PROBABILITY

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Politeknik Tuanku Sultanah Bahiyah
Kementerian Pendidikan Tinggi

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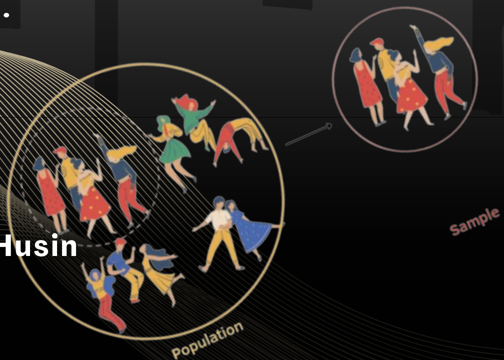
PREFACE

Probability is a fundamental concept in mathematics and statistics, providing a framework for understanding and quantifying uncertainty. Whether we realize it or not, probability influences many aspects of our daily lives, from predicting weather patterns and making decisions in business to playing games of chance and understanding risks in healthcare. This study of probability helps us to systematically analyze random events and make informed predictions about future outcomes. By learning probability, we develop the tools to assess the likelihood of events, measure risk, and make decisions under uncertainty. This text aims to introduce you to the key principles and applications of probability, starting with the basic concepts and moving toward more complex topics. The approach is both theoretical and practical, offering a balance between understanding the underlying mathematics and seeing how these principles apply to real-world situations.



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INTRODUCTION

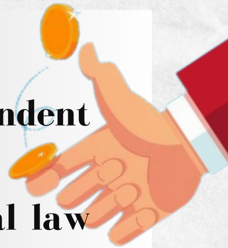
INTRODUCTION

Probability is a branch of mathematics that deals with the study of uncertainty and the likelihood of different outcomes. It provides a systematic way to quantify uncertainty and make predictions about the occurrence of various events. The concept of probability is deeply rooted in everyday life, influencing everything from weather forecasts and medical diagnoses to financial markets and games of chance.

Probability theory has a wide range of applications across many disciplines. In science, it helps researchers assess the significance of their experimental results. In finance, it is used to model risks and returns. In engineering, it plays a key role in quality control and reliability analysis. Even in everyday decisions, we use probability—whether we are aware of it or not—to make choices under uncertainty.

The study of probability involves various concepts and techniques, including the analysis of random events, the computation of probabilities using different rules, and the understanding of probability distributions. These tools allow us to model complex systems and predict the behavior of uncertain processes.

LEARNING OUTCOME



At the end of the lessons:

- Students are able to define independent event and conditional probability
- Use laws of probability using additional law of probability.
- Student are able to use laws of probability
- Solve the problems on probability

Practice mathematical knowledge and skills in different mathematics problem

Solve the mathematical problems by using appropriate mathematical techniques and solutions

Show the solution for probability problems by using appropriate mathematical methods.

3.0

NOTES & FORMULA

PROBABILITY DEFINITION IN MATHEMATICS

Probability is a measure of the likelihood of an event to occur. Many events cannot be predicted with total certainty. We can predict only the chance of an event to occur i.e., how likely they are going to happen, using it. Probability can range from 0 to 1, where 0 means the event to be an impossible one and 1 indicates a certain event. Probability for Class 10 is an important topic for the students which explains all the basic concepts of this topic.

Basic Definitions

Probability:

The likelihood or chance of an event happening. It's a value between 0 and 1.

Range: 0⩽P(E)⩽10 \leq P(E) \leq 10⩽P(E)⩽1

P(E) = 0 means the event cannot occur, and P(E) = 1 means the event is certain to occur.

Experiment:

A procedure that yields one outcome from a set of possible outcomes.

Sample Space (S):

The set of all possible outcomes of an experiment.

Example: For a coin toss, S={Heads,Tails}S = \{ \text{Heads}, \text{Tails} \}S={Heads,Tails}

Event:

Any subset of a sample space.

Example: In a coin toss, Event E={Heads}\text{Event E} = { \text{Heads} \}Event E={Heads}

Formula for Probability

Sampel Space

*Set of all the possible outcomes.

*Symbol S and set notation { }

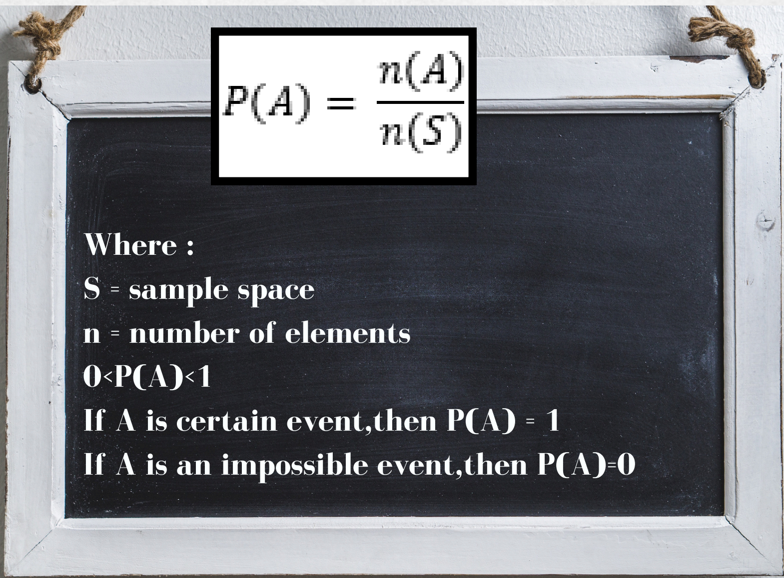
An Event

*Outcomes or set of oucomes.

*Symbol A and set notation { }

The Probability of An Event

$$= \frac{\text{number of elements in A}}{\text{number of elements in the sampel space S}}$$



Basic Probability Rules

Addition Rule (for mutually exclusive events): P(A∪B)=P(A)+P(B)P(A \cup B) = P(A) + P(B)P(A∪B)=P(A)+P(B)

General Addition Rule (for non-mutually exclusive events):
P(A∪B)=P(A)+P(B)−P(A∩B)
P(A \cup B) = P(A) + P(B)
P(A\cap B)P(A∪B)=P(A)+P(B)−P(A∩B)

Multiplication Rule (for independent events):
P(A∩B)=P(A)×P(B)
P(A \cap B) = P(A) \times P(B)
P(A∩B)=P(A)×P(B)

Conditional Probability:
The probability of event AAA given that event BBB has occurred.
P(A ∣ B)=P(A∩B)P(B)
P(A|B) = \frac{P(A \cap B)}{P(B)}
P(A ∣ B)=P(B)P(A∩B)

TYPES OF EVENTS

TYPES OF EVENTS

EXPECTATION

General terms as the product of the probability p of an event happening and the number of attempts made , n.

$$E = pn$$

DEPENDENT EVENT

The outcome of one event affects the outcome of the other.

If A and B are dependent events then the probability of both occurring is

$$P(A \text{ and } B) = P(A) \times P(B|A)$$

Probability of B given A

INDEPENDENT EVENT

The outcome of one event **does not** affect the outcome of the other.

If A and B are independent events then the probability of both occurring is

$$P(A \text{ and } B) = P(A) \times P(B)$$

CONDITIONAL PROBABILITY

the probability of an event occurring based on a previous event already taking place

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

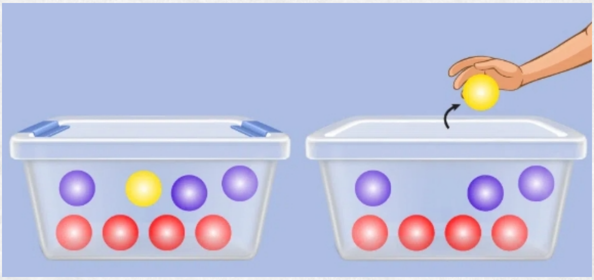
Probability of A given B; Probability of A and B; Probability of B

EXAMPLE

EXAMPLE 1

QUESTION

There are 8 balls in a container, 4 are red, 1 is yellow and 3 are blue. What is the probability of picking a yellow ball?



ANSWER

Probability is equal to the number of yellow balls in the container divided by the total number of balls in the container = 1/8

P (Y) = 1/8

EXAMPLE 2

QUESTION

A dice is rolled. What is the probability that an even number has been obtained?

ANSWER

When fair six-sided dice are rolled, there are six possible outcomes: 1, 2, 3, 4, 5, or 6.

Out of these, half are even (2, 4, 6) and half are odd (1, 3, 5).

Therefore, the probability of getting an even number is:

P(even) = number of even outcomes / total number of outcomes

P(even) = 3 / 6
P(even) = 1/2

EXAMPLE 3

QUESTION

There are 6 pillows in a bed, 3 are red, 2 are yellow and 1 is blue. What is the probability of picking a yellow pillow?

ANSWER

The probability is equal to the number of yellow pillows in the bed divided by the total number of pillows.

Sampel, S = { R,R,R,Y,Y,B}

No of Sample, n (S) = 6 n (Y) = 2

$$P(A) = \frac{n(A)}{n(S)} = \frac{2}{6} = \frac{1}{3}$$

EXAMPLE 4

QUESTION

There is a container full of coloured bottles, red, blue, green and orange. Some of the bottles are picked out and displaced. Sumit did this 1000 times and got the following results:

- No. of blue bottles picked out: 300
- No. of red bottles: 200
- No. of green bottles: 450
- No. of orange bottles: 50

a) What is the probability that Sumit will pick a green bottle?

ANSWER

Sampel, S = {300,200,450,50}
No of Sample, n (S) = 1000

a)

n(G) = 450 $\quad P(G) = \frac{n(G)}{n(S)} = \frac{450}{1000} = \frac{9}{20}$

EXAMPLE 5

QUESTION

A coin is thrown 3 times .what is the probability that at least one head is obtained?

ANSWER

Sample space =
[HHH, HHT, HTH, THH, TTH, THT, HTT, TTT]

Total number of ways = $2 \times 2 \times 2 = 8$. Fav.
Cases = 7
P (A) = 7/8

OR

P (of getting at least one head) = 1
P (no head)⇒ 1 - (1/8) = 7/8

EXAMPLE 6

QUESTION

Two dice are thrown together. What is the probability that the number obtained on one of the dice is multiple of number obtained on the other dice?

ANSWER

Total number of cases = 36

Since the number on a die should be multiple of the other, the possibilities are

(1, 1) (2, 2) (3, 3) ------- (6, 6) --- 6 ways

(2, 1) (1, 2) (1, 4) (4, 1) (1, 3) (3, 1) (1, 5) (5, 1) (6, 1) (1, 6) --- 10 ways

(2, 4) (4, 2) (2, 6) (6, 2) (3, 6) (6, 3) -- 6 ways

Favorable cases are = 6 + 10 + 6 = 22.

P (A) = 22/36 = 11/18

EXAMPLE 7

QUESTION

Three bags contain 3 red, 7 black; 8 red, 2 black, and 4 red & 6 black balls respectively. 1 of the bags is selected at random and a ball is drawn from it. If the ball drawn is red, find the probability that it is drawn from the third bag.

ANSWER

Let B1, B2, B3 and A are the events defined as follows.
B1 = First bag is chosen
B2 = Second bag is chosen
B3 = Third bag is chosen
A = Ball drawn is red

Since there are three bags and one of the bags is chosen at random,

P (B1) = P(B2) = P(B3) = 1 / 3

If B1 has already occurred, then first bag has been chosen which contains 3 red and 7 black balls.

The probability of drawing 1 red ball from it is 3/10. So, P (A/B1) = 3/10, similarly P(A/B2) = 8/10, and P(A/B3) = 4/10.

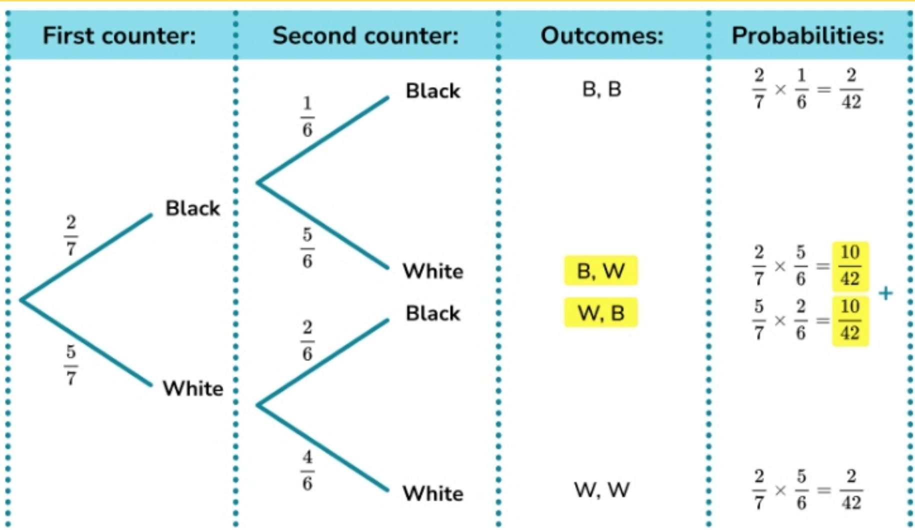
$$= \frac{\frac{1}{3} \times \frac{4}{10}}{\frac{1}{3} \times \frac{3}{10} + \frac{1}{3} \times \frac{8}{10} + \frac{1}{3} \times \frac{4}{10}} = \frac{4}{15}.$$

EXAMPLE 8

QUESTION

In a bag there are 7 counters. There are 2 black counters and the remaining counters are white.
A counter is removed and the colour noted.
The counter is NOT replaced.
A second counter is removed and the colour is noted.
Using the tree diagram.
Work out the probability that there will be one counter of each colour picked.

ANSWER



The probability that one of each colour counter will be picked is $\frac{10}{21}$.

FINAL EXAM COLLECTION

Question 1

In a class of 30 students, 15 students like basketball, 14 students like football and 9 students like both basketball and football. Calculate the probability of a chosen person at random who likes:

i. At least one of the games

ii. Basketball given that they like football

Answer

i. $P(\,Basketball\ or\ Football) = P(Basketball) + P(Football) + P(Both\ Basketball\ and\ Football)$

.

$$= \frac{6}{30} + \frac{5}{30} + \frac{9}{30}$$

$$= \frac{2}{3}$$

ii. $P\ (Basketball \mid Football) = \frac{P(Basketball\ and\ Football)}{P\ (Football)}$

$$= \frac{P(B\ \cap F)}{P\ (F)}$$

$$= \frac{9/30}{14/30}$$

$$= \frac{9}{14}$$

Question 2

A box contains of 20 red marbles, 32 blue marbles, 17 yellow marbles and 11 white marbles. A marble is picked randomly from the box. Calculate the probability of picking a red marble.

Answer

$20\ red,\quad 32\ blue,\quad 17\ yellow,\quad 11\ white,$
$n\,(S) = 80$

$$P(R) = \frac{n(R)}{n(S)}$$

$$P(R) = \frac{20}{80} = \mathbf{\frac{1}{4}}$$

Question 3

Ten pieces of paper numbered from 21 to 30 are placed in a file. A piece of paper is picked at random from the file. Calculate the probability of picking a number that is even OR divisible by 3.

Answer

21, 22, 23, 24, 25, 26, 27, 28, 29, 30

$n\,(S) = 10$

$Even\ number = 22, 24, 26, 28, 30$

$$P(E) = \frac{n(E)}{n(S)}$$

$$P(e) = \frac{5}{10}$$

$Divisible\ by\ 3 = 21, 24, 27, 30$

$$P(Divisble\ by\ 3) = \frac{n(D)}{n(S)}$$

$$P(D) = \frac{4}{10}$$

$Even\ Number\ \&\ Divisible\ by\ 3 = 24\ , 30$

$$P(E \cup D) = \frac{2}{10}$$

Probability of picking a number that is

even OR divisible by 3, $P\ (E\ \cup D) = \frac{5}{10} + \frac{4}{10} - \frac{2}{10}$

$$= \frac{7}{10}$$

Question 4

A box contains 5 yellow pens, 5 blue pens and 6 red pens. Thalia picks a pen at random and its colour is noted before she puts it back to the box. Then, a second pen is picked by her brother. Calculate the probability of getting the first picked is red and the second picked is yellow pen.

Answer

$$n(Yellow) = 5$$
$$n(Blue) = 5$$
$$n(Red) = 6$$
$$n(sample) = 16$$

$$P(Red\ Yellow) = P\ (Red) \times P(Yellow)$$
$$P(R \cap Y) = \frac{6}{16} \times \frac{5}{16}$$
$$= \frac{15}{128} \ @\ 0.117$$

Question 5

A letter is chosen at random from the word: MATHEMATICS. Calculate the probability of getting a consonant or a letter M.

Answer

$n(s) = 11$
$n(consonant) = \{M, T, H, M, T, C, S\}$
$n(c) = 7$
$n(Letter\ M) = \{M, M, \}$
$n(M) = 2$

$$P(C) = \frac{7}{11}$$

$$P(M) = \frac{2}{11}$$

$$P(C \cap M) = \frac{2}{11}$$

$$P(\ Conconant\ or\ Letter\ M) = P(C) + P(M) - P(C \cap M)$$

$$= \frac{7}{11} + \frac{2}{11} - \frac{2}{11}$$

$$= \frac{7}{11}\ @\ 0.636$$

Question 6

If a bag of balloons consists of 47 white balloons, 5 yellow balloons, and 10 black balloons, what is the approximate likelihood that a balloon chosen randomly from the bag will be black?

Answer

- White balloons = **47**
- Yellow balloons = **5**
- Black balloons = **10**

Step 1: Find total balloons

$$47 + 5 + 10 = 62$$

Step 2: Probability of choosing a black balloon

$$P(\text{black}) = \frac{\text{black balloons}}{\text{total balloons}} = \frac{10}{62}$$

Step 3: Simplify fraction

$$\frac{10}{62} = \frac{5}{31} \approx 0.1613$$

Question 7

Two coins are tossed simultaneously for 360 times. The number of times '2 Tails' appeared was three times 'No Tail' appeared and number of times '1 tail' appeared is double the number of times 'No Tail' appeared. Find the probability of getting 'Two tails'.

Answer

Total number of outcomes = 360

Let us considered the number of times 'No Tail' appeared be z

Then, number of times '2 Tails' appeared = 3z

Number of times '1 Tail' appeared = 2z

Now, z + 2z + 3z = 360

6z = 360

z = 60

Hence, the probability of getting 'two tails' = (3 x 60)/360 = 1 /2

Question 8

In a selection of school-level speech participants, candidates will be voted by members of the Speech Club where member can vote for a maximum of two candidates. The probability that Auni (A) and Hani (H) are voted to participate in the speech is and respectively.

i. Calculate the probability of Auni or Hani being voted to participate in the speech competition.

ii. If the total of votes is 64, calculate how many votes will be for other than Auni or Hani.

Answer

i. $Auni\ (A) = \frac{5}{8}$

$Hani\ (H) = \frac{1}{4}$

	Auni (A)	Hani (H)
Voted, V	$^5/_8$	$^1/_4$
Unvoted, V'	$^3/_8$	$^3/_4$

$P\ (Auni\ or\ Hani) = P(AH') + P(A'H) + P(AH)$

$= \left(\frac{5}{8} \times \frac{3}{4}\right) + \left(\frac{3}{8} \times \frac{1}{4}\right) + \left(\frac{5}{8} \times \frac{1}{4}\right)$

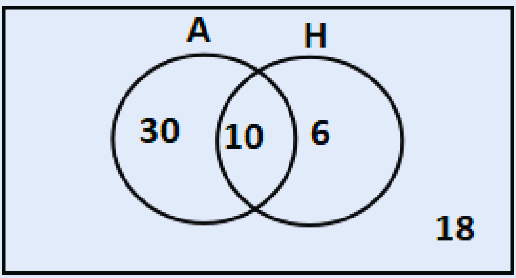
$= \frac{23}{32}$

ii. $Auni\ (A) = \frac{5}{8} \times 64 = 40$

$$Hani\ (H) = \frac{1}{4} \times 64 = 16$$

$$Auni\ \&\ Hani = \left(\frac{5}{8} \times \frac{1}{4}\right) = \frac{5}{32}$$

$$= \frac{5}{32} \times 64 = 10$$



n $(A \cup H)' = 18\ votes$

Question 9

A fair dice is rolled. Calculate the probability of getting:

i. $P(prime\ number\ \cap odd\ number)$

ii. $P(prime\ number\ \cup odd\ number)$

Answer

n(s) = {1,2,3,4,5,6}

= 6

Prime, n(P) = {2,3,5} = 3

Odd, n(O) = {1,3,5} = 3

i. $P(prime\ number\ \cap odd\ number)$

$$P(P \cap O) = \frac{2}{6}$$

$$= \frac{1}{3}$$

ii. $P(prime\ number\ \cup odd\ number)$

$$P(P\ \cup O) = \frac{3}{6} + \frac{3}{6} - \frac{2}{6}$$

$$= \frac{4}{6}$$

$$= \frac{2}{3}$$

Question 10

In a lottery box, there are 10 prizes and 25 blanks. A slip is drawn at random from the lottery box. What is the probability of getting a prize?

Answer

Total number of prize = 10

Total number of blanks = 25

So, the total number of possible outcomes (i.e., n(S)) are = 10 + 25 = 35
Total number of prizes, n(E) = 10

P(E) = n(E)/n(S) = 10/35 = 2/7

INTERACTIVE REFERENCE

Probability Distributions | Revision | MME

Probability Examples with Questions and Answers - Hitbullseye

Probability Tree Diagram - GCSE Maths - Steps, Examples & Worksheet

REFERENCE

- Nang Saruni, Zuraini < Faizatulhaida et all., (2022) DBM30033 Engineering Mathematics 3 First Edition Edition. Politeknik Tuanku Sultanah Bahiyah.
- Hines, W. W., Montgomery, D. C., Goldsman, D. M., & Borror, C. M. (2003). Probability and statistics in engineering (4th ed.). Wiley.
- Ang, A. H.-S., & Tang, W. H. (2007). Probability concepts in engineering: Emphasis on applications in civil and environmental engineering (2nd ed.). Wiley.
- Benjamin, J. R., & Cornell, C. A. (2014).Probability, statistics, and decision for civil engineers. Dover Publications.
- Ayyub, B. M., & McCuen, R. H. (2016). Probability, statistics, and reliability for engineers and scientists (4th ed.). CRC Press.
- Khan Academy. (n.d.). Probability and statistics. Khan Academy. https://www.khanacademy.org/math/statistics-probability
- MIT OpenCourseWare. (2018). Introduction to probability and statistics. Massachusetts Institute of Technology. https://ocw.mit.edu/courses/res-6-012-introduction-to-probability-spring-2018/
- Hájek, A. (2022). Interpretations of probability. In E. N. Zalta (Ed.), The Stanford encyclopedia of philosophy (Fall 2022 Edition). Stanford University. https://plato.stanford.edu/entries/probability-interpret/

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