Fluid Mechanics

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"This book is dedicated to all students and lecturers of Polytechnic Malaysia"

FLUID MECHANICS

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Preface

Fluid Mechanics books were written as a reference for Polytechnic Malaysia students especially those who have taken DJJ20073. This book provides knowledge about fluid under static and dynamic situations. Chapter 1 discussed the characteristics of fluids, while Chapter 2 used the physical properties of fluids, followed by Chapter 3 used the concept of Pascal's law in hydraulic jacks. Chapter 4 describes fluid dynamics, and finally, chapter 5 discusses energy loss in circular pipes.

In addition, the author would also like to express deepest appreciation to Norfadzillah binti Ismail for contributing some of the contents in this book.

Contents PAGE

Chapter 1: Introduction of Fluid

FLUID PROPERTIES

ABSTRACT

This topic covers the fundamental concepts of fluid properties, fluid characteristics, types of pressure, and the relationship between pressure and depth.

1.1 Fluid characteristics

1.2 Types of pressure

1.3 The pressure in fluid

1.1: Fluid Characteristic

The physical properties of a substance depend upon its physical state. Water vapor, liquid water and ice all have the same chemical properties, but their physical properties are considerably different. A fluid can be defined as

- a substance which continually deforms under an applied shear stress regardless of the magnitude of the applied stress.
- a substance that flow easily because of increased intermolecular spaces and do not have fixed shape. Liquids and gases are considered as fluids

Some example of fluids are water, air, blood, mercury, honey, gasoline and other gas or liquid.

Liquids and gases share some characteristics such as a lack of shear resistance that make them similarly useful but they also have differing characteristics that require an engineer to approach the two types of fluid differently.

COMPRESSIBILITY

Compressibility is one of the characteristics where gases and liquids vary. Gases are highly compressible. Putting gas under a lot of pressure allows a much greater mass of gas into smaller container. Therefore, pressure causes a gas to reduce in volume.

Liquids on the other hand are barely compressible. Under great pressure or forces, the liquid form will maintain a volume very close to the original volume. Often liquid has so little compressibility that they are considered incompressible in engineering calculations. **WATCH THIS VIDEO, click Figure 1.1**

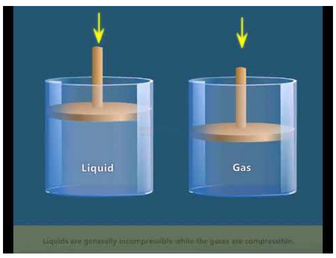


Figure 1.1: https://youtu.be/TI53_L_jdJs (@LB-kn7hf1,2020)

SHAPE AND VOLUME

In a liquid, the particles are still in close contact, so liquids have a definite volume. However, because the particles can move about each other rather freely, a liquid has no definite shape and takes a shape dictated by its container as Figure 1.2.

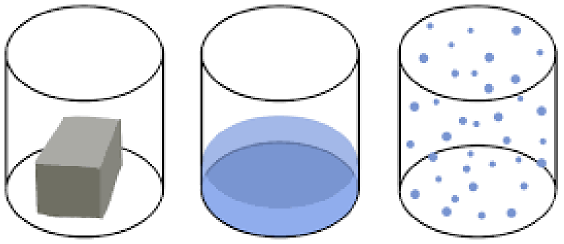


Figure 1.2: Shape and Volume

The difference is in volume. Gases will expand to fill a container's volume. The air in a balloon takes up the entire space of the balloon. Liquids will maintain a relatively constant volume so they will not necessarily fill a container's volume.

SHEAR RESISTANCE

A fluid at rest cannot resist shearing forces. Under the action of such forces it deforms continuously, however small they are. The resistance to the action of shearing forces in a fluid appears only when the fluid is in motion. This implies the principal difference between fluids and solids. In a shearing flow, adjacent layers of fluid move parallel to each other with different speeds. Viscous fluids resist this shearing motion.

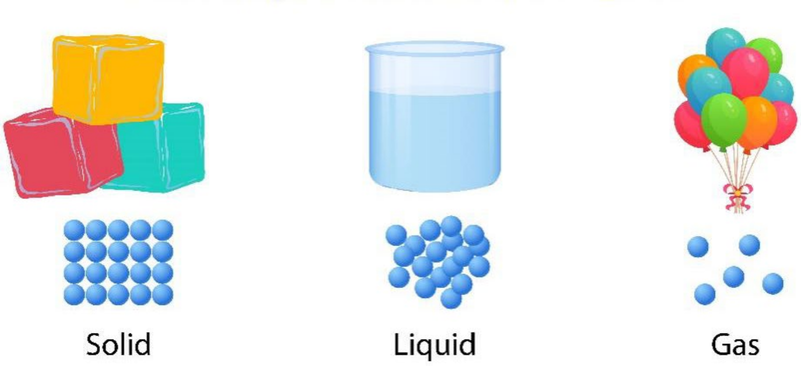
VISCOSITY

Viscosity is an absolute (dynamic) viscosity is a measure of how a fluid resists the deformation of shear stress due to its inter-molecular friction. The more viscous a fluid is, the harder it is to make a fluid flow. A viscous fluid is a real fluid that flows with some resistance in the opposite direction of its flow. Watch video below by press ctrl and click: https://youtu.be/aGxR6pj8E0A

MOLECULAR SPACING

What is the molecular spacing arrangement in solid liquid and gas? Particles in a:

- **solid** are tightly packed, usually in a regular pattern
- **liquid** is close together with no regular arrangement
- **gas** is well separated with no regular arrangement



DIFFERENCES BETWEEN SOLID,LIQUID AND GAS : STATE OF MATTER

Solid	Liquid	Gas
Not very compressible	Not very compressible	Highly compressible
High denisty	High density	Low density
Definite volume	Definite volume	Fills container competely
Retains its own shape	Assumes shape of container	Assumes shape of container
Motion limited to vibrational movement	Slow diffusion – particles can slip past each other	Rapid diffusion
Low expansion on heating	Low expansion on heating	High expansion on heating

1.2: TYPES OF PRESSURE

PRESSURE

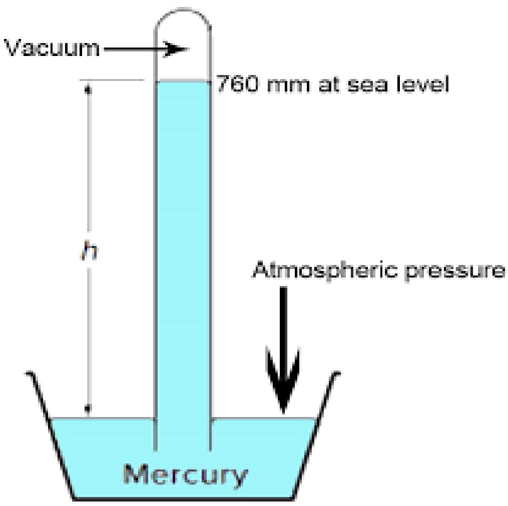
Pressure is defined as a normal force exerted by a fluid per unit area. **Pressure is the force applied perpendicular to the surface of an object** per unit area over which that force is distributed. Gauge pressure is the pressure relative to the ambient pressure. Various units are used to express pressure. The basic formula for pressure is F/A (Force per unit area). Unit of pressure is Pascals (Pa)

Formula of pressure (*P*): $\frac{Force}{Area} = \frac{F}{A}$

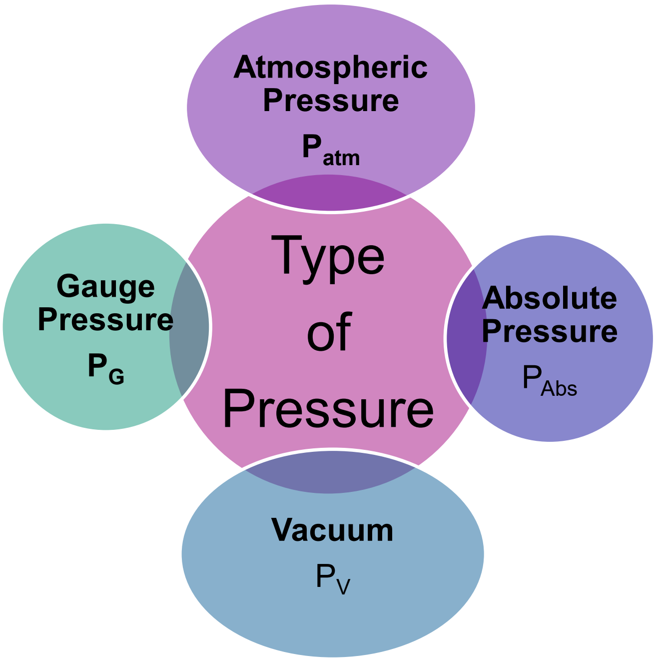
CONVERSION VALUES FOR THE MOST COMMON PRESSURE

	Pa	bar	atm	Torr
1 Pa	1	10^{-5}	9.87×10^{-6}	7.5×10^{-3}
1 bar	10^{5}	1	0.987	750.06
1 mbar	10^{2}	10^{-3}	0.967×10^{-3}	0.75
1 atm	1.013×10^{5}	1.013	1	760
1 Torr	133.32	1.33×10^{-3}	1.32×10^{-3}	1

The cause of this pressure is due to acceleration, gravity, or by forces that are outside the closed container. Gauge pressure is the pressure measured relative to atmospheric pressure.



TYPES OF PRESSURE



Atmospheric Pressure

- Atmospheric pressure, also known as air pressure or barometric pressure (after the barometer), is the pressure within the atmosphere of Earth. The standard atmosphere is a unit of pressure defined as 101,325 Pa, which is equivalent to 1,013.25 millibars, 760 mm Hg, 29.9212 inches Hg, or 14.696 psi.

Absolute Pressure

- The pressure measured with a vacuum taken as reference is known as the absolute pressure. This is independent of the atmospheric pressure and other environmental conditions. This type of pressure is mostly used for scientific experiments where the requirement of pressure independent from other factors is required. It is measured with a special type of pressure gauge known as the absolute pressure gauge which has a vacuum attached to it to be used as reference. The correct absolute pressure formula is:

$$P_{Abs}=P_{atm}+P_{gauge}$$

Gauge Pressure

- Gauge Pressure is defined as the pressure measured with the atmospheric pressure as the reference. It is found out using the formula:

$$P_{Gauge} = P_{Abs} - P_{atm}$$

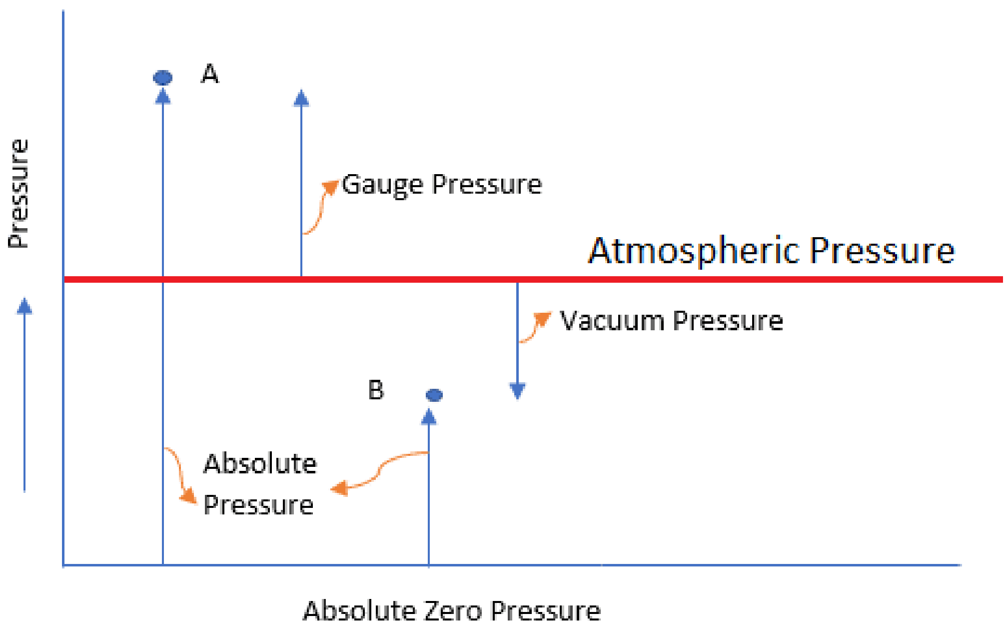
Where, P = Pressure recorded on the pressure gauge scale

- Thus, Gauge Pressure can be considered as the difference between the pressure measured and the atmospheric pressure. The gauge pressure can be positive or negative depending upon the pressure of the system being measured. Positive pressure means overpressure whereas negative pressure is partial vacuum.
- The gauge pressure varies with the location of measurement, due to different atmospheric pressure on different locations of the Earth. It is the most common type of pressure measured by mostly all pressure gauges

Vacuum Pressure

- Vacuum Pressure is the pressure inside a sealed vessel which is under vacuum conditions. The absolute pressure inside the vacuum is zero. In real-life situations, it is difficult to obtain absolute zero pressure but it is lower than the atmospheric pressure. The pressure is measured with the sensor inside the sealed vessel to make it free from the effect of environmental pressure and other factors like hydrostatic pressure

PRESSURE DIAGRAM



Examples

Example 1: A force of 400 Newtons is applied to an area of 0.04 square meters. Calculate the pressure exerted in Pascals

Given:
Force (F) = 500 N   Area (A) = 0.05 m^2

Using the pressure formula:
Pressure = Force / Area
Substituting the given values:
Pressure = 400 N / 0.04 m^2
Pressure = 10,000 N/ m^2 @ (Pa)

Therefore, the pressure exerted is 10,000 pascals.

Example 2: The mass of an object is 50 kgs and the object is accelerating at 2 m/sec2. This object strikes a surface of area 5 m2. Find the pressure exerted by the object on the surface

Given:
Mass = 50kgs   Acceleration (a)= 2 m/sec^2 Area of surface (A) = 5 m^2

Using force formula,
Force (F) = Mass × Acceleration
F = 50 × 2
F= 100 N

Now using the Pressure formula as,
P = Force / Area
P = 100/5
P = 20 N/m^2

The pressure exerted by object on surface the surface is, 20 N/m^2 @ (Pa)

Example 3: If the air in a cylinder has a gauge pressure of 250 kPa, the absolute pressure is?

Given: P_g = 250 kPa

Using force formula,
$P_{Abs} = P_g + P_{atm}$
P_{Abs} = 250 + 101.3
Pabs = 351.3 kPa = 350 kPa.

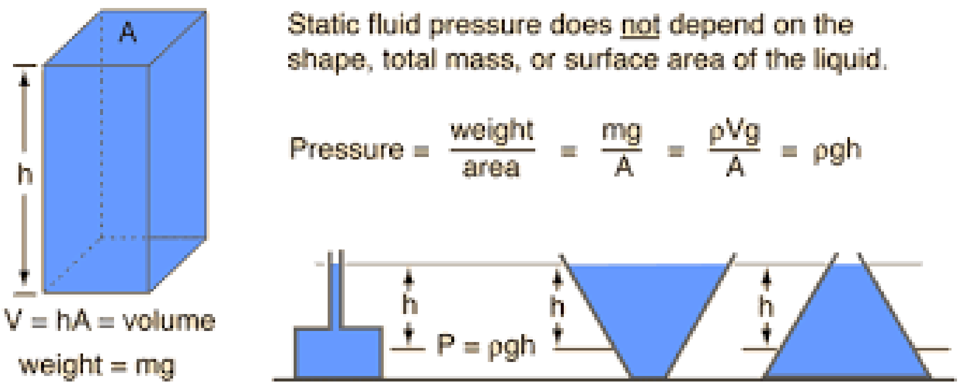
Example 4: A vacuum gauge attached to a chamber indicates a pressure of 40 kPa. At the location, the atmospheric pressure is 100 kPa. The absolute pressure in the chamber is then measured**.**

We know, Absolute Pressure = Vacuum Pressure – Atmospheric pressure
Vacuum pressure = 40 kPa, Atmospheric pressure = 100 kPa

Absolute pressure = 100 – 40 = 60 kPa

1.3: PRESSURE RELATED TO DEPTH

The deeper the object is placed in the fluid, the more pressure it experiences. This is because is the weight of the fluid above it. The denser the fluid above it, the more pressure is exerted on the object that is submerged, due to the weight of the fluid. The amount of pressure that it exerts depends on the height of the water. The higher the height of the water the greater the pressure exerted at the base of that container or pipe. For example, the water in the glass above exerts a pressure on all sides of the glass including the bottom.



To understand more about PRESSURE related to DEPTH,
Press Ctrl + Click link https://youtu.be/sEVMbcYu8rM?si=FI2EI82sPc75bC5U

Pressure formula: $$P = \rho g h$$

where
ρ – density
g – acceleration of gravity
h - height of the fluid

Example 1: The pressure at the top of a pipe full of water is 101 pascals. What is the change in pressure between the top and the bottom of the pipe, 3.4 meters lower?

As known, $\rho_{water} = 1000\ \mathrm{kg/m^3}$, $g = 9.81\ \mathrm{m/s^2}$
Given h = 3.4 m

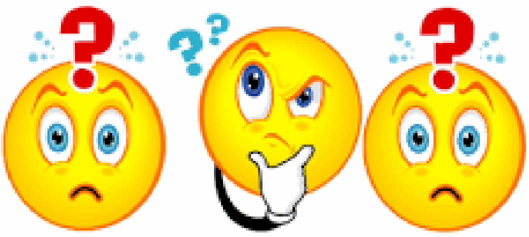
Use formula $P = \rho g h$
= 1000 X 9.81 X 3.4
= 3.3 X 10^4 Pa

Example 2: One end of a 50-meter-long hose is attached to the bottom of a large basin full of water. How many meters below the top of the basin must the hose outlet be positioned for the water pressure at the outlet to be 18,000 pascals?

As known, ρ_{water} = 1000 kg/m3 , g = 9.81 m/s2
Given P = 18000 Pa

Use formula $P = \rho g h$, to find h

$h = P / \rho g$
= 18000 / (1000) X (9.81)
= 1.8 m



Test yourself

Press Ctrl + Click link

https://homework.study.com/learn/pressure-questions-and-answers.html

EXERCISES

QUESTION 1:

Assume the density of water to be 1000 Kg/m^3 at atmospheric pressure 101 KN/m^2, what will be :

(a) The Gauge Pressure,
(b) The absolute pressure of water at a depth of 2000 m below the free surface ?

QUESTION 2:

Calculate the absolute pressure of air in the compressor cylinder if the pressure gauge is 2500 N/m^2. (Assume atmospheric gauge is 101.3 KN/m^2).

QUESTION 3:

Head of a gas that shows by a pressure measurement tool is 68 mm water while head mercury of atmospheric pressure is 750 mm mercury. What is the value of absolute pressure in KN/m^2 for water & Mercury?

QUESTION 4:

What is the pressure gauge of air in the cylinder, if the atmospheric gauge is 101.3 KN/m^2 and absolute pressure is 500 KN/m^2.

Chapter 2: Physical Properties of Fluid

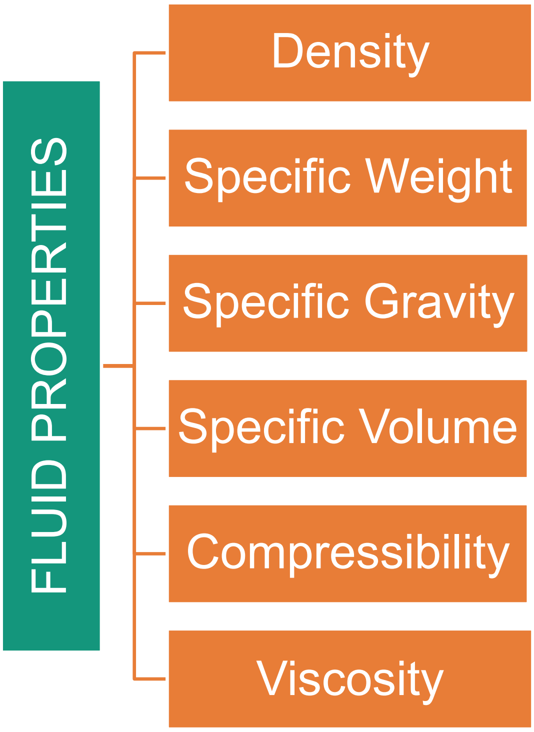
PHYSICAL PROPERTIES

ABSTRACT

This topic covers This topic covers the physical properties of fluid, determination of effectiveness on physical properties etc

Physical Properties of Fluid

In general, the physical properties of Fluids are Density, Viscosity and Surface tension among the other fluid properties. The important properties of fluids along with Density, Viscosity and surface tension. **The properties of fluids specifically are density, temperature, internal energy, pressure, specific volume and specific weight**. The intermolecular forces determine the physical properties of the liquid.



DENSITY OR MASS DENSITY

The density or mass of a fluid is defined as the mass of a fluid to its volume. Thus, mass per unit volume of a fluid is called density. Density of liquids may be considered a constant while gases change with the variation of pressure and temperature.

The formula of mass density (ρ) is written as

ρ = Mass of fluid / Volume of fluid
= m/V

** The value of the density of water is gm/cm^3 or 1000 kg/m^3*

SPECIFIC WEIGHT OR WEIGHT DENSITY

Specific weight or weight density of the fluid is the ratio between the weight of a fluid and its volume. Thus, weight per unit volume of a fluid is called weight density and it is denoted by the symbol w.

The formula of specific weight **(ω)** is written as

ω = Weight of fluid / Volume of fluid

$$\omega = \frac{\text{(Mass of fluid × Acceleration but to gravity)}}{\text{Volume of fluid}}$$

$$\frac{\text{(Mass of fluid) × (Acceleration but to gravity)}}{\text{Volume of fluid}}$$

ω = ρ×g

** The value of specific weight density (ω) for water is 9810 Newton /m³ in SI units*

SPECIFIC VOLUME

The specific volume of a fluid is defined as the volume of fluid occupied by a unit mass or volume per unit mass of fluid called specific volume.

The formula of specific weight **(v)** is written as

v = Volume of fluid / Mass of fluid

The above expression can be written as reversing formula of density = 1 / (Mass of fluid /Volume of fluid)

v = 1/ρ

Thus, the specific volume is the reciprocal of mass density.

The SI units for Specific Volume is m^3/kg.

**Specific volume is commonly applied to gases.*

SPECIFIC GRAVITY

Specific gravity is defined as the ratio of weight density (or density) of fluid to the weight density (or density) of a standard fluid. As for liquids, the standard fluid is taken water and for gases, the standard fluid is taken air. Specific gravity is also called relative density. It is a dimensionless quantity and is denoted by the symbol S.

The formula of specific weight (S) is written as,

$$S\ (\text{liquid}) = \frac{\text{Weight density (density) of } \textit{liquid}}{\text{Weight density (density) of } \textit{water}}$$

$$S\ (\text{gases}) = \frac{\text{Weight density (density) of gas}}{\text{Weight density (density) of air}}$$

Thus, weight density of a liquid = S × Weight density of water = S × 1000 × 9.81 N/m^3

The density of a liquid = S × Density of water = S × 1000 kg/m^3.

If the specific gravity of fluid is known, then the density of the fluid will be equal to the specific gravity of fluid multiplied by the density of water.

For example, the specific gravity of mercury is 13.6,

$$S\ (\text{liquid}) = \frac{\text{(density) of mercury}}{\text{(density) of water}}$$

$$13.6 = \frac{\text{density of mercury}}{1000}$$

Hence, density of mercury =13.6 ×1000= 13600 kg/m^3

COMPRESSIBILITY

Compressibility is a measure of the relative volume change of a fluid or solid as a response to a pressure (or mean stress) change.

$$\beta = -\frac{1}{V}\frac{\partial V}{\partial p}$$

The volume of real fluids changes when they are expanded or compressed by an external force or the change of pressure or temperature. The property of volume change is called compressibility and a fluid whose volume changes is called compressible fluid. Watch this

Press Ctrl + CLICK https://youtu.be/-vZqSliZzSs?si=jPSpa_46_iE-dINX

VISCOSITY

Viscosity is a property of a fluid that determines its flow behaviour. The higher the viscosity the less easily the fluid can flow. Air has a low viscosity compared to water, and water a low viscosity compared to an automotive gear oil. It is the internal resistance to flow possessed by a liquid. The liquids which flow slowly, have high internal resistance. This is because of the strong intermolecular forces. Therefore, these liquids are more viscous and have high viscosity. To understand more about viscosity, watch this video

Press Ctrl + CLICK https://youtu.be/9NYs3Y-IjGw?si=86xJMOSykj6b6Tgh

PROBLEM EXAMPLES

Example 1: Calculate the specific weight, density and specific gravity of a liquid which weighs 7N.

Given: Volume (V) =1 litre =1/1000 m^3
Weighs of liquid (W) = 7N
Therefore,

(i) Specific weight (ω) = W / V
= 7N / [1/1000]m^3
= 7000 N/m^3.
(ii) Density (ρ) = ω / g
= 7000 / 9.81
= 731.5 kg /m^3
(iii) Specific gravity (S) = density of liquid / density of water
= 731.5 / 1000
= 0.731.5

Example 2: Calculate the density, specific weight and weight of one litre of specific gravity = 0.7

Given: Volume (V) = 1 litre = 1×1000 cm^3 = 1000 cm^3 = 0.001 m^3
Specific gravity (S) = 0.7

(i) Density (ρ) = S × Density of water
= S ×1000
= **0.7 ×1000 = 700 kg/m^3**
(ii) Specific weight (ω) = ρ×g
= 700 x 9.81
= **6867 N/m^3**
(iii) Weight (W) = Specific weight(ω) × Volume of fluid
= 6867 × 0.001
= **6.867 N**

EXERCISES

QUESTION 1

What is the specific weight "ω" of fluid in KN/m^3 if the weight of fluid is 15 N and the volume is 750 cm^3. After get the answer, find:

(a) Specific Gravity
(b) Specific Volume
(c) Mass in gram

QUESTION 2:

Oil in a cylinder with temperature 40℃ has its volume and mass 5.5 m^3 and 8500kg respectively.

Calculate:
(a) Mass density
(b) Relative density
(c) Specific weight
(d) Specific volume

QUESTION 3:

If the mass and the respective volume of air is 0.007 Tan and 717 cm^3, calculate:

(a) Mass density
(b) Specific weight
(c) Specific Volume
(d) Relative density of the air

QUESTION 4:

The weight of 1 litre of fluid is 7.05 N. Calculate:-

(a) Mass density
(b) Specify weight
(c) Specific volume
(d) Specific gravity

Chapter 3: FLUID STATICS

3.0 INTRODUCTION

1. Fluid statics are fluids that are at rest, while fluids in motion are called fluid dynamics.

2. A good example is when you drink using a straw: you reduce the pressure at the top of the straw, and the atmosphere pushes the liquid up the straw and into your mouth.

Figure 1: Blaise Pascal

3. Pascal's law was discovered by Blaise Pascal. Born in France, 19 August 1623. Since childhood he has not attended school because he is sick. Even so, he is still diligent in studying from home. At the age of 16, Pascal was able to write a small book on cones. This enabled him to create the world's first digital calculator at the age of 18.

4. Pascal's law states that pressure applied on one point of liquid transmits equally in all direction.

Application of Pascal's Law

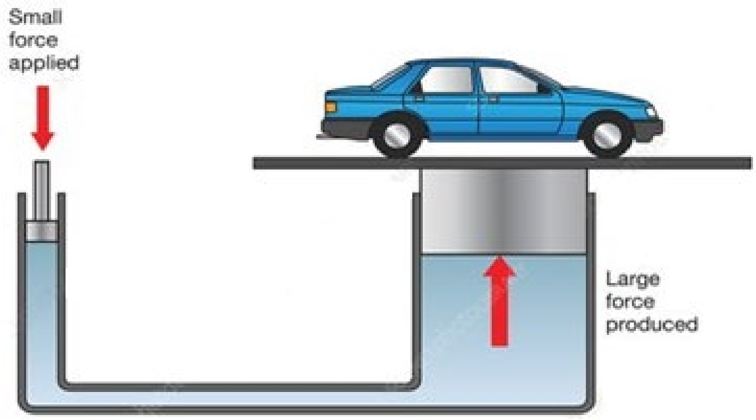


Figure 2: Simple hydraulic lift

1. Hydraulic Lift- Figure 3 is the principle of working of hydraulic lift. It works based on the principle of equal pressure transmission throughout a fluid.
2. The construction is such that a narrow cylinder (in this case A – Figure 2) is connected to a wider cylinder (in this case B – Figure 2). They are fitted with airtight pistons on either end. The inside of the cylinders is filled with fluid that cannot be compressed.
3. Pressure applied at piston A (Figure 2) is transmitted equally to piston B without diminishing, on the use of the fluid that cannot be compressed. Piston B effectively serves as a platform to lift heavy objects like big machines or vehicles.
4. A few more applications include a hydraulic jack and hydraulic press and forced amplification is used in the braking system of most cars.

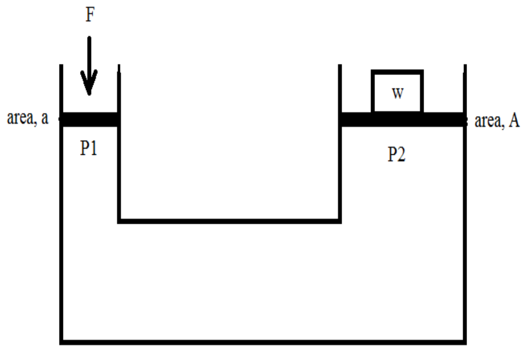
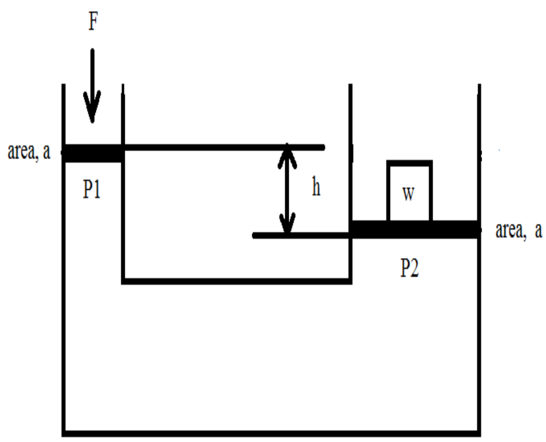
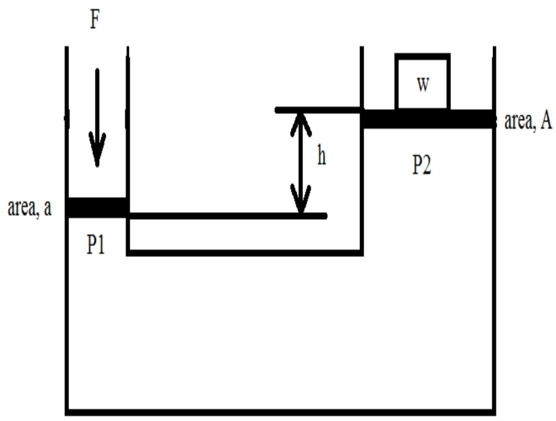
$$P_1 = P_2$$

But since $P_2 = \frac{F_2}{A_2}$, we see that,

$$\frac{F_1}{A_1} = \frac{F_2}{A_2}$$

3.1 HYDRAULIC JACK

TABLE 1: SMART NOTES FOR HYDRAULIC JACK

CASE	FIGURE	FORMULAS
SAME LEVEL		$P_1 = \frac{F}{a}$, $P_2 = \frac{W}{A}$ $\frac{F}{a} = \frac{W}{A}$ $W = \frac{FA}{a}$ or $W = mg$
LEFT ⬆ HIGH THAN RIGHT		$P_2 = P_1 + \rho g h$ $P_1 = \frac{F}{a}$, $P_2 = \frac{W}{A}$ $\frac{W}{A} = \frac{F}{a} + \rho g h$, $W = \left(\frac{F}{a} + \rho g h\right) A$ or $W = mg$
LEFT ⬇ LOW THAN RIGHT		$P_1 = P_2 + \rho g h$ $P_1 = \frac{F}{a}$, $P_2 = \frac{W}{A}$ $\frac{F}{a} = \frac{W}{A} + \rho g h$, $W = \left(\frac{F}{a} - \rho g h\right) A$ or $W = mg$

Example 1:

A force, F of 650N is applied to the small cylinder of a hydraulic jack. The area of a small piston is 15cm^2 and the area, A of a larger piston is 150cm^2. What load can be lifted on the larger piston if,

I. The pistons are at a same level.

ii. The larger piston is 0.65m below the smaller piston.

iii. The smaller piston is 0.40m below the larger piston.

Solution:

The pistons are at a same level.

$$P_1 = P_2$$

$$\frac{F}{a} = \frac{W}{A}$$

$$\frac{650}{15 \times 10^{-4}} = \frac{W}{150 \times 10^{-4}}$$

$$W = 6500N$$

The larger piston is 0.65m below the smaller piston.

$$P_1 + \rho gh = P_2$$

$$\frac{F}{a} + \rho gh = \frac{W}{A}$$

$$\frac{650}{15 \times 10^{-4}} + (1000 \times 9.81 \times 0.65) = \frac{W}{150 \times 10^{-4}}$$

$$W = 6595N$$

The smaller piston is 0.40m below the larger piston.

$$P_1 = P_2 + \rho gh$$

$$\frac{F}{a} = \frac{W}{A} + \rho gh$$

$$\frac{650}{15 \times 10^{-4}} = \frac{W}{150 \times 10^{-4}} + (1000 \times 9.81 \times 0.65)$$

$$W = 6441N$$

Example 2:

In Figure 3 below, the areas of plunger A and cylinder B are 66 mm^2 and 666 mm^2, respectively, and the weight of B is 9600 kN. The channel connecting A and B filled with oil having a specific gravity of 0.88. What force, F is required for equilibrium?

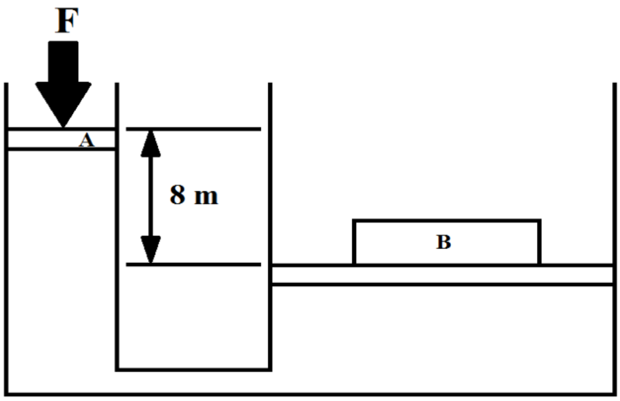


Figure 3

Given:

$A_A = 66\, mm^2 = 66\ x\ 10^{-6}\, m^2$

$A_B = 666\, mm^2 = 666\ x\ 10^{-6}\, m^2$

$W_B = 9600\, kN$

$S_O = 0.88$

h = 8m

Solution:

$$P_2 = P_1 + \rho_O g h$$

$$P_1 = \frac{F}{A_A} \quad , \quad P_2 = \frac{W_B}{A_B}$$

$$\frac{W_B}{A_B} = \frac{F}{A_A} + \rho_O g h$$

$$\frac{9600\ x\ 10^3}{666\ x\ 10^{-6}} = \frac{F}{66\ x\ 10^{-6}} + (0.88)(1000)(9.81)(8)$$

$$14414414.41 = \frac{F}{66\ x\ 10^{-6}} + 69062.4$$

$$F = 946.793\, N$$

Example 3:

A hydraulic press has a diameter ratio between the two piston 8:1. The diameter of the larger piston is 600 mm and it is required to support mass of 3500 kg. The press is filled with a hydraulic fluid of specific gravity 0.8. Calculate the force required on the smaller piston, when it position is 2.6 m below the larger piston.

Given:
D : d
8 : 1
600mm: 75mm

Solution:

$$A = \frac{\Pi D^2}{4}$$

$A = \pi (600 \times 10^{-3})^2/4$
$= 0.282 \text{ m}^2$

$a = \pi d^2/4$
$= \pi (75 \times 10^{-3})^2/4$
$= 4.417 \times 10^{-3} \text{ m}^2$

$P_1 = F/a$
$P_2 = W/A$

$P_1 = P_2 + \rho.g.h$

$F/a = W/A + \rho.g.h$

$F = (W/A + \rho.g.h)\ a$

$= (mg/A + \rho.g.h)\ a$

$= [\ (3500 \times 9.81/0.282) + (0.8 \times 1000 \times 9.81 \times 2.6)]\ 4.417 \times 10^{-3}$

$= 627.92$ N

3.2 PRESSURE MEASUREMENT

Manometer

A manometer usually limited to measuring pressures near to atmospheric. The term manometer is often used to refer specifically to liquid column hydrostatic instruments.

Three types of manometer:

i.Simple U-tube manometer

ii.Differential U-tube manometer

iii.Inverted U-tube manometer

Simple U-tube Manometer

A simple manometer consists of a tubular arrangement where one end of the tube is connected to the point in the fluid, whose pressure is to be determined and the other end is kept open to the atmosphere. Simple manometers can be used to determine the gauge pressure or vacuum pressure.

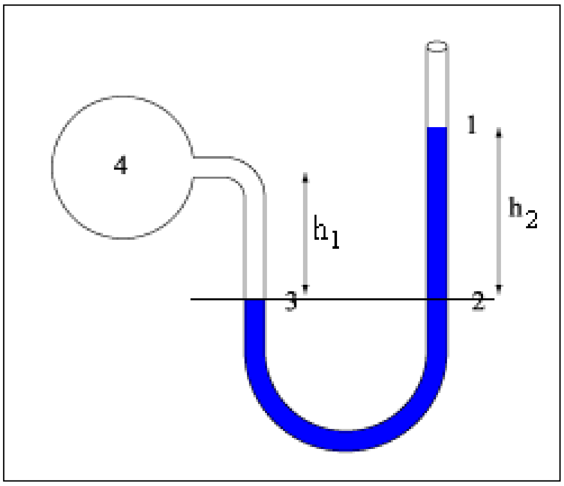


Figure 4: Simple U-tube manometer

$$P_3 = P_2$$

Left limb	Right limb
$P_3 = P_4 + \rho g h_1$	$P_2 = P_1 + \rho g h_2$

Example 4:

A U-tube manometer in figure 5 is used to measure the gauge pressure of a fluid P of density 1000 kg/m^3. If the density of the liquid Q is 13600 kg/m^3, what will be the gauge pressure at A if h_1 = 0.15 m and h_2 = 0.25 m above the BC. Take into consideration P_{atm} = 101.3 kN/m^2.

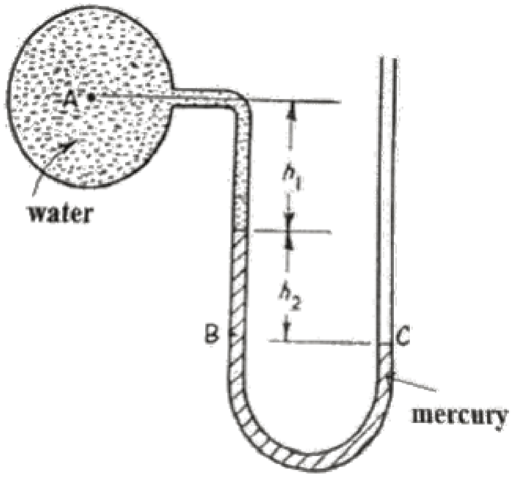


Figure 5

Solution

Left:

$$
\begin{aligned}
P_B &= P_A + \rho_{water} g h_1 + \rho_{Hg} g h_2 \\
&= P_A + (1000 \times 9.81 \times 0.15) + (13600 \times 9.81 \times 0.25) \\
&= P_A + 1471.5 + 33354 \\
&= P_A + 34825.5
\end{aligned}
$$

Right:

$$P_C = P_{atm} = 101.3 \ kN/m^2$$

$$P_B = P_C$$

$$P_A + 34825.5 = 101.3 \times 10^3$$

$$P_A = 66474.5$$

$$= 66.47 \, kN/m^2$$

Example 5:

What is the gauge pressure of the water at A if h_1 = 0.6 m and the mercury in the right hand limb, h_2 = 0.9 m as shown in the Figure 6.

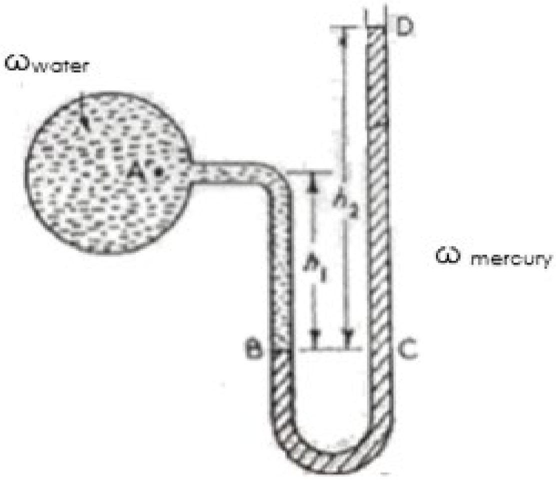


Figure 6 : Simple manometer

Solution

Left:

$$\begin{aligned} P_B &= P_A + \omega_{water} h_1 \\ &= P_A + (9810 \times 0.6) \\ &= P_A + 5885 \end{aligned}$$

Right:

$$\begin{aligned} P_C &= P_D + \omega_{Hg} h_2 \\ &= P_D + (133416 \times 0.9) \\ &= P_D + 120074.4 \end{aligned}$$

$$P_B = P_C$$

$$P_A + 5886 = P_D + 120074.4$$

$$\begin{aligned} P_A &= 120074.4 - 5886 \\ &= 114188.4\, N/m^2 \\ &= 114.19\, kN/m^2 \end{aligned}$$

Example 6:

A U-tube manometer like Figure 7 is used to measure the gauge pressure of water (mass density = 1000 kg/m^3). If the density of mercury is13600 kg/m^3, what will be the gauge pressure at A if h_1 is 0.45 m, and D is 0.7 m above the BC.

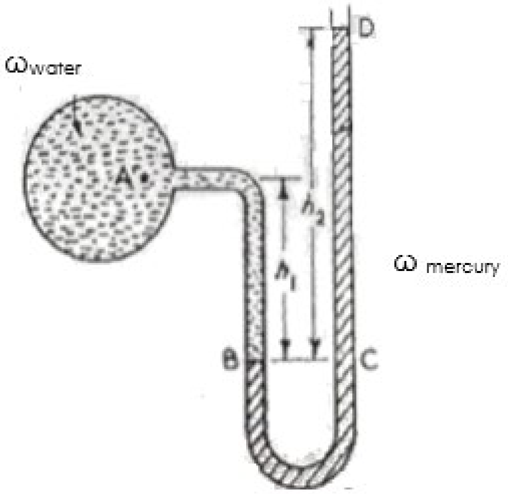


Figure 7 : Simple manometer

Solution

Left:

$$\begin{aligned} P_B &= P_A + \omega_{water} h_1 \\ &= P_A + (9810 \times 0.45) \\ &= P_A + 4414.5 \end{aligned}$$

Right:

$$\begin{aligned} P_C &= P_D + \omega_{Hg} h_2 \\ &= P_D + (133416 \times 0.7) \\ &= P_D + 93391.2 \end{aligned}$$

$$P_B = P_C$$

$$\begin{aligned} P_A + 4414.5 &= P_D + 93391.2 \\ P_A &= 93391.2 - 4414.5 \\ &= 88976.7\, N/m^2 \\ &= 88.98\, kN/m^2 \end{aligned}$$

Differential U-tube Manometer

It is used to measure pressure difference at two points in a pipe or between two pipes at different levels.

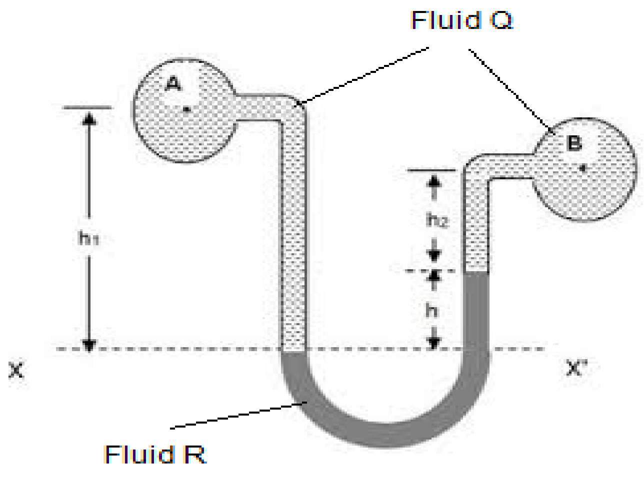


Figure 8: Differential U-tube manometer

$$P_X = P_{X'}$$

Left limb	Right limb
$P_X = P_A + \rho_Q g h_1$	$P_{X'} = P_B + \rho_Q g h_2 + \rho_R g h$

Example 7:

A U-tube manometer contains oil, mercury, and water as shown in Figure 9. For the column heights indicated what is the pressure differential between pipes A and B?

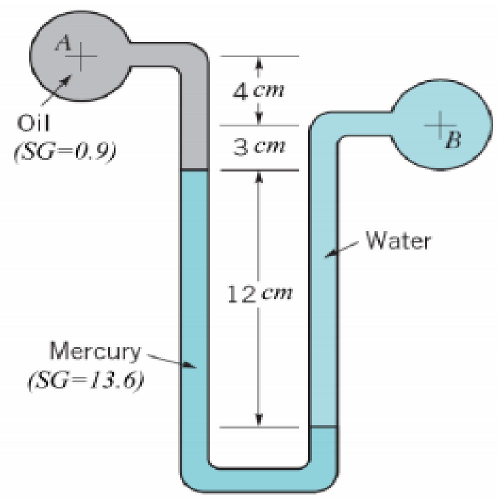


Figure 9: Differential manometer

Solution

Left:

$$
\begin{aligned}
P_C &= P_A + \rho_{oil} g h_1 + \rho_{Hg} g h_2 \\
&= P_A + (0.9 \times 1000 \times 9.81 \times 0.07) + (13.6 \times 1000 \times 9.81 \times 0.12) \\
&= P_A + 618.03 + 16900.92 \\
&= P_A + 166627.95
\end{aligned}
$$

Right:

$$
\begin{aligned}
P_D &= P_B + \rho_{water} g h_3 \\
&= P_B + (1000 \times 9.81 \times 0.15) \\
&= P_B + 1471.5
\end{aligned}
$$

Therefore:

$$
\begin{aligned}
P_C &= P_D \\
P_A + 166627.95 &= P_B + 1471.5 \\
P_A - P_B &= 1471.5 - 166627.95 \\
&= -165156.45
\end{aligned}
$$

$$\therefore P_B - P_A = 165156.45\, N/m^2 = 165.2\, kN/m^2$$

Example 8:

Figure 10 shows a differential manometer of water (fluid P) and mercury (fluid Q) is used. If pressure difference between point A and B is 47 kN/m2, h = 1.2 m and a = 42 cm, find the level value of b. (Specific gravity of mercury = 13.6)

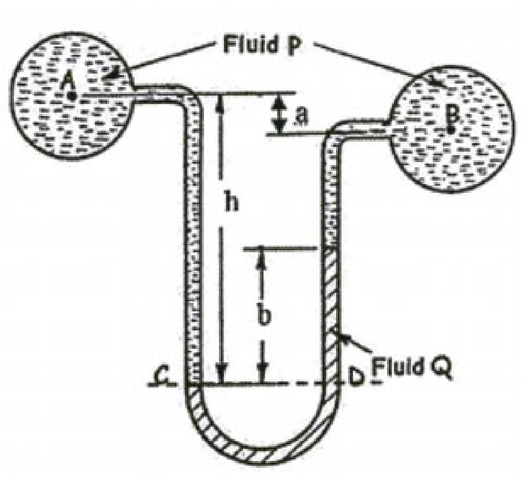


Figure 10: Differential manometer

Solution

Given;

$$P_A - P_B = 47\ kN/m^2$$

$$h = 1.2\ m;\ a = 42\ cm = 0.42\ m;\ \ b = ?$$

$$\begin{aligned} P_C &= P_A + \rho_{water} g h \\ &= P_A + (1000 \times 9.81 \times 1.2) \\ &= P_A + 11772 \end{aligned}$$

$$\begin{aligned} h &= a + x + b \\ 1.2 &= 0.42 + x + b \\ x + b &= 0.78 \\ b &= 0.78 - x \end{aligned}$$

$$\begin{aligned} P_D &= P_B + \rho_{water} g a + \rho_{Hg} g b \\ &= P_B + (1000 \times 9.81 \times 0.42) + (13600 \times 9.81 \times b) \\ &= P_B + 4120.2 + 133416b \\ &= P_B + 4120.2 + 133416(0.78 - x) \\ &= P_B + 4120.2 + 104064.48 - 133416x \end{aligned}$$

$$P_C = P_D$$

$$\begin{aligned} P_A + 11772 &= P_B + 4120.2 + 104064.48 - 133416x \\ P_A - P_B &= 4120.2 - 11772 + 104064.48 - 133416x \\ 47 \times 10^3 &= 96412.68 - 133416x \\ -49412.68 &= -133416x \\ x &= 0.37\ m \end{aligned}$$

$$\begin{aligned} x + b &= 0.78 \\ b &= 0.78 - 0.37 \\ &= 0.41\ m \end{aligned}$$

Example 9:

A U-tube manometer contains oil, mercury, and water as shown Figure 11. For the column heights indicated what is the pressure differential between pipes A and B?

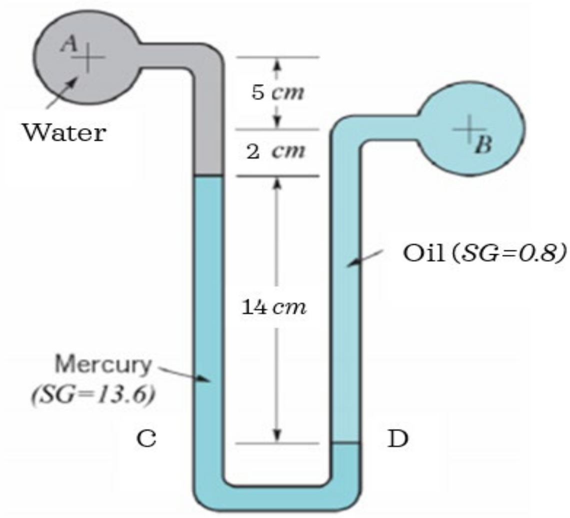


Figure 11: U-tube manometer

Solution

Note:

h_1= *5 cm* + 2 *cm* = *7* cm + 0.07 *m*

h_2= 14 *cm* = 0.14 *cm*

h_3= 14 *cm* + 2 *cm* = 16 *cm*= 0.16 *m*

Left:

$$\begin{aligned} P_C &= P_A + \rho_{water} g h_1 + \rho_{Hg} g h_2 \\ &= P_A + (1000 \times 9.81 \times 0.07) + (13.6 \times 1000 \times 9.81 \times 0.14) \\ &= P_A + 618.03 + 18678.24 \\ &= P_A + 19296.27 \end{aligned}$$

Right:

$$\begin{aligned} P_D &= P_B + \rho_{oil} g h_3 \\ &= P_B + (0.8 \times 1000 \times 9.81 \times 0.15) \\ &= P_B + 1177.2 \end{aligned}$$

$$\begin{aligned} P_C &= P_D \\ P_A + 19296.27 &= P_B + 1177.2 \\ P_A - P_B &= 1177.2 - 19296.27 \\ &= -18119.07 \end{aligned}$$

$$\therefore P_B - P_A = 18119.07\, N/m^2 = 18.12\, kN/m^2$$

Inverted U-tube Manometer

Inverted U-tube manometer is used for measuring pressure differences in liquids. The space above the liquid in the manometer is filled with air which can be admitted or expelled through the tap on the top, in order to adjust the level of the liquid in the manometer. For inverted U - tube manometer the manometric fluid is usually air.

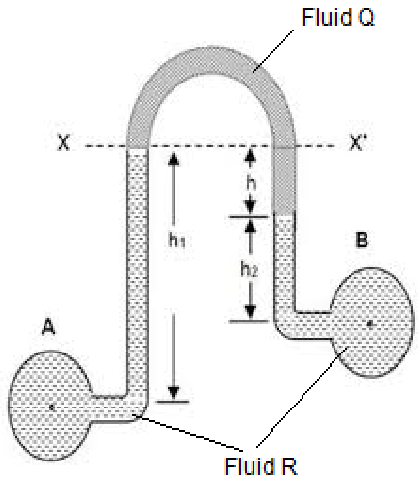


Figure 12: Inverted U-tube manometer

$$P_X = P_{X'}$$

Left limb	Right limb
$P_X = P_A - \rho_R g h_1$	$P_{X'} = P_B - \rho_R g h_2 -$

Example 10:

An inverted tube differential manometer in Figure 13 having an oil of specific gravity 0.9 is connected to two different pipes carrying water under pressure. Determine the pressure in the pipe B. The pressure in pipe A is 20 kN/m^2.

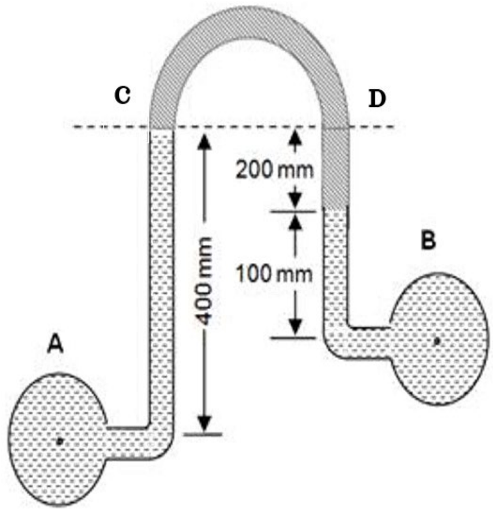


Figure 13: Inverted U-tube manometer

Solution

Given;

$$P_A = 20 \ kN/m^2$$

$$\begin{aligned} P_C &= P_A - \rho_{water} gh \\ &= 20 \times 10^3 - (1000 \times 9.81 \times 0.4) \\ &= 16076 \end{aligned}$$

$$\begin{aligned} P_D &= P_B - \rho_{water} gh - \rho_{oil} gh \\ &= P_B - (1000 \times 9.81 \times 0.1) - (900 \times 9.81 \times 0.2) \\ &= P_B - 981 - 1765.8 \\ &= P_B - 2746.8 \end{aligned}$$

$$P_C = P_D$$

$$\begin{aligned} 16076 &= P_B - 2746.8 \\ P_B &= 16076 + 2746.8 \\ P_B &= 18822.8 \, N/m^2 \\ P_B &= 18.82 \, kN/m^2 \end{aligned}$$

EXERCISES

1) A hydraulic jack fluid containing water has a small diameter of 80 mm piston surface and piston surface diameter of 650 mm. The surface of the small piston 6 m above the large piston. What is the force, F is required on the small piston to lift the load of 3500 kg on the piston?

2) The diameters of plunger and ram of a hydraulic press are 30mm and 200mm respectively. Find the weight lifted by the hydraulic press when the force applied at the plunger is 400N and the difference level between plunger and ram is 0.5m. Give ρ fluid is $1065 kg/m^3$.

3) In Figure 14, the areas of the plunger A and cylinder B are $60 mm^2$ and $600 mm^2$, respectively, and the weight of B is 9000 kN. The channel connecting A and B filled with oil having a specific gravity of 0.75. What force, F is required for equilibrium?

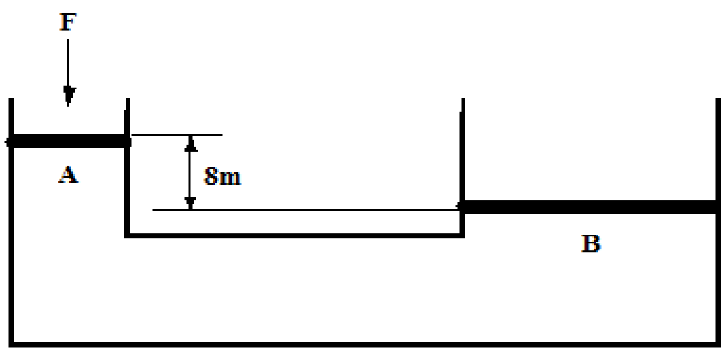


Figure 14

4) A force, F of 475 N is applied to the smaller cylinder of a simple hydraulic jack. The diameter of large piston is 18cm and the diameter of smaller piston is 7 cm. Calculate the load, W which can be lifted on the large piston if it is 0.75 m put below the smaller piston. (Assume the specific gravity oil used is 0.85)

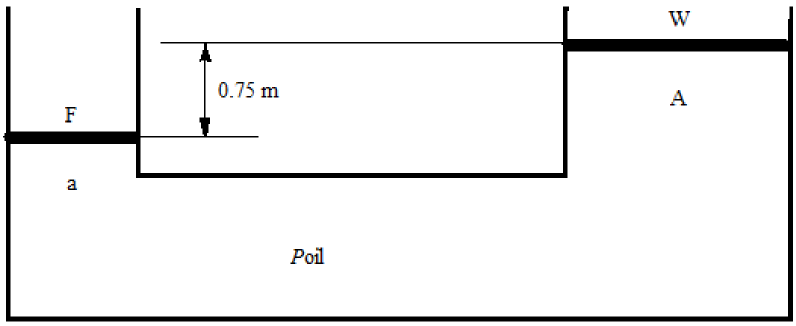


Figure 15

5. Find the pressure difference between A and B in KN/m^2 for figure 16.

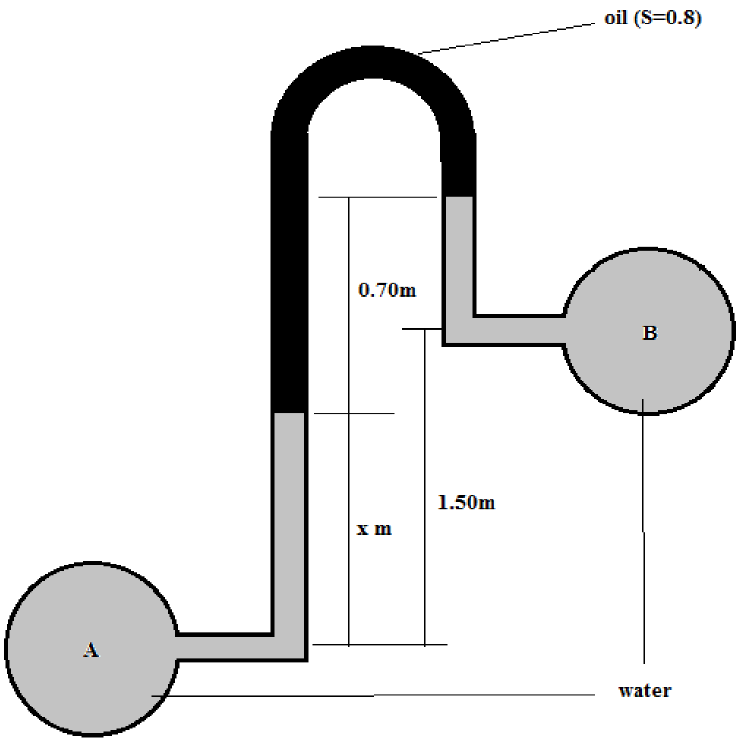


Figure 16

Chapter 4: FLUID DYNAMICS

4.0 INTRODUCTION

Fluid dynamics is "the branch of applied science that is concerned with the movement of liquids and gases," according to the American Heritage Dictionary. Fluid dynamics is one of two branches of fluid mechanics, which is the study of fluids and how forces affect them. (The other branch is fluid statics, which deals with fluids at rest.)

Types of Flow

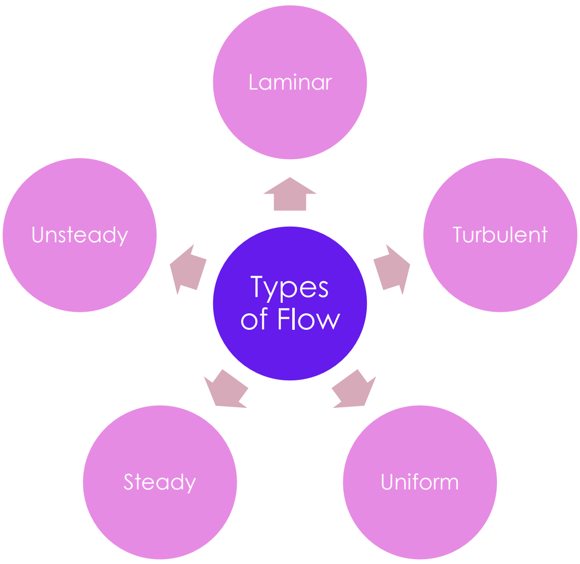


Figure 1: Types of flow

Laminar flow:

Fluid flow in which the fluid travels smoothly or in regular paths. The velocity, pressure, and other flow properties at each point in the fluid remain constant. Examples include the flow of oil through a thin tube and blood flow through capillaries.

Turbulent flow:

Fluid flow in which the fluid undergoes irregular fluctuations, or mixing. The speed of the fluid at a point is continuously undergoing changes in magnitude and direction, which results in swirling and eddying as the bulk of the fluid moves in a specific direction. Common examples of turbulent flow include atmospheric and ocean currents, blood flow in arteries, oil transport in pipelines, lava flow, flow through pumps and turbines, and the flow in boat wakes and around aircraft wing tips.

Uniform flow:

Flow of a fluid in which each particle moves along its line of flow with constant speed and in which the cross section of each stream tube remains unchanged.

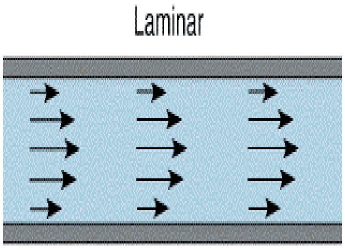
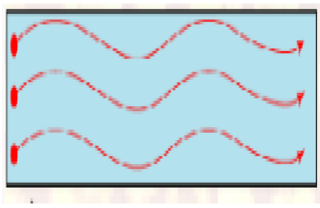
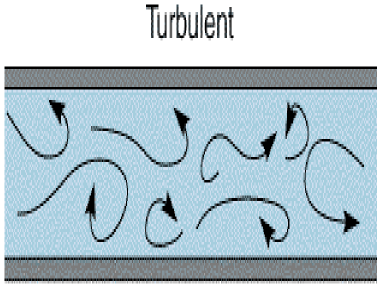
Steady flow:

Conditions does not change with position or time. The velocity and cross-sectional area of the stream of fluid are the same at each cross-section: for example, flow of a liquid through a pipe of uniform bore running completely full at constant velocity.

Unsteady flow:

At a given instant of time the velocity at every point is the same, but this velocity will change with time: for example, accelerating flow of a liquid through a pipe of uniform bore running full, such as would occur when a pump is started up.

Table 1: The differences between laminar, transition and turbulent

Laminar flow	Transition	Turbulent flow
		
Renault number less than 2300	2300<Re<4000	Renault number more than 4000
Laminar have parallel streamlines where fluid particles do not cut the paths of others	The particles followed wavy but parallel path that was not parallel to the boundaries of the tube	Turbulent particles are in zig zag way, they cut paths of each other’s
Laminar have low velocity, inelastic steady flow	Transitional flow is a mixture of laminar and turbulent flow, with turbulence in the centre of the pipe, and laminar flow near the edges. Each of these flows behave in different manners in terms of their frictional energy loss while flowing and have different equations that predict their behaviour	Turbulent have high velocity

4.1 FLUID FLOW

Flow rate is referring to flow measurement which is a quantification of bulk fluid movement.

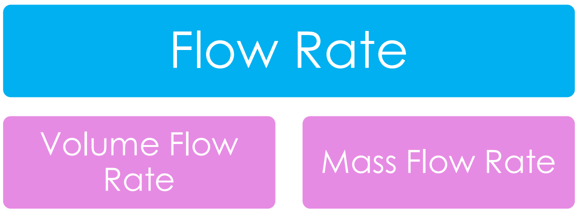


Figure 2: Types of flow rate

4.1.1 Volume Flow Rate

Volume flow rate is the volume of fluid which passes through a given surface per unit time. It is usually represented by the symbol, Q. It is measured in cubic meter per second (m^3/s).

Volume Flow Rate Formula

Given an area, A, and a fluid flowing through it with uniform velocity, v, the flow rate is:

$$Q = Av$$

Where:

Q is the volumetric flow rate

v is the velocity

A is the flow area

Example 1:

If the diameter, d = 15cm and the mean velocity, v = 3 m/s. Calculate the actual discharge in the pipe.

Solution:

$$Q = Av$$

$$= \frac{\pi(0.15)^2}{4} \times 3$$

$$= 0.053\ m^3/s$$

4.1.2 Mass Flow Rate

Mass flow rate is the mass of substance which passes through a given surface per unit time. It is usually represented by the symbol, $\dot{m}$. It is measured in kilogram per second (kg/s). Mass flow rate can be calculated from the density of the substance, the cross-sectional area through which the substance is flowing, and its velocity relative to the area of interest.

Mass Flow Rate Formula

$$\dot{m} = \rho\, v\, A$$

Where:

$\dot{m}$ is the mass flow rate
ρ is the density

v is the velocity

A is the flow area

This is equivalent to multiplying the volumetric flow rate by the density.

$$\dot{m} = \rho \cdot Q$$

Where:

ρ is the density

Q is the volumetric flow rate

Example 2:

Oil flows through a pipe at a velocity of 1.6 m/s. The diameter of the pipe is 8cm. Calculate the discharge and mass flow rate of oil. Consider s_{oil} =0.85.

Solution

$Q = Av$

$= \frac{\pi(0.08)^2}{4} \times 1.6$

$= 8.042 \times 10^{-3}\ m^3/s$

$\dot{m} = \rho Q$

$= (0.85 \times 1000) \times 8.042 \times 10^{-3}$

$= 6.836\ kg/s$

4.2 CONTINUITY EQUATION

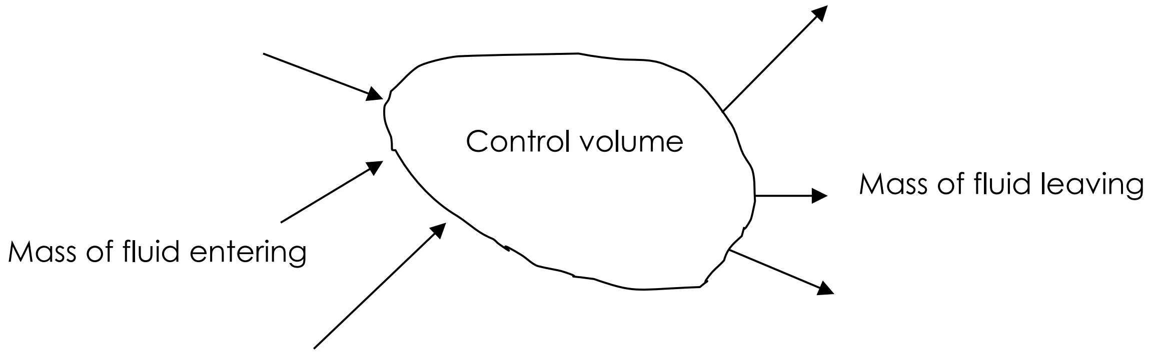


Figure 3: Continuity of flow

For continuity of flow in any system of fluid flow, the total amount of fluid entering the system must equal the amount leaving the system. This occurs in the case of steady flow and uniform flow.

$$total\ amount\ of\ fluid\ entering\ the\ system = total\ amount\ of\ fluid\ leaving\ the\ system$$

$$Q_1 = Q_2$$

$$\dot{m}_1 = \dot{m}_2$$

The continuity equation can also be applied to determine the relation between the flows into and out of a junction.

$$total\ inflow\ to\ junction = total\ outflow\ from\ junction$$
$$Q_1 = Q_2 + Q_3$$

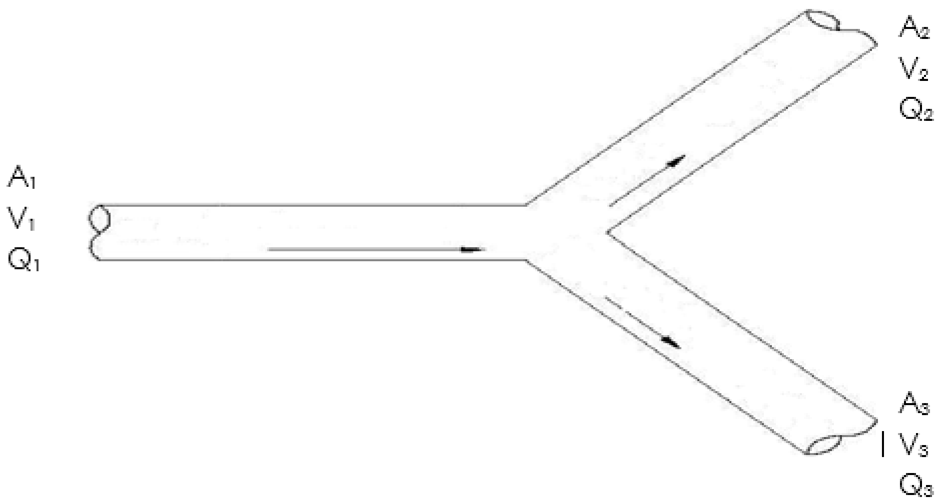


Figure 4: Application of the continuity equation

Example 3:

If the area, $A_1 = 10 \times 10^{-3} m^2$and $A_2 = 3 \times 10^{-3} m^2$and the upstream mean velocity, $v_1 = 2.1\ m/s$, calculate the downstream mean velocity.

Solution:

$$v_2 = \frac{A_1 v_1}{A_2}$$

$$= \frac{(10 \times 10^{-3}) \times 2.1}{3 \times 10^{-3}}$$

$$= 7 m/s$$

4.3 BERNOULLI THEOREM

Bernoulli's theorem is a statement on the law of the conservation energy. It states that energy can be neither created nor destroyed, but merely changed from one form to another. Bernoulli's principles state that:

In steady flow of fluid, the pressure of the fluid decreases when the velocity of the fluid increases.

Bernoulli's principle is very important as it is used in the design of airplanes, boat hulls, fan blades and cars.

Bernoulli's Theorem Formula

$$z + \frac{P}{\omega} + \frac{v^2}{2g} = constant$$

Where;

H = Head in m

z = Datum (or elevation) energy

$\frac{P}{\omega}$ = Pressure energy

$\frac{v^2}{2g}$ = Kinetic energy

Bernoulli's equation limitation in its applicability, they are:

a. Flow is steady
b. Density is constant (fluid is incompressible)
c. Friction losses is negligible
d. The equation relates the states at two points along a single streamline, (not conditions on two different streamlines)

Types of Heads (Energies) of a Liquid in Motion

There are three types of energies or heads of flowing liquids.

i. Potential head or potential energy

This is due to configuration or position above some suitable datum line. It is denoted by z.

ii. Velocity head or kinetic energy

This is due to the velocity of flowing liquid and is measured as $\frac{v^2}{2g}$ where *v* is the velocity of flow and 'g' is the acceleration due to gravity.

iii. Pressure head or pressure energy

This is due to the pressure of liquid and reckoned as $\frac{P}{\omega}$ where *P* is the pressure and ω is the weight density of the liquid.

Total head/energy

$$H = z + \frac{P}{\omega} + \frac{v^2}{2g}$$

Example 4:

In a pipe of 90 mm diameter water is flowing with a mean velocity of 2 *m/s* and at a gauge pressure of 350 kN/m^2. Determine the total head, if the pipe is 8 m above the datum line. Neglect friction.

Solution

Total head of water, H:

$$H = z + \frac{P}{\omega} + \frac{v^2}{2g}$$

$$= 8 + \frac{350 \times 10^3}{9.81 \times 10^3} + \frac{2^2}{2 \times 9.81}$$

$$= 43.88\ m$$

4.3.1 Application of Bernoulli's Equation

Application 1: Horizontal Venturimeter

Venturimeter: It is a device used for measuring the rate of flow of a non-viscous, incompressible fluid in non-rotational and steady-stream lined flow. Although venturi meters can be applied to the measurement of gas, they are most used for liquids. The following treatment is limited to incompressible fluids.

A venturimeter consists of the following three parts:

i. A short converging part,
ii. Throat or neck
iii. Diverging part.

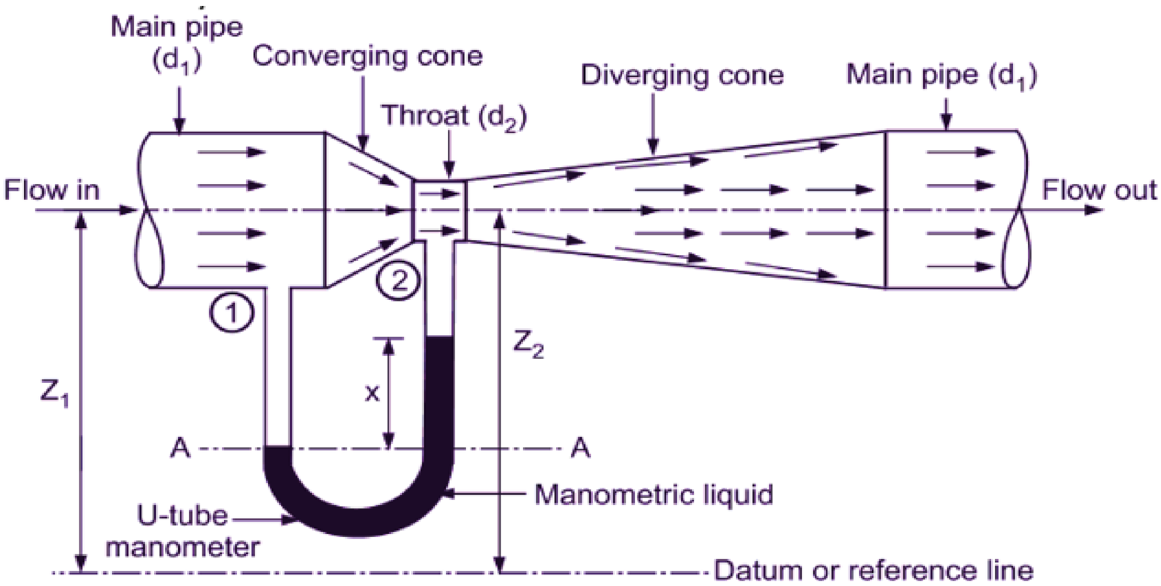


Figure 5: Horizontal Venturimeter

Example 5:

A venturi tube tapers from 300 *mm* in diameter at the entrance to 100 *mm* in diameter at the throat; the discharge co-efficient is 0.98. A differential mercury U-tube gauge is connected between pressure tapping at the entrance at throat. If the meter is used to measure the flow of water and the water fills the leads to the U-tube and is in contact with the mercury, calculate the discharge when the difference of level in the U-tube is 55 *mm*.

Solution

$$Q_{actual} = c_d A_1 \sqrt{\frac{2gH}{m^2 - 1}}$$

$$\text{Actual discharge, } Q_{actual} = 0.98 \times 0.0706 \sqrt{\frac{2 \times 9.81 \times 0.693}{81 - 1}}$$

$$Q_{actual} = \underline{0.0285 m^3 / s}$$

Example 6:

A horizontal venturi meter measures the flow of oil of specific gravity 0.9 in a 75 *mm* diameter pipeline. If the difference of pressure between the full bore and the throat tapping is 34.5 kN/m^2 and the area ratio, *m* is 4, calculate the rate of flow, assuming a coefficient of discharge is 0.97.

Solution

$$Q_{actual} = c_d A_1 \sqrt{\frac{2gH}{m^2 - 1}}$$

So,

$$\text{Actual discharge, } Q_{actual} = 0.97 \times 0.00441 \sqrt{\frac{2 \times 9.81 \times 3.92}{16 - 1}}$$

$$Q_{actual} = \underline{0.0106 \, m^3 / s}$$

Application 2: Inclined Venturimeter

Example 7:

An inclined venturimeter measures the flow of oil of specific gravity 0.82 and has an entrance of 125 *mm* diameter and throat of 50 *mm* diameter. There are pressure gauges at the entrance and at the throat, which is 300 *mm* above the entrance. If the co-efficient for the meter is 0.97 and pressure difference is 27.5 kN/m^2, calculate the actual discharge in m^3/s.

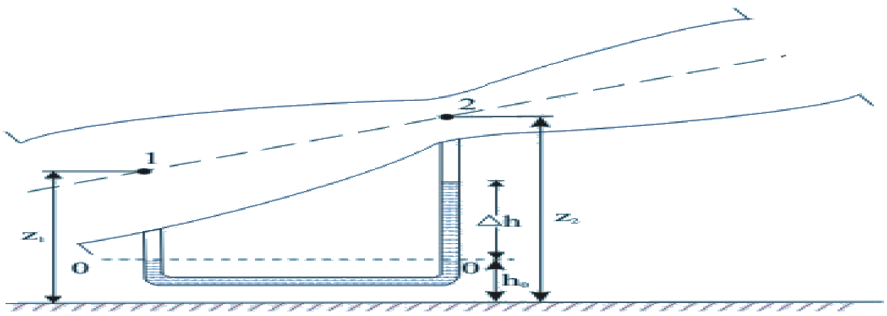


Figure 6: Incline Venturimeter

Solution

$$Q_{actual} = \frac{C_d \times A_1}{\sqrt{(m^2 - 1)}} \sqrt{\left[2g\left\{\left(\frac{p_1 - p_2}{\omega}\right) + (z_1 - z_2)\right\}\right]}$$

So,

$$A_1 = \frac{3.142(0.125)^2}{4} = 0.01226\, m^2$$

$$p_1 - p_2 = 27.5 \times 10^3\ kN/m^2$$
$$\omega = 0.82 \times 9.81 \times 10^3\ N/m^2$$
$$z_1 - z_2 = -0.3\,m$$
$$\mathrm{m} = \frac{d_1^2}{d_2^2} = \left(\frac{125}{50}\right)^2 = 6.25$$
$$C_d = 0.97$$

Therefore,

$$Q_{actual} = \frac{C_d \times A_1}{\sqrt{(m^2 - 1)}} \sqrt{\left[2g\left\{\left(\frac{p_1 - p_2}{\omega}\right) + (z_1 - z_2)\right\}\right]}$$

$$Q_{actual} = \frac{0.97 \times 0.01226}{\sqrt{((6.25)^2 - 1)}} \sqrt{\left[2 \times 9.81\left(\frac{27.5 \times 10^3}{0.82 \times 9.81 \times 10^3} - 0.3\right)\right]} = \underline{\underline{0.01535 m^3/s}}$$

EXERCISES

1) If the diameter d = 15 cm and the mean velocity, v = 3 m/s, calculate the actual discharge in the pipe.

2) Oil flows through a pipe at a velocity of 1.6 m/s. The diameter of the pipe is 8 cm. Calculate discharge and mass flow rate of oil. Take into consideration. S_{oil} = 0.85.

3)

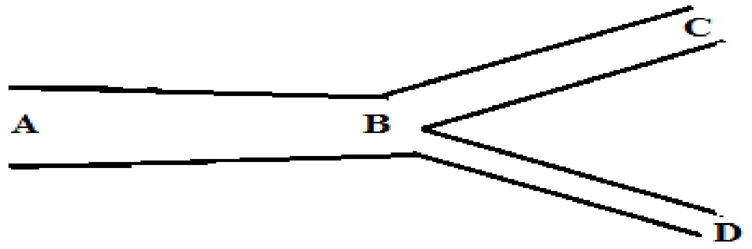


Figure 7

A pipe AB split into 2 pipes which are BC and BD. The following information is given:

diameter pipe AB at A = 0.45 m
diameter pipe AB at B = 0.3 m
diameter pipe BC = 0.2 m
diameter pipe BD = 0.15 m

Calculate:

(i) Discharge at section A if V_A= 2 m/s.
(ii) Velocity at section B and section D if velocity at section C = 4 m/s.

4) A horizontal venture meter 250 mm diameter at inlet and 150 mm diameter at the throat. Differential mercury manometer is connected to the venture meter shows the reading of 55 mm difference in mercury levels. Calculate the coefficient of discharge if the actual of flowing discharge is 0.063 m^3/ s.

5) A horizontal of the venture meters horizontally with a throat diameter of 150 mm is installed at the water main pipe diameter 300 mm. The coefficient of the venture flow meter is 0.982. Calculate the manometer height difference. if the actual flow rate is 0.142 m^3 / s.

Chapter 5: ENERGY LOSS IN PIPELINES

5.0 INTRODUCTION

Energy Loss in Pipelines the terms pipe, duct, and conduit are usually used interchangeably for flow sections. In general, flow sections of circular cross section are referred to as pipes (especially when the fluid is a liquid), and flow sections of noncircular cross section as ducts (especially when the fluid is a gas). Small diameter pipes are usually referred to as tubes.

5.1 ROUND PIPE SYSTEM

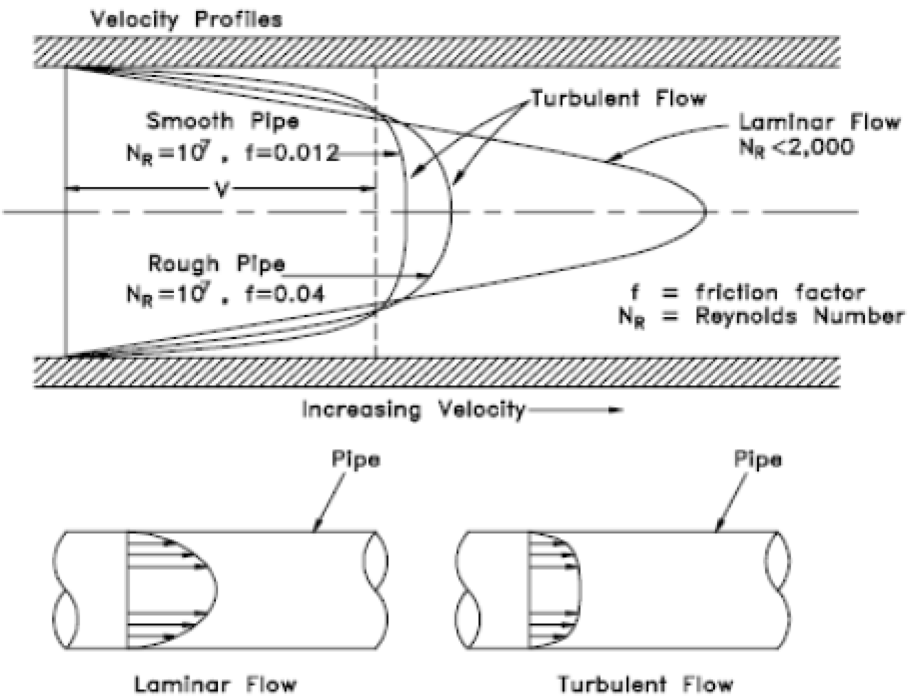


Figure 1: Velocity diagram in the round pipe system

Types of head loss energy in the round pipeline

Head Losses Energy/head Losses in the Round Pipelines cause by:

1. Sudden enlargement
2. Sudden contraction
3. Friction
4. entry loss
5. exit loss

- **Sudden Enlargement**

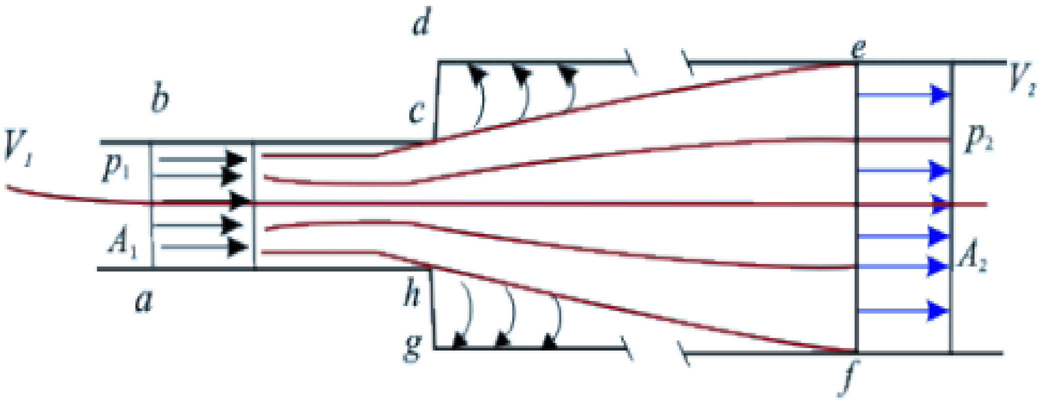


Figure 2: Flow through abrupt but finite enlargement

$$h_L = \frac{(v_1 - v_2)^2}{2g}$$

Where,

v_1 = velocity in small pipe

v_2 = velocity in large pipe

- **Sudden Contraction**

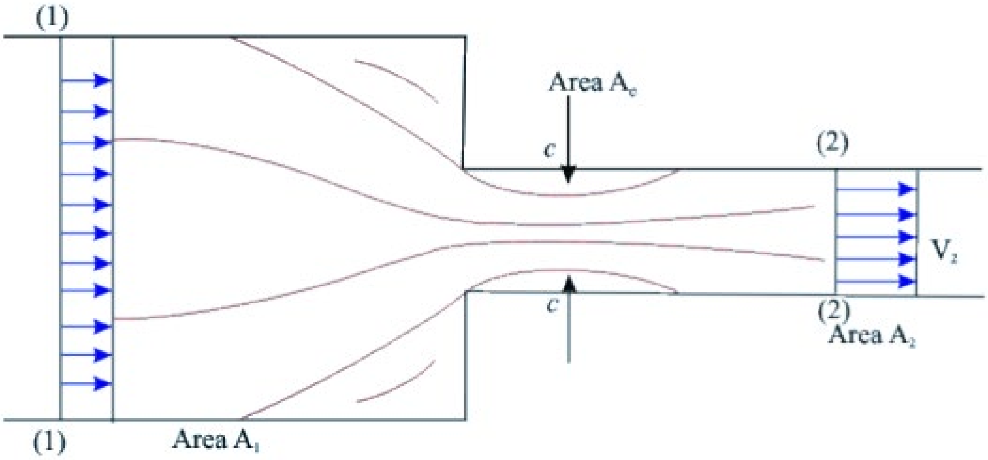


Figure 3: Flow through a sudden contraction

$$h_C = \left(\frac{1}{C_c} - 1\right)^2 \frac{v_2^{\,2}}{2g} \qquad , \qquad C_C = \frac{a_C}{v_C}$$

Where,

V_2 = velocity in small pipe

C_c = contraction co-efficient

- **Friction**

$$h_f = \frac{4fL}{d}\frac{v^2}{2g}$$

Where,

v = velocity in pipe

L = pipe length

d = diameter of pipe

f = friction co-efficient

- **Entry Loss**

$$h_C = \frac{1}{2}\left(\frac{v^2}{2g}\right)$$

Where,

v = velocity in pipe

- **Exit loss**

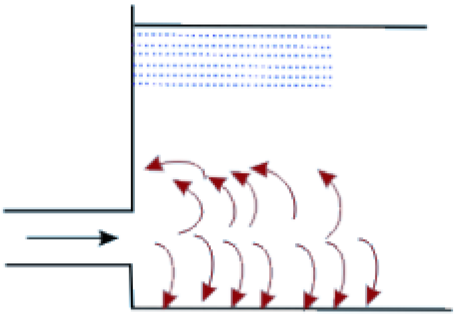


Figure 4: Flow at Infinite enlargement (Exit Loss)

$$h_L = \frac{v^2}{2g}$$

Where,

v = velocity in pipe

Example 1

A pipe carrying 1800 ℓ/min of water increases suddenly from 10 cm to 15 cm diameter. Find:

a) the head loss due to the sudden enlargement

b) the difference in pressure in kN/m^2 in the two pipes

Solution

a) 1 liter = 0.001 m^3
1800 liter = 1.8m^3

Q = 1.8m^3 / min = 0.03 m^3/s
$Q_A = A_A v_A = A_B v_B$

Therefore,

$$v_A = Q/A_A = \frac{0.03}{\pi\frac{(0.1)^2}{4}} = \underline{\underline{3.8917\ m/s}}$$

$$v_B = Q/A_B = \frac{0.03}{\pi\frac{(0.15)^2}{4}} = \underline{\underline{1.697\ m/s}}$$

Head loss of enlargement,

$$h_L = \frac{(v_A - v_B)^2}{2g} = \frac{(3.8197 - 1.697)^2}{2(9.81)} = \underline{0.2294\ m\ of\ water}$$

b) Difference in pressure

$$\frac{p_A}{\rho g} + \frac{v_A^2}{2g} + z_A = \frac{p_B}{\rho g} + \frac{v_B^2}{2g} + z_B + h_L$$

$z_A = z_B$

$$\frac{(p_A - p_B)}{g} = \frac{v_B^2 - v_A^2}{2g} + h_L$$

$$p_A - p_B = \frac{(1.697^2 - 3.8197^2)}{19.62} + 0.596(9810) = \underline{3602.56\ N/m^2}$$

Example 2

Determine the loss of head due to friction in a pipe 14 m long and 2 m diameter which carries 1.5 m/s oil. Take into consideration f = 0.05.

Solution

$$
\begin{aligned}
h_f &= \frac{4fL}{d}\frac{v^2}{2g} \\
&= \frac{4(0.05)(14)}{2} \times \frac{1.5^2}{2(9.81)} \\
&= \underline{\underline{0.16\, m\ of\ oil}}
\end{aligned}
$$

Example 3

A pipe carrying 0.06 m^3/s suddenly contracts from 200 mm to 150 mm diameter. Assuming that the vena contracta is formed in the smaller pipe, calculate the coefficient of contraction if the pressure head at a point upstream of the contraction is 0.655 m greater than at a point just downstream of the vena contracta.

Solution

Inserting this expression for the loss of head in Bernoulli's equation,

$$
\begin{aligned}
\frac{p_1}{\rho g} + \frac{v_1^2}{2g} &= \frac{p_2}{\rho g} + \frac{v_2^2}{2g} + \frac{v_2^2}{2g}\left(\frac{1}{C_C} - 1\right)^2 \\
\frac{p_1 - p_2}{\rho g} &= \frac{v_2^2}{2g}\left[1 + \left(\frac{1}{C_C} - 1\right)^2\right] - \frac{v_1^2}{2g}
\end{aligned}
$$

Given,

$$\frac{p_1 - p_2}{\rho g} = 0.655$$

Using the continuity of flow Q = Av where velocity v_1 and v_2

$$
\begin{aligned}
v_1 &= \frac{Q}{A_1} \\
&= \frac{0.06 \times 4}{\pi(0.2)^2} \\
&= \underline{1.91 m/s}
\end{aligned}
\qquad
\begin{aligned}
v_2 &= \frac{Q}{A_2} \\
&= \frac{0.06 \times 4}{\pi(0.15)^2} \\
&= \underline{3.4 m/s}
\end{aligned}
$$

Thus,

$$0.655 = \frac{(3.4)^2}{2 \times 9.81}\left[1 + \left(\frac{1}{C_C} - 1\right)^2\right] - \frac{(1.91)^2}{2 \times 9.81}$$

$$12.86 = 11.6\left[1 + \left(\frac{1}{C_C} - 1\right)^2\right] - 3.65$$

$$\left[1 + \left(\frac{1}{C_C} - 1\right)^2\right] = \frac{16.51}{11.6} = 1.39$$

$$\left(\frac{1}{C_C} - 1\right)^2 = 0.39$$

$$\left(\frac{1}{C_C}\right) - 1 = 0.625$$

$$\frac{1}{C_C} = 1.625$$

Coefficient of contraction, C_c = 0.615

5.2 PIPELINE PROBLEM

All pipeline problems should be solved by applying Bernoulli's theorem between points for which the total energy is known and including expressions for any loss of energy due to shock or to friction. Thus,

$$\frac{p_1}{\rho g} + \frac{v_1^2}{2g} + z_1 = \frac{p_2}{\rho g} + \frac{v_2^2}{2g} z_2 + shock\ loss + frictional\ loss$$

Three types of problem:

1. Discharge to atmosphere

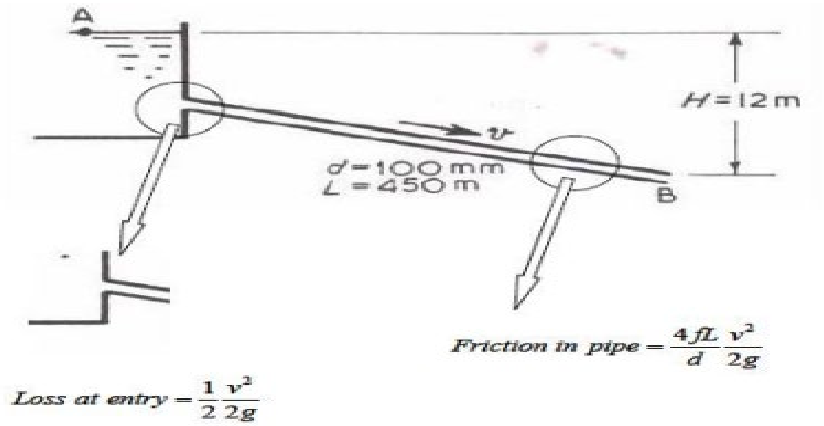
2. Pipes in series

3. Hydraulic gradient

Discharge to atmosphere

Example 4

Water from a large reservoir is discharged to atmosphere through a 100 mm diameter pipe 450 m long. The entry from the reservoir to the pipe is sharp and the outlet is 12 m below the surface level in the reservoir. Taking f = 0.01 in the Darcy formula calculate the discharge.



Solution

Apply Bernoulli's Teorem to A and B, then assuming the velocity at A is Zero (0), where P_A = P_B = atmospheric pressure

Total energy at A = Total energy at B + loss at entry + frictional loss

$$H = \frac{v^2}{2g} + \frac{1}{2}\frac{v^2}{2g} + \frac{4fL}{d}\frac{v^2}{2g}$$

By defining H = 12m, *f* = 0.01, L = 450m, d = 100mm~ 0.1m

$$H = \frac{v2}{2g}\left(1 + \frac{1}{2} + \frac{4fL}{d}\right)$$

$$12 = \frac{v^2}{2g}\left(1.5 + \frac{4 \times 0.01 \times 450}{0.1}\right)$$

$$= 181.5\frac{v^2}{2g}$$

$$\underline{\underline{v = 1.14\, m/s}}$$

Discharge

$$: = \frac{\pi d^2}{4} v$$

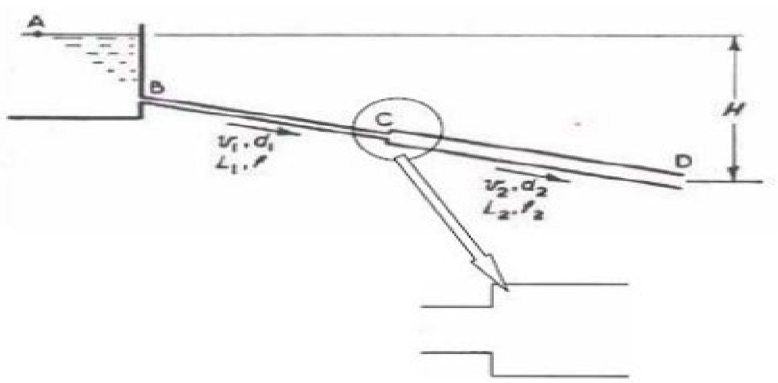
$$= \frac{\pi (0.1)^2}{4} (1.14)$$

$$= \underline{\underline{8.96 \times 10^{-3} \ m^3/s}}$$

Pipes in Series

Example 5

Water is discharged from a reservoir into the atmosphere through a pipe 39 m long. There is a sharp entrance to the pipe and the diameter is 50 mm for the first 15 m from the entrance. The pipe then enlarges suddenly to 75 mm in diameter for the remainder of its length. Considers the loss of head at entry and at the enlargement, calculate the difference of level between the surface of the reservoir and the pipe exit which will maintain a flow of 2.8 dm^3/s. Take f as 0.0048 for the 50 mm pipe and 0.0058 for the 75 mm pipe.



$$Loss \ at \ enlargement = \frac{(v_1 - v_2)^2}{2g}$$

Solution

$$Q = \frac{1}{4}\pi d_1^{\,2} v_1 = \frac{1}{4}\pi d_2^{\,2} v_2$$

$$v_1 = \frac{4Q}{\pi d_1^{\,2}}$$
$$= \frac{4 \times 2.8 \times 10^{-3}}{\pi (0.05)^2}$$
$$= \underline{\underline{1.426\ m/s}}$$

$$v_2 = \frac{4Q}{\pi d_2^{\,2}}$$
$$= \frac{4 \times 2.8 \times 10^{-3}}{\pi (0.075)^2}$$
$$= \underline{0.634\ m/s}$$

By applying Bernoulli's Teorem to A and B, where P_A = P_B = atmospheric pressure and vA = 0 for unit weight

Total energy at A = Total energy at D + shock loss at B + frictional loss at BC + shock loss at C + frictional loss at CD

* No shock loss at D as discharge to atmosphere

1. Total energy at D,

$$= \frac{v_2^{\,2}}{2g}$$
$$= \frac{(0.634)^2}{2(9.81)}$$
$$= \underline{\underline{0.020\ m}}$$

2. Loss at entry B,

$$= \frac{1}{2}\frac{v_1^{\,2}}{2g}$$
$$= \frac{1}{2}\frac{(1.426)^2}{2(9.81)}$$
$$= \underline{\underline{0.052\ m}}$$

3. Frictional loss in BC,

$$= \frac{4fL_1}{d_1}\frac{v_1^{\,2}}{2g}$$
$$= \frac{4(0.0048)(15)}{0.050}\frac{(1.426)^2}{2(9.81)}$$
$$= \underline{0.597\ m}$$

4. Shock Loss at C,

$$= \frac{(v_1 - v_2)^2}{2g}$$
$$= \frac{(1.426 - 0.634)^2}{2(9.81)}$$
$$= \underline{\underline{0.032\ m}}$$

5. Frictional loss in CD,

$$= \frac{4fL_2}{d_2}\frac{v_2^2}{2g}$$

$$= \frac{4(0.0058)(24)}{0.075}\frac{(0.634)^2}{2(9.81)}$$

$$= \underline{\underline{0.152\ m}}$$

6. **Difference of level (H)**

= 0.02+0.052+0.597+0.032+0.152

= 0.853

Hydraulic Gradient

Example 6

Two reservoirs are connected by a pipeline which is 150 mm in diameter for the first 6 m and 225 mm in diameter for the remaining 15 m. The entrance and exit are sharp and the change of section is sudden. The water surface in the upper reservoir is 6 m above that in the lower. Tabulate the losses of head which occur and calculate the rate of flow in m^3/s. Friction coefficient f is 0.01 for both pipes.

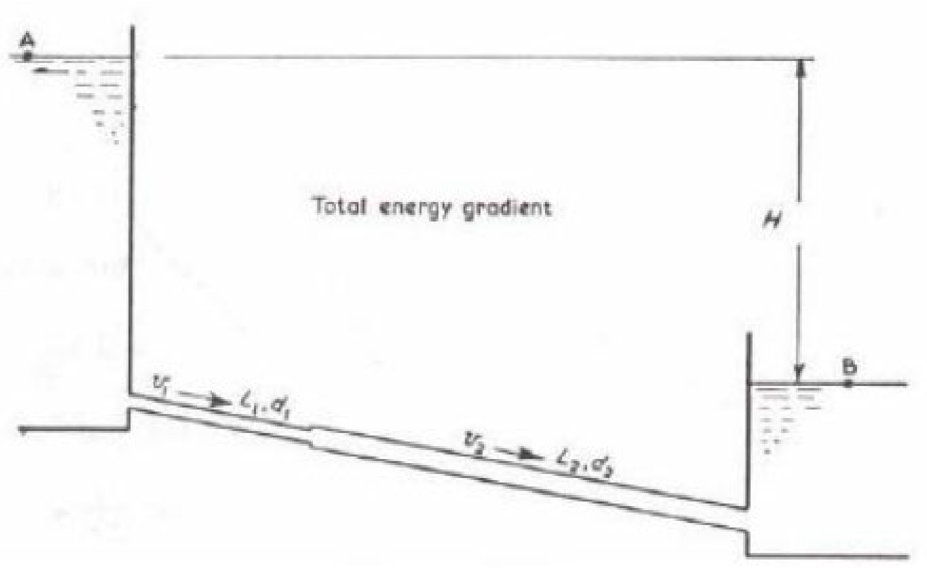


Figure 7

Solution

Since d_1 = 150mm and d_2 = 225mm

$$v_1 = \left(\frac{225}{150}\right)^2 v_2 = \left(\frac{9}{4}\right) v_2$$

Loss at entry $= \frac{1}{2}\frac{v_1^2}{2g}$

$$= \frac{1}{2}\left(\frac{9}{4}\right)^2 \frac{v_2^2}{2g}$$

$$= 2.53\frac{v_2^2}{2g}$$

Frictional loss in 6m pipe $= \frac{4fL_1}{d_1}\frac{v_1^2}{2g}$

$$= \frac{4(0.01)(6)}{0.15}\frac{v_1^2}{2g}$$

$$= 1.6\frac{v_1^2}{2g}$$

$$= 1.6\left(\frac{9}{4}\right)^2 \frac{v_2^2}{2g}$$

$$= 8.1\frac{v_2^2}{2g}$$

Shock loss at enlargement $= \frac{(v_1 - v_2)^2}{2g}$

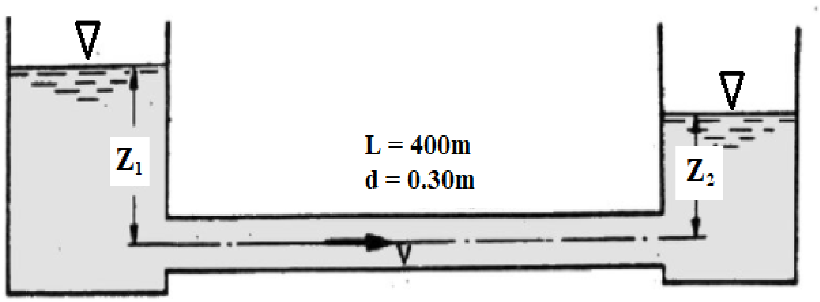
$$= \frac{v_2^2}{2g}\left(\frac{9}{4} - 1\right)^2$$

$$= 1.56\frac{v_2^2}{2g}$$

EXERCISES

1. Two reservoirs were connected by a pipeline which is 170 mm in diameter for the first 6 m and 230 mm in diameter for the remaining 15 m. The entrance and exit are sharp and the change of section is sudden. The water surface in the upper reservoir is 6 m above that in the lower. Tabulate the losses of head which occur and calculate the rate of flow in m^3/s. Friction coefficient *f* is 0.01 for both pipes.

2. Water is flow at a rate of 300 L/s through a horizontal pipe. The pipe diameter is 30 cm and length 400 m. Calculate the difference elevation between the water surfaces in the two tanks. Assume sharp-edged entrance and exit for the pipe. Take the value of $f = 0.032$.



3. A horizontal pipeline 40 m long is connected to a water tank at one end and discharges freely into the atmosphere at the other end. For the first 25 m of its length from the tank, the pipe is 150 mm diameter and its diameter suddenly enlarged to 300 mm. The height of water level in the tank is 8 m above the center of the pipe. Considering all losses of head occur:

   i) Determine the rate of flow.

   ii) Draw the hydraulic gradient.

   (take *f* = 0.01 for both section of pipes

4. Water is discharged form a reservoir into the atmosphere through a pipe 80 m long. There is a sharp entrance to the pipe and the diameter is 250 mm for the first 50 m. The pipe then enlarges suddenly to 450 mm in diameter for the remainder of its length. The outlet is 35 m below the water surface level in the reservoir. Take f = 0.004 for both pipes. Calculate the discharge.

5. Two reservoirs have a different in level of 9 *m* and are connected by a pipeline, which is 38 *mm* in diameter for the first 13 *m* and 23 *mm* for the remaining 6 *m.* Take f = 0.01 for both pipes and C_C = 0.66. Calculate the discharge.

6. Two opened tanks filled with water are connected by series pipe AB and BC. AB pipe has a diameter of 10cm and BC pipe is 6cm. The length of pipe for AB is 200m and for pipe BC is 150m. The flow rate of water entering the pipe is 0.007m3/s and coefficient of contraction is 0.62. If the energy losses ONLY occur due to the shock loss at sudden contraction and friction, calculate the level difference of the two tanks. Given f = 0.04 for both pipes.

7. Two tanks filled with water connected by a serial pipe. AB pipe has a diameter of 12 cm while the diameter of BC pipe is 6 cm. The flow rate of water entering the pipe is 0.009 m3/s and the coefficient of contraction is 0.65. Consider energy losses because of sudden contraction and friction only, calculate level difference between the two tanks. Given friction coefficient, f = 0.004 for both pipes.

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