

Phasor Diagram for a Parallel RLC Circuit

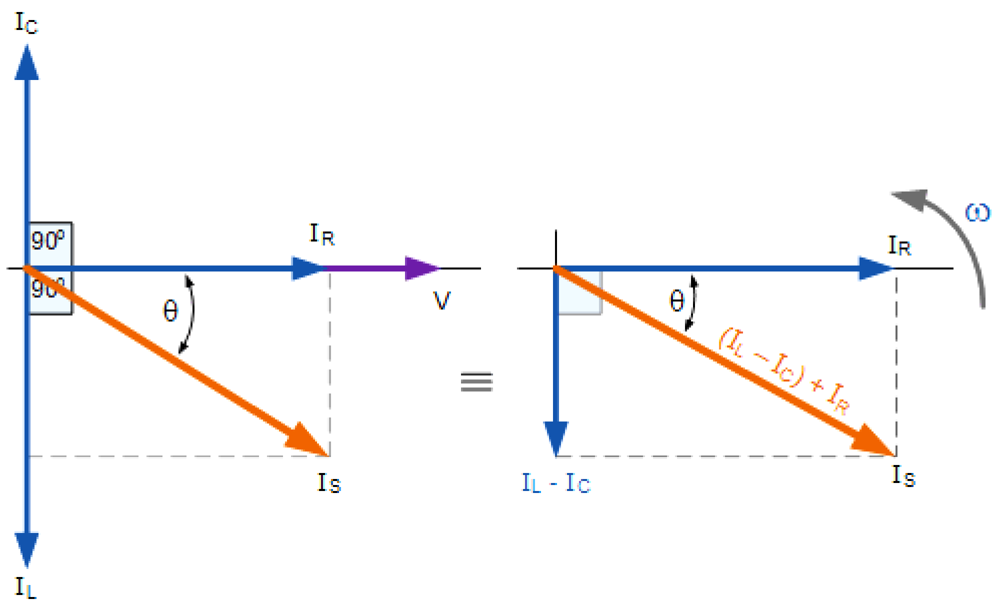


Figure 4.2: Phasor Diagram for a Parallel RLC Circuit

We can see from the phasor diagram on the right hand side above that the current vectors produce a rectangular triangle, comprising of hypotenuse I_S, horizontal axis I_R and vertical axis $I_L - I_C$ Hopefully you will notice then, that this forms a Current Triangle. We can therefore use Pythagoras's theorem on this current triangle to mathematically obtain the individual magnitudes of the branch currents along the x-axis and y-axis which will determine the total supply current I_S of these components as shown.

Current Triangle for a Parallel RLC Circuit

$$I_S^2 = I_R^2 + \left(I_L - I_C\right)^2$$

$$I_S = \sqrt{I_R^2 + \left(I_L - I_C\right)^2}$$

$$\therefore \quad I_S = \sqrt{\left(\frac{V}{R}\right)^2 + \left(\frac{V}{X_L} - \frac{V}{X_C}\right)^2} = \frac{V}{Z}$$

$$\text{where:} \quad I_R = \frac{V}{R}, \quad I_L = \frac{V}{X_L}, \quad I_C = \frac{V}{X_C}$$

Since the voltage across the circuit is common to all three circuit elements, the current through each branch can be found using Kirchhoff's Current Law, (KCL). Remember that Kirchhoff's current law or junction law states that "the total current entering a junction or node is exactly equal to the current leaving that node". Thus, the currents entering and leaving node "A" above are given as:

$$\text{KCL: } I_S - I_R - I_L - I_C = 0$$

$$I_S - \frac{V}{R} - \frac{1}{L}\int v\,dt - C\frac{dv}{dt} = 0$$

Taking the derivative, dividing through the above equation by C and then re-arranging gives us the following Second-order equation for the circuit current. It becomes a second-order equation because there are two reactive elements in the circuit, the inductor and the capacitor.

$$I_S - \frac{d^2V}{dt^2} - \frac{dV}{RCdt} - \frac{V}{LC} = 0$$

$$\therefore\ I_{S(t)} = \frac{d^2V}{dt^2} + \frac{dV}{dt}\frac{1}{RC} + \frac{1}{LC}V$$

The opposition to current flow in this type of AC circuit is made up of three components: X_L X_C and R with the combination of these three values giving the circuits impedance, Z. We know from above that the voltage has the same amplitude and phase in all the components of a parallel RLC circuit. Then the impedance across each component can also be described mathematically according to the current flowing through, and the voltage across each element as.

Impedance of a Parallel RLC Circuit

$$R = \frac{V}{I_R} \qquad X_L = \frac{V}{I_L} \qquad X_C = \frac{V}{I_C}$$

$$Z = \frac{1}{\sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{X_L} - \frac{1}{X_C}\right)^2}}$$

$$\therefore \frac{1}{Z} = \sqrt{\left(\frac{1}{R}\right)^2 + \left(\frac{1}{X_L} - \frac{1}{X_C}\right)^2}$$

Applications

The Resistors, Inductors and Capacitors may be normal and simple components but when they are combined to gather to form circuits like RC/RL and RLC circuit they exhibit complex behaviour which makes it suitable for a wide range of application.

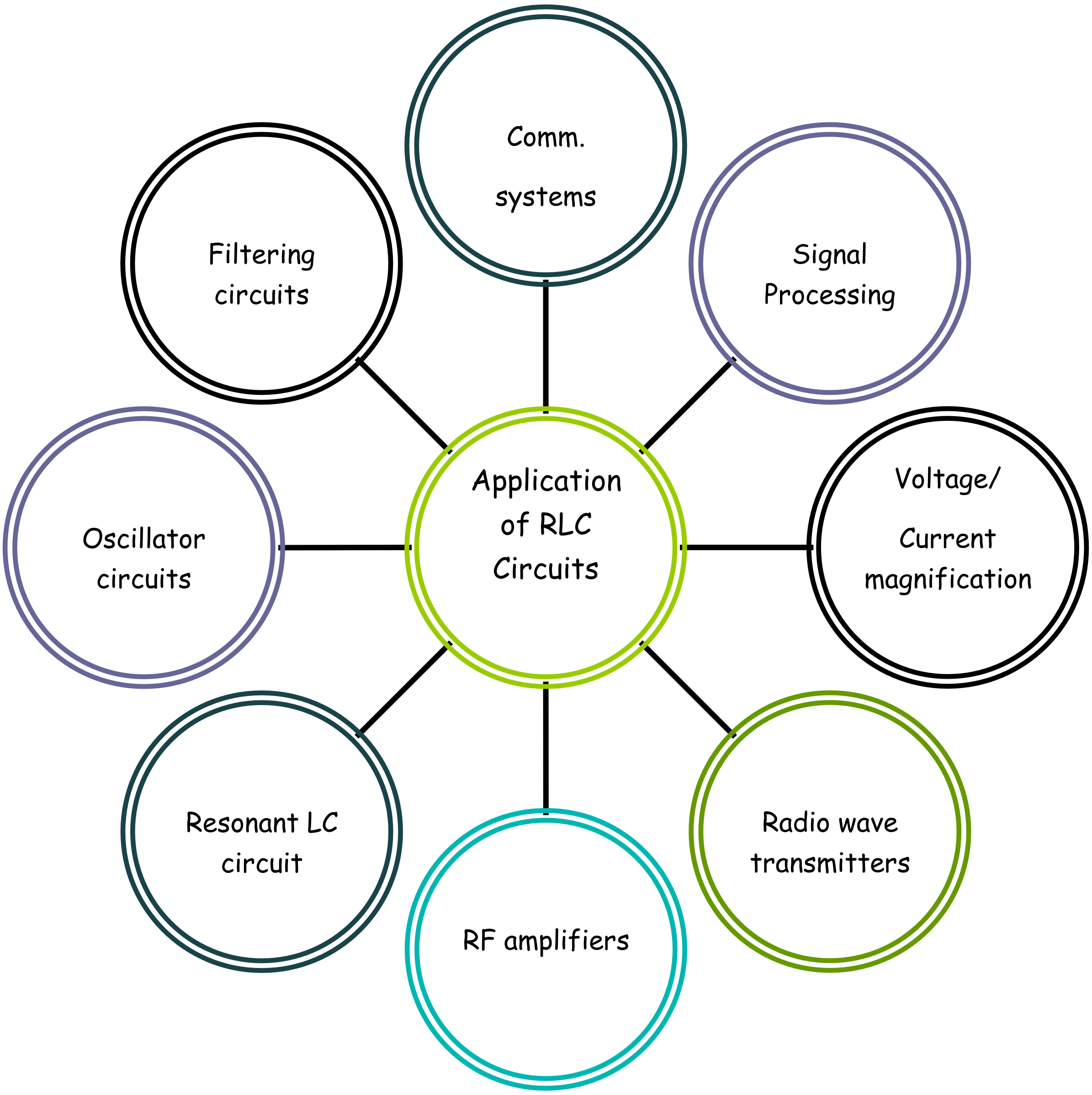


Figure 4.3: Application of RLC Circuits

OHM'S LAW IN R-L-C CIRCUITS

Ohm's law∗ states that the current I flowing in a circuit is directly proportional to the applied voltage V and inversely proportional to the resistance R, provided the temperature remains constant.

Ohm’s Law can be used to validate the static values of circuit components, current levels, voltage supplies, and voltage drops. If, for example, a test instrument detects a higher than normal current measurement, it could mean that resistance has decreased or that voltage has increased, causing a high-voltage situation. This could indicate a supply or circuit issue.

In direct current (dc) circuits, a lower than normal current measurement could mean that the voltage has decreased, or circuit resistance has increased. Possible causes for increased resistance are poor or loose connections, corrosion and/or damaged components.

Loads within a circuit draw on electrical current. Loads can be any sort of component: small electrical devices, computers, household appliances or a large motor. Most of these components (loads) have a nameplate or informational sticker attached. These nameplates provide safety certification and multiple reference numbers.

Technicians refer to nameplates on components to learn standard voltage and current values. During testing, if technicians find that customary values do not register on their digital mustimeters or clamp meters, they can use Ohm's Law to detect what part of a circuit is faltering and from that determine where a problem may lie.

Named for German physicist Georg Ohm (1789-1854), Ohm's Law addresses the key quantities at work in circuits:

Table 4.4: Ohm's Law Quantities Table

QUANTITY	**OHM'S LAW SYMBOL**	**UNIT OF MEASURE (ABBREVIATION)**	**ROLE IN CIRCUITS**	**IN CASE YOU'RE WONDERING:**
VOLTAGE	E	Volt (V)	Pressure that triggers electron flow	E = electromotive force (old-school term)
CURRENT	I	Ampere, amp (A)	Rate of electron flow	I = intensity
RESISTANCE	R	Ohm (Ω)	Flow inhibitor	Ω = Greek letter omega

TOPIC: 5

PROBLEM SOLVING USING COMPLEX NUMBER METHOD

USING CALCULATOR TO COMPUTE VALUE OF REACTANCE,IMPEDANCE : ADDITION OF COMPLEX NUMBERS

Example 1 :

Determine the impedance of a coil in series circuit which has the resistance of 12Ω and a reactance of 16Ω.

Solution :

Given R = 12Ω, XL = 16Ω, Z = ?

Step of using calculator

1. Press $\boxed{\text{MODE}}$ button and change mode to 'CMPLX'
2. Enter the value to be calculated

   Example : R + jXL = 12 + j16

   Enter number 12 as usual, then press $\boxed{+}$ button.

   Enter number 16 as usual,then press $\boxed{\text{SHIF}}$ $\boxed{\text{ENG}}$ button.
3. Press $\boxed{=}$ button once (*but this is not the answer yet!*)
4. Press $\boxed{\text{SHIF}}$ $\boxed{+}$ $\boxed{=}$ to get the magnitude value of Z,
5. Press $\boxed{\text{SHIF}}$ $\boxed{=}$ to get the phase angle value of Z.

So the answer of Example 1 are Z = R + jXL

= 12 + j16

Z = 20∟53.13°Ω

magnitude phase angle

Example 2 :

A resistance of 50Ω is connected in series with the capacitance of 20µF. If the supply of 200V, 100Hz is connected across the arrangement find the circuit impedance.

Solution :

Given R = 50Ω, C = 20µf, V = 200V, f = 100Hz, Z = ?

Compute Capacitive reactance value,

$X_C = 1/2\pi fC = 1/2\pi(100)(20\mu)$

$= \mathbf{79.58\Omega}$

Step of using calculator

1. Press $\boxed{\text{MODE}}$ button and change mode to 'CMPLX'
2. Enter the value to be calculated

   Example : R - jXC = 50 - j79.58

   Enter number 50 as usual, then press $\boxed{-}$ button.

   Enter number 79.58 as usual,then press $\boxed{\text{SHIF}}$ $\boxed{\text{ENG}}$ button.
3. Press $\boxed{=}$ button once (*but this is not the answer yet!*)
4. Press $\boxed{\text{SHIF}}$ $\boxed{+}$ $\boxed{=}$ to get the magnitude value of Z,
5. Press $\boxed{\text{SHIF}}$ $\boxed{=}$ to get the phase angle value of Z.

So the answer of Example 2 are Z = R - jXC

= 50 – j79.58

$\mathbf{Z = 93.98\angle{-57.86°}\Omega}$

magnitude phase angle

USING CALCULATOR TO COMPUTE VALUE OF CURRENT : DIVISION OF COMPLEX NUMBERS

Example 3 :

A resistance of 50Ω is connected in series with the capacitance of 1µF. If the supply of 200V, 1kHz is connected across the arrangement, find the circuit current.

Solution :

Given R = 50Ω, C = 1µf, V = 200V, f = 1kHz, Z = ?

Compute Capacitive reactance value,

$XC = 1/2\pi fC = 1/2\pi(1k)(1\mu)$

$= \mathbf{159.15\Omega}$

Compute impedance value,

$Z = R - jXC$

$= 50 - j159.15$

$\mathbf{Z = 166.82\angle{-72.56}°\Omega}$

Then, compute current value,

$I = V/Z = 200\angle 0°/166.82\angle{-72.56}°$

Step of using calculator

1. Enter the value to be calculated

   Example :

   ÷

   $V/Z = 200\angle 0°/166.82\angle{-72.56}°$

   (–)

200 devide by 166.82 = 1.20

0° substract with -72.56°= 72.56°

So the answer for Example 1 are

$I = V/Z = 200\angle 0°/166.82\angle{-72.56}°$

$\mathbf{I = 1.20\angle 72.56°A}$

Example 4 :

An RLC series circuit has a 40Ω resistor, a 3mH inductor, and a 5µF capacitor. Find the circuit's impedance at 60Hz. If the voltage source has Vrms=120V, what is Irms at that frequency?

Solution :

Given R = 40Ω, C = 5µf, L = 3mH, V = 120V, f = 60kHz, Z = ? and I = ?

Compute Capacitive reactance value,

$$XC = 1/2\pi fC = 1/2\pi(60k)(5\mu)$$

$$\mathbf{XC = 530.52m\Omega}$$

Compute Inductive reactance value,

$$XL = 2\pi fL = 2\pi(60k)(3m)$$

$$\mathbf{XL = 1.13k\Omega}$$

Compute impedance value,

$$Z = R + jXL - jXC$$

$$= 40 + j1.13k - j530.52$$

$$\mathbf{Z = 600.81\angle 86.18°\Omega}$$

Then, compute current value,

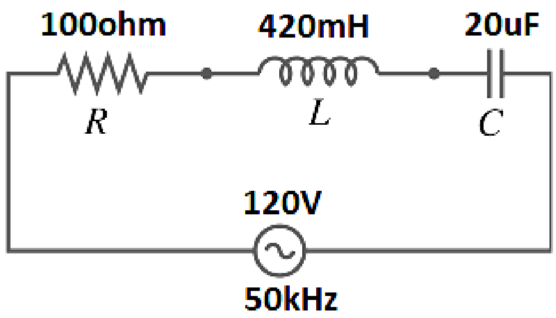
$$I = V/Z = 120\angle 0°/600.81\angle 86.18°$$

$$\mathbf{I = 199.73\angle -86.18° mA}$$

USING CALCULATOR TO COMPUTE VALUE OF VOLTAGE DROP : MULTIPLYING OF COMPLEX NUMBERS

Example 5 :

The image below depicts an RLC series circuit with the following components: A 120 V, 50 Hz AC supply, A 100-ohm resistor, A 20μF capacitor, A 420mH inductor. Calculate voltage drops across all three impedances.



Solution :

Given R = 100Ω, C = 20µf, L = 420mH, V = 120V, f = 50kHz, VR =? VC =? VL =?

Compute Capacitive reactance value,

$XC = 1/2\pi fC = 1/2\pi(50k)(20\mu)$

$XC = 0.16\Omega$

Compute Inductive reactance value,

$XL = 2\pi fL = 2\pi(50k)(420m)$

$XL = 131.95k\Omega$

Compute impedance value,

$Z = R + jXL - jXC$

$= 100 + j131.95k - j0.16$

$Z = 131.95\angle 89.96°k\Omega$

Then, compute current value,

$I = V/Z = 120\angle 0°/131.95\angle 89.96°k$

$I = 909.44\angle -89.96°\mu A$

Finally, compute voltage drop value for each impedance

a) Voltage drop across resistor,

Multiply magnitude value

$VR = I \times R = (909.44\angle -89.96°\mu A) \times (100\angle 0°A)$

Add the angle value

$\mathbf{VR = 0.09 \angle -89.96\ V}$

b) Voltage drop across Inductor,

$VL = I \times jXL = (909.44\angle -89.96°\mu A) \times (131.95 \angle 90°A)$

$\mathbf{VL = 0.12\angle 0.04\ V}$

c) Voltage drop across capacitor.

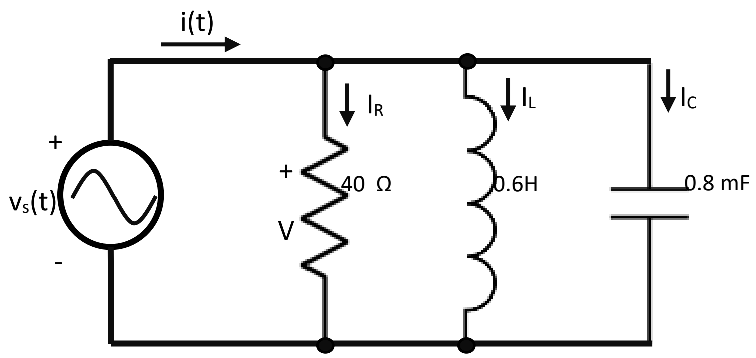
$VL = I \times (-jXC) = (909.44\angle -89.96°\mu A) \times (0.16 \angle -90°A)$

$\mathbf{VC = 145.51\angle -179.96\ \mu V}$

USING CALCULATOR TO COMPUTE VALUE OF REACTANCE, IMPEDANCE : USING OF COMPLEX NUMBERS

Example 1 :

Determine the total impedance, Z_T, total current i(t), current through the resistor I_R, current through the inductor, I_L and current through the capacitor, I_C for circuit shown below.
Given Vt(t) = 100 cos 100t V



Solution :

Given R = 40 Ω, L = 0.6H C = 0.8mF

Vt(t) = 100 cos 100t V

ω

Step of using calculator

1. Press $\boxed{\text{MODE}}$ button and change mode to 'CMPLX'

2. Find Z_L and Z_C.

$Z_L = j\omega L$

$= \boxed{\text{ENG}}\ \boxed{(}\ \boxed{100}\ \boxed{)}\ \boxed{(}\ \boxed{0.6}\ \boxed{)}$

= j60 Ω

$Z_C = -\frac{j}{wc}$

$= \boxed{(-)}\ \boxed{\text{ENG}}\ \boxed{\div}\ \boxed{(}\ \boxed{100}\ \boxed{\times}\ \boxed{0.8}\ \boxed{\text{SHIFT}}\ \boxed{5}\ \boxed{)}$

= - j12.5

3. Find Z_T

$Z_T = R \parallel j60 \parallel -j12.5$

$$\frac{1}{ZT} = \frac{1}{R} + \frac{1}{ZL} + \frac{1}{ZC}$$

$$Z_T = \frac{1}{\frac{1}{R} + \frac{1}{ZL} + \frac{1}{ZC}}$$

$$Z_T = \frac{1}{\frac{1}{40} + \frac{1}{j60} + \frac{1}{-j12.5}}$$

$$Z_T = \frac{1}{0.025 + j0.063}$$

$Z_T = 5.39 - j13.67$

$Z_T =$ **14.69 ∟-68.46 ° Ω**

Find Z_T by using calculator

$Z_T =$ [1] [÷] [(] [(] [1] [÷] [40] [)] [+] [(] [1] [÷] [60] [ENG] [)]

[+] [(] [1] [÷] [(] [(-)] [12.5] [ENG] [)] [)] [=]

Answer in rectangular

[5.39] [SHIFT] [=] [-13.67 i]

To change answer in polar

[SHIFT] [+] [=]

Answer in polar

[14.69] [SHIFT] [=] [∟-68.46]

So the answer of Example 1 are $Z_T = R + jX$

$Z_T = 5.39 - j13.67$

Z_T = 14.69 ∟- 68.46°Ω

magnitude phase angle

3. Find total current, i(t)

$$i(t) = \frac{V}{ZT}$$

$$i(t) = \frac{100 \angle 0^\circ}{5.39 - j13.67}$$

$i(t) = 2.496 + j\,6.33$ A

Find i(t) by using calculator

i(t) = $\boxed{100}$ $\boxed{\div}$ $\boxed{(}$ $\boxed{5.39}$ $\boxed{-}$ $\boxed{13.67}$ $\boxed{\text{ENG}}$ $\boxed{)}$ $\boxed{=}$

Answer in rectangular

$\boxed{2.496}$ $\boxed{\text{SHIFT}}$ $\boxed{=}$ $\boxed{6.33\ i}$

To change answer in polar

$\boxed{\text{SHIFT}}$ $\boxed{+}$ $\boxed{=}$

Answer in polar

$\boxed{6.81}$ $\boxed{\text{SHIFT}}$ $\boxed{=}$ $\boxed{\angle 68.46}$

So the answer of Example 1 are

$i(t) = 2.496 + j6.33$ A

$i(t) = 6.81 \angle 68.46^\circ$A

magnitude phase angle

Find total current, I_R

$I_R = \frac{V}{R}$

$I_R = \frac{100 \angle 0^\circ}{40}$

$I_R = 2.5\ A \angle 0°A$

Find I_R by using calculator

$I_R =$ $\boxed{100}$ $\boxed{\div}$ $\boxed{40}$ $\boxed{=}$

Answer in rectangular

$\boxed{2.5}$ $\boxed{\text{SHIFT}}$ $\boxed{=}$ $\boxed{0}$

To change answer in polar

$\boxed{\text{SHIFT}}$ $\boxed{+}$ $\boxed{=}$

Answer in polar

$\boxed{2.5}$ $\boxed{\text{SHIFT}}$ $\boxed{=}$ $\boxed{\angle 0}$

So the answer of Example 1 are

$I_R = 2.5 + j\,0\ A$

$I_R = 2.5 \angle 0°A$

phase angle

magnitude

3. Find total current, I_L

$I_L = \frac{V}{ZL}$

$I_L = \frac{100 \angle 0^\circ}{j60}$

$I_L = -j1.67$ A

Find I_L by using calculator

I_L = | 100 | ÷ | 60 | ENG | = |

Answer in rectangular

| -1.67i |

To change answer in polar

| SHIFT | + | = |

Answer in polar

| 1.67 | SHIFT | = | ∟-90 |

So the answer of Example 1 are

$I_L = -\,j1.67$ A

$I_L = 1.67 \angle -90^\circ$A

phase angle

magnitude

4. Find total current, I_C

$I_C = \frac{V}{ZC}$

$I_C = \frac{100 \angle 0^\circ}{-j12.5}$

$I_C = j8$ A

Find I_L by using calculator

$I_C =$ $\boxed{100}$ $\boxed{\div}$ $\boxed{(}$ $\boxed{(-)}$ $\boxed{12.5}$ $\boxed{ENG}$ $\boxed{)}$ $\boxed{=}$

Answer in rectangular

$\boxed{8i}$

To change answer in polar

$\boxed{SHIFT}$ $\boxed{+}$ $\boxed{=}$

Answer in polar

$\boxed{8}$ $\boxed{SHIFT}$ $\boxed{=}$ $\boxed{\angle 90}$

So the answer of Example 1 are

$I_L = j8$

$I_L = 8 \angle 90^\circ A$

phase angle

magnitude

EXERCISES

QUESTIONS & ANSWER

QUESTION AND ANSWER

Question 1

An alternating current is given by e= 70 sin (314t +30^0) volt. Determine the voltage if this wave is measured by voltmeter.

ANSWER: VRMS = 49.49V

Question 2

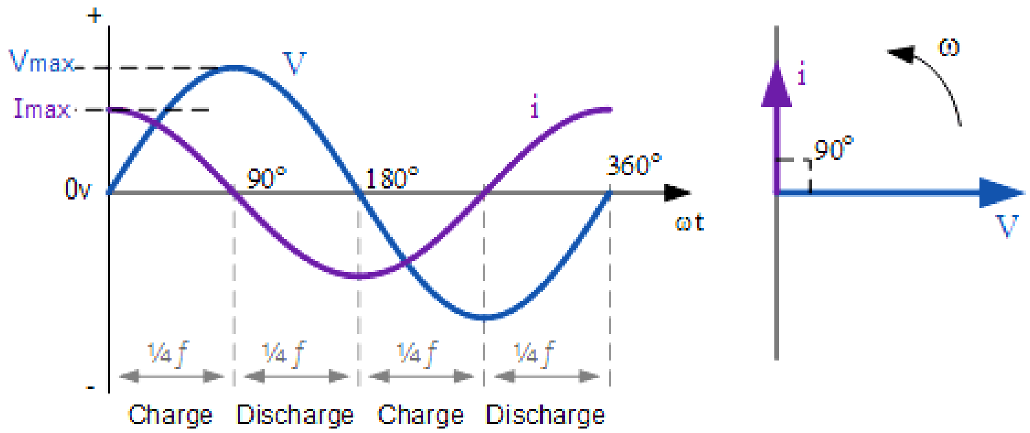
A 10Ω resistance, 90mH inductance and a 0.015uF capacitance are connected in series across an AC source. Calculate the impedance magnitude at frequency 1.2KHz.

ANSWER: Z = 1.617KΩ

Question 3

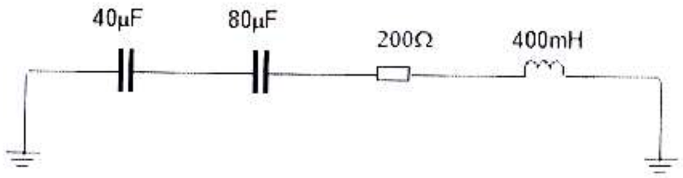
With the aid of a diagram, state the relationship between the voltage and the current for pure capacitive circuit.

ANSWER:



Question 4

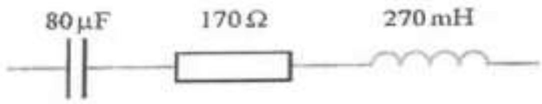
With reference to Figure below, determine the total impedance, Z_T for the series circuits which has a frequency of 60Hz.



ANSWER: Z_T = 206.48Ω

Question 5

Determine the complex impedance of the following series arrangement at a frequency of 60Hz



ANSWER: Z = 170 + j68.631Ω

Question 6

A sinusoidal current of 5A peak and 60 Hz flows through a capacitor of 20mF. What voltage will appear across the capacitor?

ANSWER: VC = 663.15mV

Question 7

What is the applied voltage for a series RLC circuit when I_T = 3mA, V_L = 30V, V_C = 18V and R = 100ohms?

ANSWER:
V_T = 12.37V

Question 8

A serial RCL circuit has a 75.0Ω resistor, a 20.0μF capacitor and a 55.0mH inductor connected across an 800-volt rms AC generator operating at 128Hz.

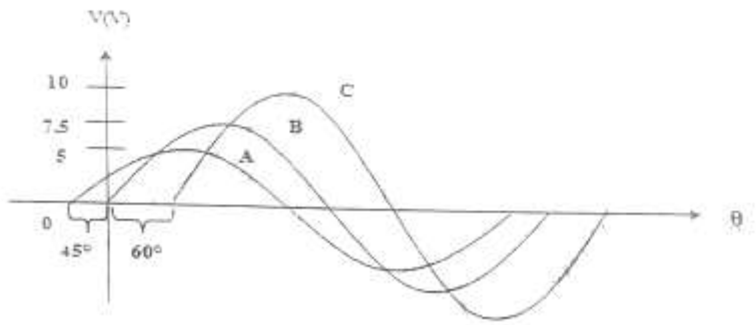
i. Is the load on the circuit inductive, capacitive or resistive?
ii. What is the phase angle Ø
iii. What is the rms current in the circuit
iv. Write the formula for the current in the circuit as a function of time
v. Find the rms voltage across each element
vi. Find the average power delivered to the circuit by the generator

ANSWER:

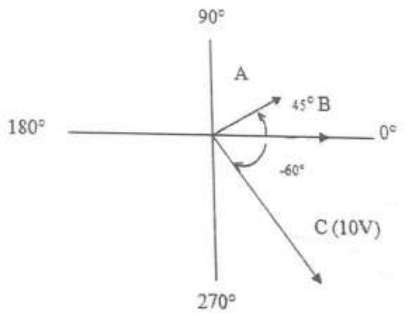
i. The Load is capacitive
ii. Phase Angle Ø = -13.5
iii. RMS current = 10.37A
iv. i = 14.7 sin (804t + 0.223)
v. V_{Rrms} = 777.75 V_{Lrms} = 458.35V V_{Crms} = 645.01V
vi. Average Power, P = 8.11kW

Question 9

Draw a phasor diagram to represent the sine wave in Figure below



ANSWER



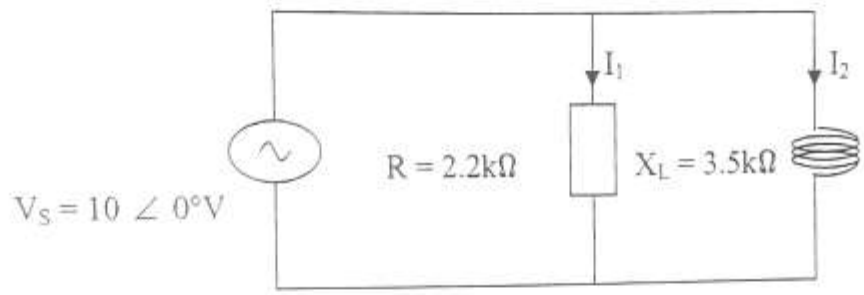
Question 10

A circuit having a resistance of 12Ω, an inductance of 0.15H and a capacitance of 100mF in series, is connected across a 100V, 50Hz supply. Calculate the total impedance, Z_T of RLC circuit

ANSWER: $Z_T = 12 + j15.7$

Question 11

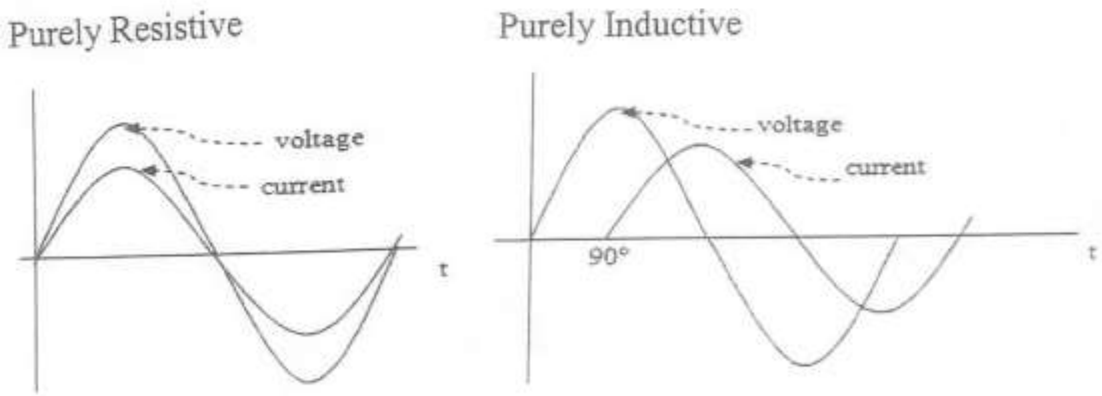
Find the current of I_1 and I_2 in Figure below:

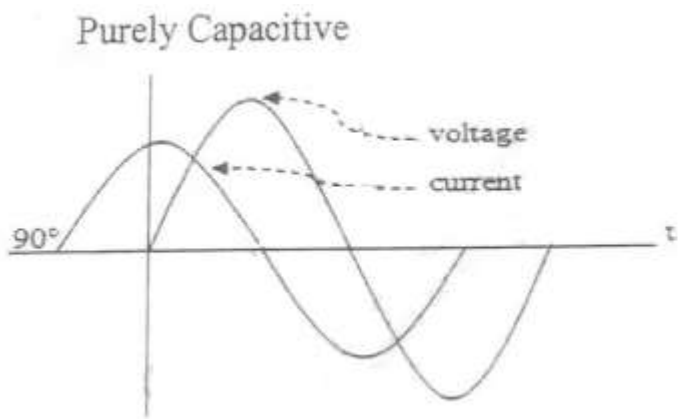


ANSWER: $I_1 = 4.55mA$ $I_2 = 4.55mA$

Question 12

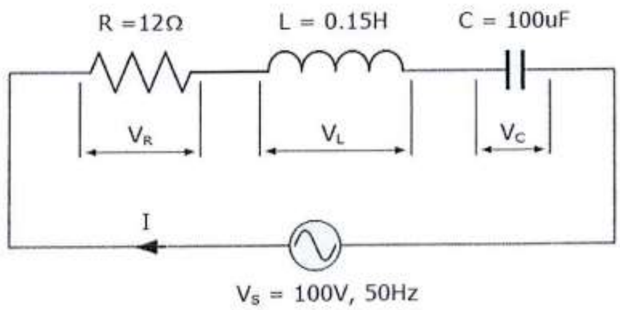
Sketch the voltage and current waveform in a purely resistance circuit, inductive circuit and capacitive circuit.





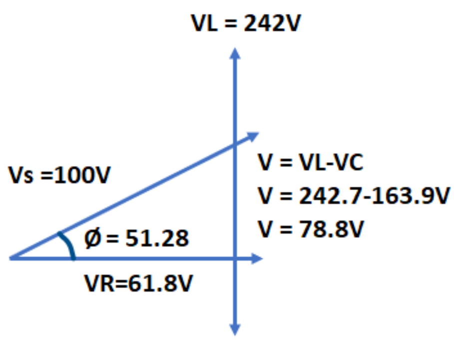
Question 13

With reference to Figure below, determine thee total current flow(I) and sketch the voltage phasor diagram of the circuit.



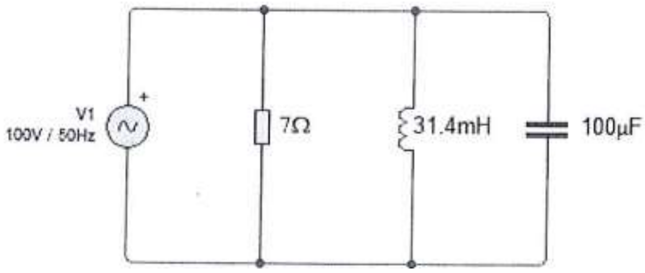
ANSWER:

i. **$X_L = 47.13\ \Omega$**
ii. **$X_C = 31.83\ \Omega$**
iii. **$Z_T = 19.4\ \Omega$**
iv. ***$I_T = 5.15A$***
v. **$V_R = 61.8V$ $V_L = 244.7V$ $V_C = 163.9V$**
vi. **$Ø = 51.76^0$**
vii. **Phasor diagram**



Question 14

With reference to Figure below, a 7Ω resistor is connected in parallel with a 31.4mH inductor and a 100uF capacitor. An AC sinusoidal waveform 100V, 50Hz is used as a supply to the circuit. Find the value of the total current, I_T flowing in the circuit.

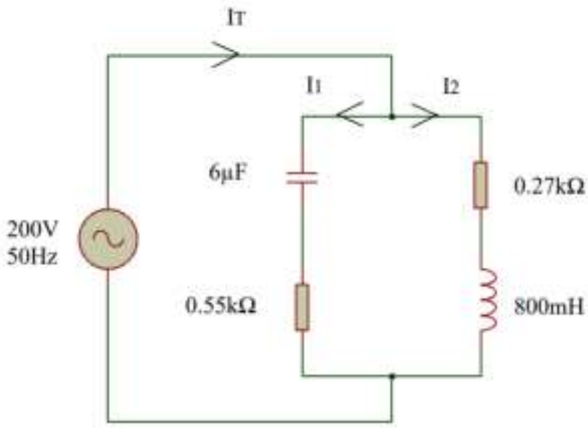


ANSWER:

i. **$X_L = 9.86\Omega$**
ii. **$X_C = 31.83\Omega$**
iii. **$Z_T = 6.286\Omega$**
iv. ***$I_R = 14.29A$***
v. **$I_L = 10.41A$**
vi. **$I_C = 3.142A$**
vii. **IT = 16.03A^0**

Question 15

Referring to Diagram below, calculate current I1, I2 and IT and draw the phasor diagram of currents. Calculate also the power factor, true power and the apparent power.

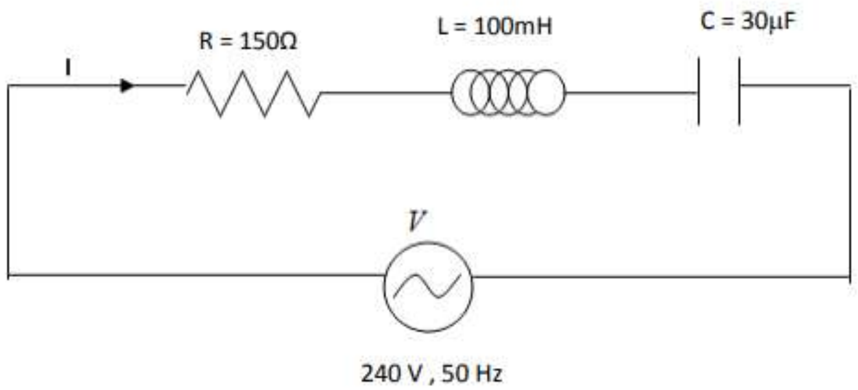


ANSWER:

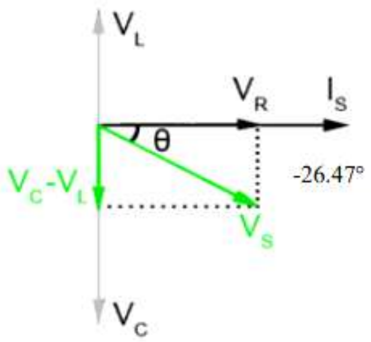
i. $X_L = 251.3\Omega$
ii. $X_C = 530.5\Omega$
iii. $Z_T = 248.8\Omega$
iv. *$I_1 = 261.7mA$*
v. $I_2 = 0.5A$
vi. $I_T = 0.83A$

Question 16

Figure below shows the RLC series circuit. Calculate the total impedance, current flowing in the circuit, the potential difference at each component, the phase angle, the value of active power, the value of reactive power and draw the voltage phasor diagram

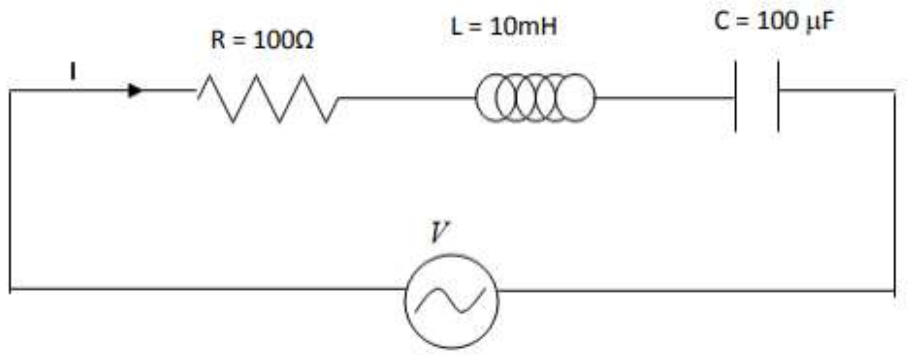


ANSWER :
i. $X_L = 31.42\Omega$
ii. $X_C = 106.12\Omega$
iii. $Z_T = 167.56\Omega$
iv. *$I_T = 1.43A$*
v. $V_L = 44.93V$
vi. $V_R = 214.5V$
vii. $V_C = 16.03A^0$
viii. $Ø = -26.47^0$
ix. P = 308.88 WATT **cos Ø= 0.9**
x. P = 147.58 VAR sin **Ø = 0.43**
xi. VOLTAGE PHASOR DIAGRAM



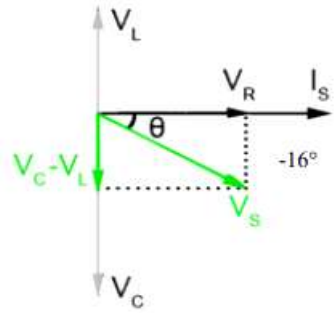
Question 17

Figure 2 shows the RLC series circuit. Calculate the total impedance, the current flowing in the circuit, the potential difference at each component, the phase angle, the value of active power, the value of reactive power and draw the voltage phasor diagram.



ANSWER :

i. **X_L = 3.14Ω**
ii. **X_C = 31.83Ω**
iii. **Z_T = 104.03Ω**
iv. ***I_T = 2.31A***
v. **V_L = 7.25V**
vi. **V_R = 231V**
vii. **V_C = 73.53V**
viii. **Ø = -16^0**
ix. P = 532.22 WATT **cos Ø= 0.96**
x. P = 155.23 VAR sin **Ø = 0.28**
xi. VOLTAGE PHASOR DIAGRAM

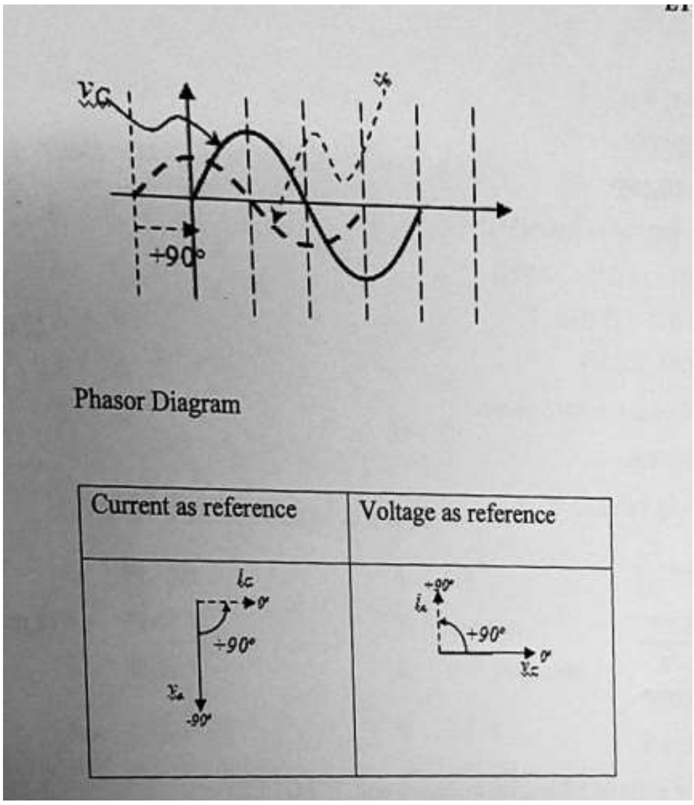


Question 18

Describe the characteristics of Purely Capacitive Circuits with aid of correct waveform and phasor diagram.

ANSWER:

Purely capacitive circuit, the voltage V lag Current, I with 90^0 phase difference or the current, I lead voltage, V with 90^0 phase difference waveforms



Question 19

A capacitor of 10mF is connected in series with a 200 Ω resistor to a 240V, 50Hz supply. Calculate the total impedance of circuit.

ANSWER:

i. **X_C = 0.318Ω**
ii. **Z_T = 200<-0.091^0**

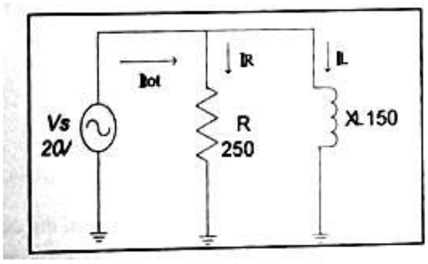
Question 20

A 120Ω resistor is in parallel with a capacitor with a capacitive reactance of 40Ω. Both components are across a 12V ac source. What is the magnitude of the total impedance.

ANSWER: Z_T = 37.9 Ω

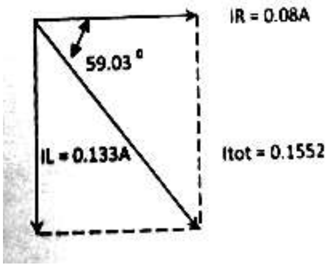
Question 21

Determine the value of each current in Figure below, and describe the phase relationship of each with the source voltage. Draw the current phasor diagram, current and voltage waveform.



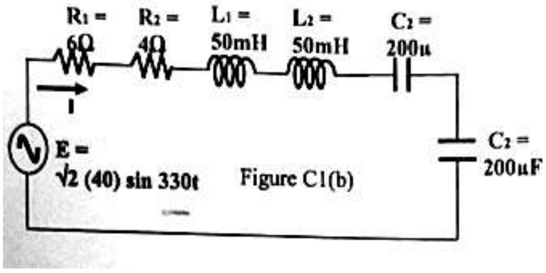
ANSWER :

i. **I_R = 0.08A**
ii. **I_L = 0.133A**
iii. **I_T = 0.1552A**
iv. **Ø = 59.03^0**
v. **PHASOR DIAGRAM**



Question 22

By referring to figure below

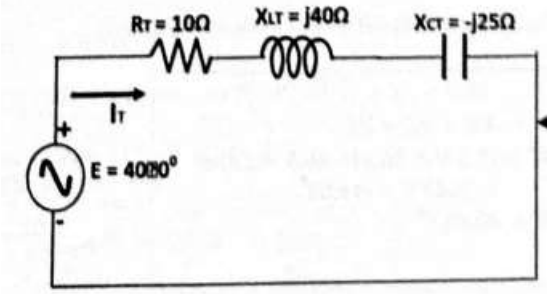


i. Find total resistor and each Reactance element in the circuit
ii. Redraw the circuit using phasor notation
iii. Calculate the Total Impedence (Z_T)
iv. Calculate the total current flow in the circuit (I_T) and current flow at L_1 and C_1.
v. Determine Voltage drop at V_R , V_L, V_C

ANSWER:

i. **R_T = 10Ω LT = 100mH XLTj40ΩΩ CT = 100µF XCT = -j25**

ii. **Redraw the circuit in the phasor form**



iii. **Z_T = 10 + j15Ω**

iv. **I_T = 3.86A<-15.1^0 I_C = I_L = I_T = 2.22A <-56.31^0**

v. **V_R = 22.2V < -56.31^0 V_L = 88.8V<33.6^0 V_C = 55.47V <-146.3^0 V_T = 40<00**

Question 23

A 5Ω resistance, a 120mH inductance and a 100µF capacitance are connected in series to a 240V, 50Hz voltage supply. Calculate the current flowing through the circuit and the voltage across each capacitor and inductor.

ANSWER :

i. **X_L = 37.7Ω**
ii. **X_C = 031.83Ω**
iii. **Z_T = 7.71Ω<49.58^0**
iv. **IT = 31.13A**
v. **VC = 990.87V VL = 182.73V**

Question 24

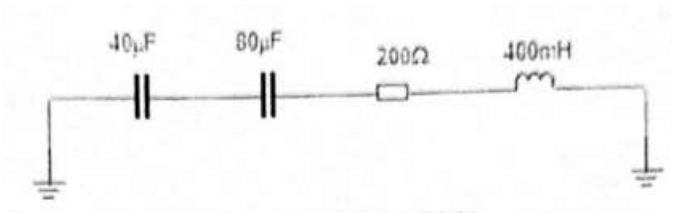
A 20Ω resistor is connected in parallel with an inductance of 20mH across a 240V, 1KHz voltage supply. Calculate the current in each branch, the total current, true power, apparent power and reactive power.

ANSWER :

i. **X_L = 125.56Ω**
ii. **I_R = 12A I_L = 1.91A I_T = 12.15A <-9.04^0**
iii. **True Power , P = 2.88KW**
iv. **Reactive Power, Q = 458.17 Var**
v. **Apparent power , S = 2.916KVA**

Question 25

Referring to diagram below, determine total impedence, Z_T for series circuit which has a frequency of 60Hz.

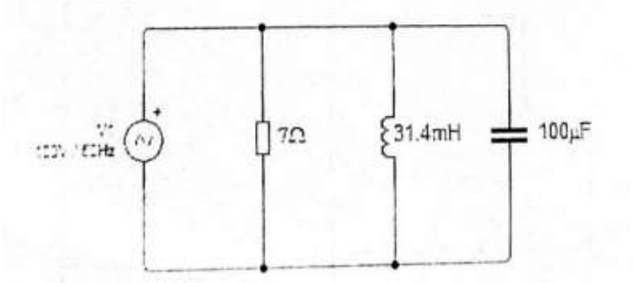


ANSWER :

i. **X_{C1} = 66.31Ω**
ii. **X_{C2} = 33.61Ω**
iii. **X_L = 150.80Ω**
iv. **ZT = 200 + j51.337 Ω**

Question 26

A 7Ω resistor is connected in parallel with a 31.4mH inductor and a 100µF capacitor. An AC sinusoidal waveform 100V, 50Hz is used as a supply to this circuit. Find the value of the total current, IT flowing in the circuit.



ANSWER :

i. X_C = 31.83Ω

ii. X_L = 9.86Ω

iii. $I_R = 14.3<0^0$A

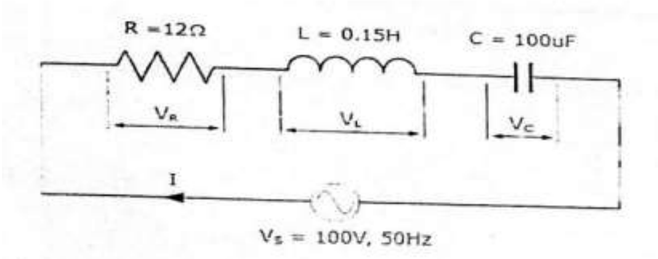
iv. $I_L = 10.14 <-90^0$A

v. $I_C = 3.14 < +90^0$A

vi. I_T = 14.3 – j7A @ $15.92<-26.08^0$A

Question 27

Refer to diagram below, determine the total current flow (I) and then sketch the voltage phasor diagram of the circuit.



ANSWER :

i. X_C = 31.83Ω

ii. X_L = 47.13Ω

iii. Z_T = 19.4Ω

iv. I_T = 5.15A

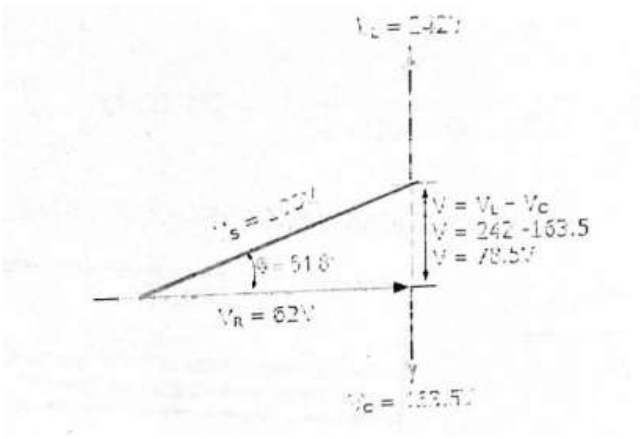
v. V_R = 61.7V

vi. V_L = 242.2V

vii. V_C = 163.5V

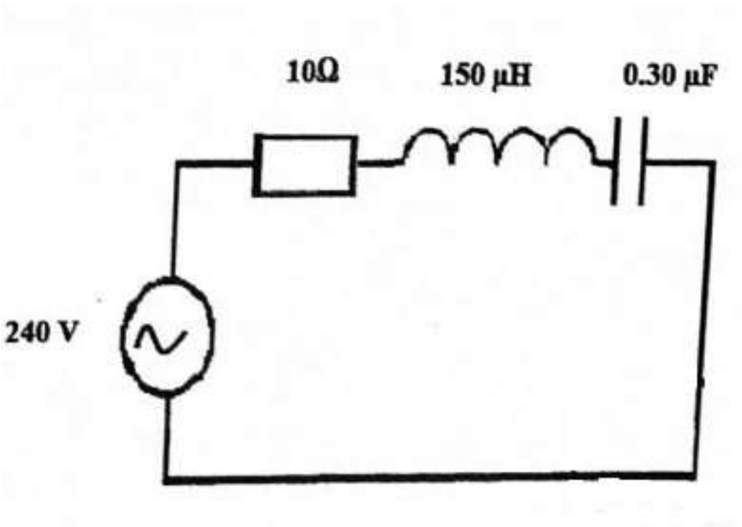
viii.Power Factor = 0.619 @ 51.8^0

ix. Phasor Diagram



Question 27

Referring to the diagram below, determine the total impedence, ZT for the series circuit which has a frequency of 30KHz.



ANSWER :

i. **RT = 10Ω**

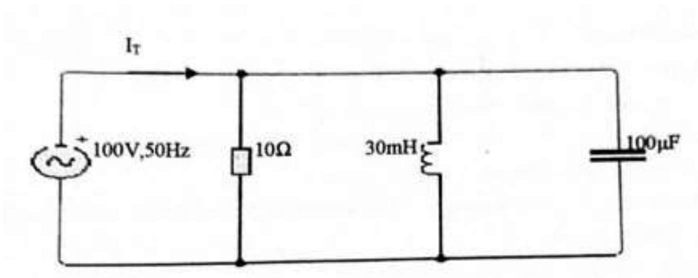
ii. **X_C = 17.68Ω**

iii. **X_L = 28.27Ω**

iv. **Z_T = 10 + j10.59Ω @ 14.56Ω <46.64^0**

Question 28

With reference to the Diagram below, Find the value of the total current, I_T flowing in the circuit.



ANSWER :

i. $X_C = 31.85\Omega$

ii. $X_L = 9.42\Omega$

iii. $I_R = 10.0 < 0^0 A$

iv. $I_L = 10.62 <-90^0 A$

v. $I_C = 3.14 < 90^0 A$

vi. $I_T = 10.0 – j7.48A$ @ $12.49 < -36.8^0$ A

Question 28

A 90Ω resistor, 0.3H inductor, 10µF capacitor and 1H inductor are connected in series to 100V, 50Hz supply. Calculate the voltage across each component and real power dissipation of the circuit.

ANSWER :

i. $X_C = 318.3\Omega$

ii. $X_L = 408.4\Omega$

iii. $Z = 127.35 <45^0\Omega$

iv. $I_T = 0.79 < -45^0 A$

v. $V_R = 71.1 <-45^0 V$

vi. $V_L = 322.64 < 45^0 V$

vii. $V_C = 251.5 < -135^0 V$

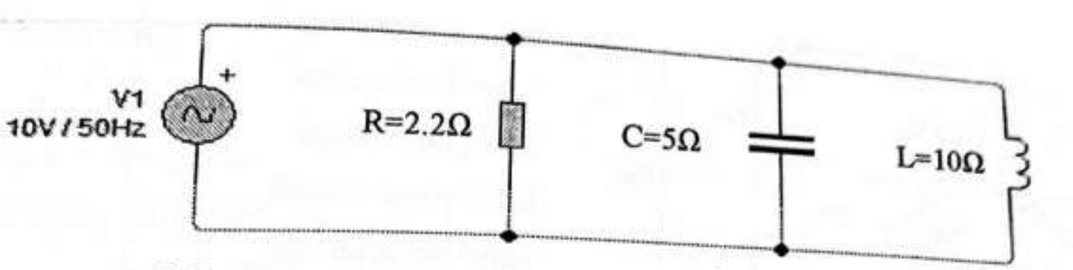
viii. $\emptyset = 45.03^0$

ix. **Power Factor = Cos 45.03^0**

x. **Real Power Dissipation = 55.83 Watt**

Question 29

By referring to figure below, calculate the total impedence and current at each branch.

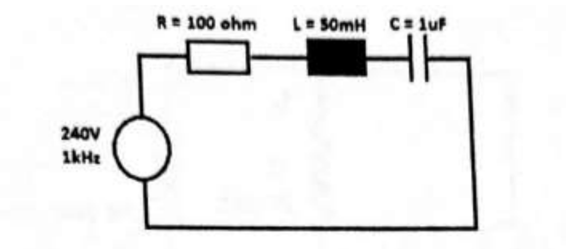


ANSWER :

i. $I_C = 2A$
ii. $I_L = 1A$
iii. $I_R = 4.55A$
iv. $I_T = 2.698A$
v. $Z = 3.71\ \Omega$

Question 30

By referring to figure below, calculate the total impedence, total current and voltage across inductor, L and capacitor, C.

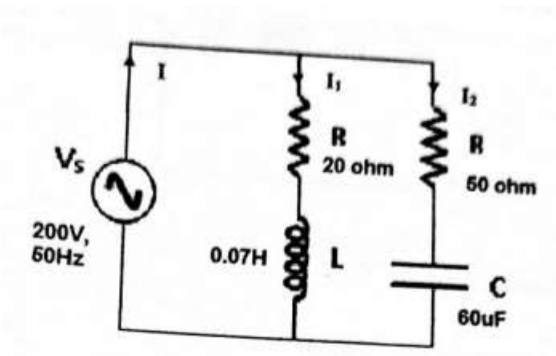


ANSWER :

i. $X_C = 159.15\ \Omega$
ii. $X_L = 314.15\ \Omega$
iii. $Z_T = 184.45\ \Omega < 57.17^0$
iv. $I_T = 1.3\ A < -57.17^0$
v. $V_C = 206.89V$
vi. $V_L = 408.39V$

Question 31

By referring to figure below, calculate the line current I1, I2, IT and draw the phasor diagram of current. Also find the power factor, true power and the apparent power.



ANSWER :

i. **X_C = 53.1 Ω**

ii. **X_L = 22 Ω**

iii. **Z_1 = 29.73 Ω < 47.7^0**

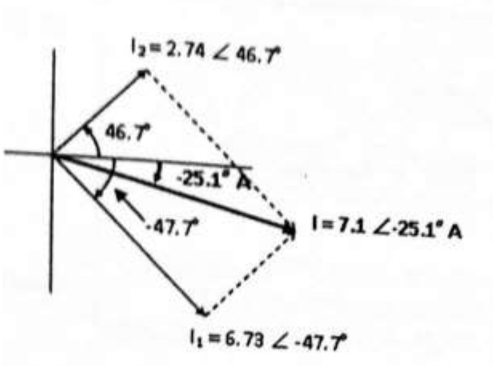
iv. **Z_2 =73 Ω < -46.7^0**

v. **I_1 = 6.73 < 47.7^0A**

vi. **I_2 = 2.74 < 46.7^0A**

vii. **I_T = 7.1 <-25.1^0A**

viii. **Current Phasor Diagram**



ix. **Power Factor = kos 25.1 @ 0.9 lag**

x. **True Power = 1278Watt**

xi. **Apparent Power, S = 1420VA**

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