

ENGINEERING

Mathematics 3

-----SIMPLEX METHOD-----

MATHEMATICS, SCIENCE & COMPUTER DEPARTMENT

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SIMPLEX METHOD

FIRST EDITION

2025

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PREFACE

This e-book on the Simplex Method is designed to provide a clear, step-by-step understanding of this powerful optimization algorithm.

This e-book aims to simplify the complexities of the Simplex Method. We start with an introduction to linear programming concepts, progressively building the foundation needed to understand the Simplex Method. Through detailed explanations, practical examples, and intuitive illustrations, we will guide you through the algorithm's core principles, including how to set up a linear programming problem, perform the iterative steps of the Simplex algorithm, and interpret the final results.

By the end of this e-book, you will have a solid grasp of the Simplex Method and its role in solving optimization problems. Whether you're working through textbook exercises or real-world challenges, this knowledge will empower you to make data-driven decisions that maximize efficiency and minimize costs.

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INTRODUCTION

The Simplex Method is a fundamental algorithm in mathematical optimization used to solve linear programming problems. This method was introduced by American mathematician George Dantzig in 1947 while he was working at the Statistical Control Department of the U.S. Air Force during World War II. Dantzig is widely recognized as the father of linear programming and the inventor of the Simplex Method.

The method is based on principles of geometry and linear algebra, where it moves from one basic solution to another through pivot operations, evaluating improvements in the objective function at each step. In 1947, Dantzig introduced the Simplex Method to reduce the number of extreme points that needed to be examined in order to find the optimal solution.

DEFINITION

Simplex Method is an alternative approach other than graphical method to solve linear programming problems by using slack variables, tableaus and pivot variables as a means to find the optimal solution of an optimization problem.

FRAMEWORK OF SIMPLEX METHOD

OPTIMAL SOLUTION

■ Z_{max}

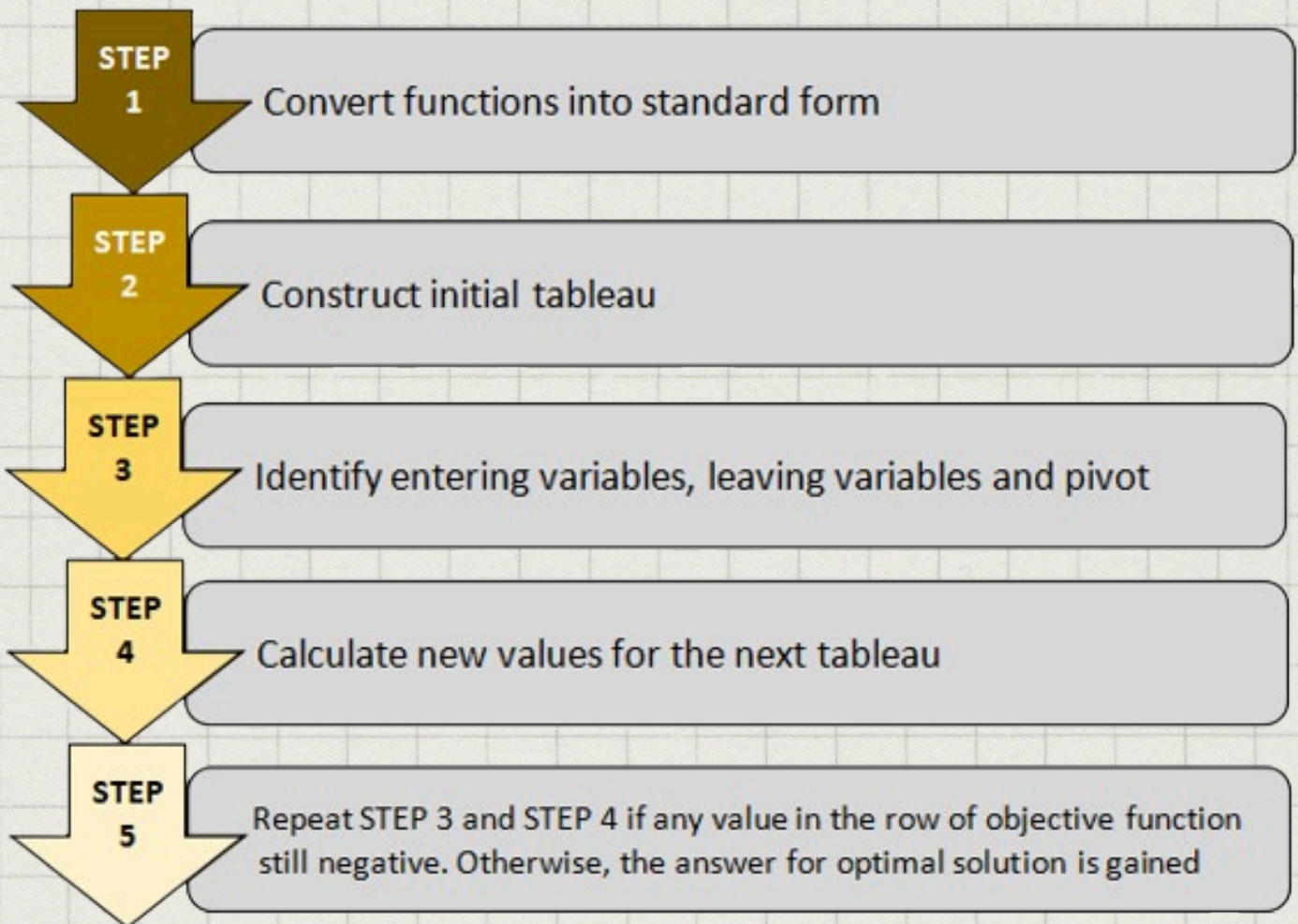
CONSTRUCT TABLEAU

- Entering Variable
- Leaving Variable
- Pivot

STANDARD FORM

- Objective Function
- Constraint (slack value)

STEP OF SOLUTION



KEYNOTES

- **Entering variables:** The COLUMN with the MOST NEGATIVE of the row represents objectives function
- **Leaving variables:** The ROW with the SMALLEST POSITIVE value of the ratio for constant to entering variable
- **Pivot :** Interception between Entering Variables and Leaving Variables. The value of pivot must be 1. Otherwise, the whole row must be divided with pivot value to become 1

EXAMPLE :

$$Z_{max} = 2x + 7y$$

Subject to constraints:

$$x - y \leq 5$$

$$2x + y \leq 4$$

STEP 1: Convert functions into standard form:

$$Z_{max} - 2x - 7y = 0$$

$$x - y + s_1 = 5$$

$$x + y + s_2 = 4$$

STEP 2: Construct initial tableau

Basic	x	y	s ₁	s ₂	Constant
Z	-2	-7	0	0	0
s ₁	1	-1	1	0	5
s ₂	1	1	0	1	4

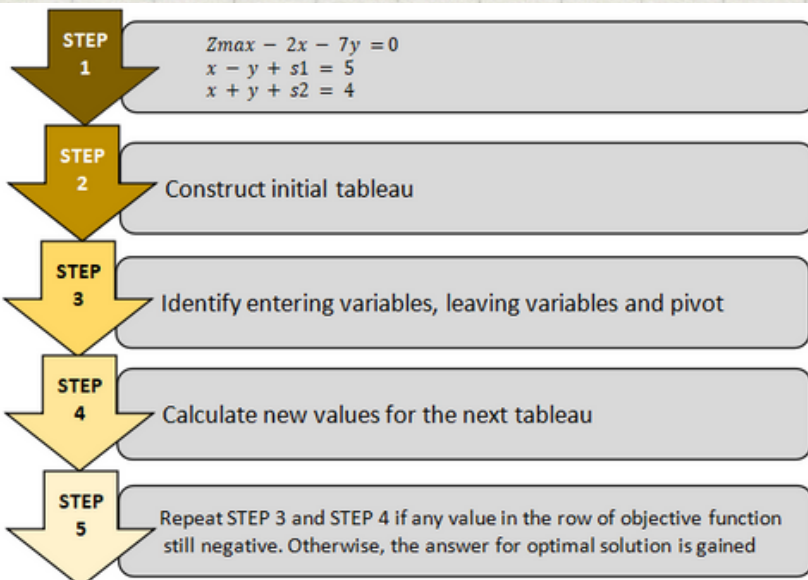
STEP 3: Identify entering variables, leaving variables and pivot

Entering Variables

Basic	x	y	s ₁	s ₂	Constant	Constant/Entering variable
Z	-2	-7	0	0	0	0
s ₁	1	-1	1	0	5	-5
s ₂	1	1	0	1	4	4

Pivot

Leaving Variables



EXAMPLE 1 :

Find the optimal solution for the following linear programming problem by using the Simplex Method;

$$\text{Maximize, } Z = 3x_1 + 2x_2$$

Subject to;

$$-x_1 + 2x_2 \leq 4$$

$$3x_1 + 2x_2 \leq 14$$

$$x_1 - x_2 \leq 3$$

$$x_1, x_2 \geq 0$$

SOLUTION :

Converting inequalities to equalities (Standard Form)

$$Z - 3x_1 - 2x_2 = 0$$

$$-x_1 + 2x_2 + S_1 = 4$$

$$3x_1 + 2x_2 + S_2 = 14$$

$$x_1 - x_2 + S_3 = 3$$

Where; S_1, S_2 and S_3 are slack variables.

Initial Tableau

Basic	Problem Variables		Slack Variables			Constant / Solution
	X_1	X_2	S_1	S_2	S_3	
Z	-3	-2	0	0	0	0
S_1	-1	2	1	0	0	4
S_2	3	2	0	1	0	14
S_3	1	-1	0	0	1	3

Calculating values for the index rows Z, S_1 & S_2 for the Second Tableau.

Row Z	Row S_1	Row S_2
$-3 + 3(1) = 0$	$-1 + 1(1) = 0$	$3 - 3(1) = 0$
$-2 + 3(-1) = -5$	$2 + 1(-1) = 1$	$2 - 3(-1) = 5$
$0 + 3(0) = 0$	$1 + 1(0) = 1$	$0 - 3(0) = 0$
$0 + 3(0) = 0$	$0 + 1(0) = 0$	$1 - 3(0) = 1$
$0 + 3(1) = 3$	$0 + 1(1) = 1$	$0 - 3(1) = -3$
$0 + 3(3) = 9$	$4 + 1(3) = 7$	$14 - 3(3) = 5$

CONT...

Second Tableau

Basic	Problem Variables		Slack Variables			Constant / Solution
	X ₁	X ₂	S ₁	S ₂	S ₃	
Z	0	-5	0	0	3	9
S ₁	0	1	1	0	1	7
S ₂	0	5	0	1	-3	5
X ₁	1	-1	0	0	1	3



Calculating values for the index rows Z, S₂ & X₁ for the Third Tableau.

Row Z	Row S ₂	Row X ₁
$0 + 5(0) = 0$	$0 - 5(0) = 0$	$1 + 1(0) = 1$
$-5 + 5(1) = 0$	$5 - 5(1) = 0$	$-1 + 1(1) = 0$
$0 + 5(1) = 5$	$0 - 5(1) = -5$	$0 + 1(1) = 1$
$0 + 5(0) = 0$	$1 - 5(0) = 1$	$0 + 1(0) = 0$
$3 + 5(1) = 8$	$-3 - 5(1) = -8$	$1 + 1(1) = 2$
$9 + 5(7) = 44$	$5 - 5(7) = -30$	$3 + 1(7) = 10$

Third Tableau

Basic	Problem Variables		Slack Variables			Constant / Solution
	X ₁	X ₂	S ₁	S ₂	S ₃	
Z	0	0	5	0	8	44
X ₂	0	1	1	0	1	7
S ₂	0	0	-5	1	-8	-30
X ₁	1	0	1	0	2	10

So, the **optimal solution**, $Z = 44$, $X_1 = 10$ and $X_2 = 7$

Checking,

$$Z = 3x_1 + 2x_2$$

$$Z = 3(10) + 2(7)$$

$$Z = 44$$



EXAMPLE 2 :

Find the optimal solution by using the Simplex Method for the following;

Maximize, $Z = x + 4y$

subject to;

$$5x + 4y \leq 40$$

$$-x + 2y \leq 6$$

$$x, y \geq 0$$

SOLUTION :

Converting inequalities into Standard Form.

$$Z - x - 4y = 0$$

$$5x + 4y + S_1 = 40$$

$$-x + 2y + S_2 = 6$$

Where; S_1 and S_2 are slack variables.

Initial Tableau

Basic	Problem Variables		Slack Variables		Constant / Solution
	X	Y	S_1	S_2	
Z	-1	-4	0	0	0
S_1	5	4	1	0	40
S_2	-1	2	0	1	6

Calculating values for the index rows Z & S_1 for the Second Tableau

Row Z	Row S_1
$-1 + 4(-1/2) = -3$	$5 - 4(-1/2) = 7$
$-4 + 4(2/2) = 0$	$4 - 4(2/2) = 0$
$0 + 4(0/2) = 0$	$1 - 4(0/2) = 1$
$0 + 4(1/2) = 2$	$0 - 4(1/2) = -2$
$0 + 4(6/2) = 12$	$40 - 4(6/2) = 28$



CONT...

Second Tableau

Basic	Problem Variables		Slack Variables		Constant / Solution
	X	Y	S ₁	S ₂	
Z	-3	0	0	2	12
S ₁	7	0	1	-2	28
Y	-1/2	2/2	0/2	1/2	6/2

Calculating values for the index rows Z & Y for the Third Tableau

Row Z	Row Y
$-3 + 3(7/7) = 0$	$(-\frac{1}{2}) + (\frac{1}{2})(7/7) = 0$
$0 + 3(0/7) = 0$	$1 + (\frac{1}{2})(0/7) = 1$
$0 + 3(1/7) = 3/7$	$0 + (\frac{1}{2})(\frac{1}{7}) = 1/14$
$2 + 3(-2/7) = 8/7$	$(\frac{1}{2}) + (\frac{1}{2})(-\frac{2}{7}) = 5/14$
$12 + 3(28/7) = 24$	$(\frac{6}{2}) + (\frac{1}{2})(28/7) = 5$

Third Tableau

Basic	Problem Variables		Slack Variables		Constant / Solution
	X	Y	S ₁	S ₂	
Z	0	0	3/7	8/7	24
X	7/7	0/7	1/7	-2/7	28/7 = 4
Y	0	1	1/14	5/14	5

So, the optimal solution, $Z = 24$, $X = 4$ and $Y = 5$

Checking,

$$Z = x + 4y$$

$$Z = (4) + 4(5)$$

$$Z = 24$$



EXAMPLE 3 :

Find the optimal solution by using the Simplex Method for the following;

$$\text{Maximize, } Z = 8x_1 + 10x_2 + 7x_3$$

$$\text{Subject to; } x_1 + 3x_2 + 2x_3 \leq 10$$

$$x_1 + 5x_2 + x_3 \leq 8$$

$$x_1, x_2, x_3 \geq 0$$

SOLUTION :

$$Z - 8x_1 - 10x_2 - 7x_3 = 0$$

$$x_1 + 3x_2 + 2x_3 + S_1 = 10$$

$$x_1 + 5x_2 + x_3 + S_2 = 8$$

Initial Tableau

Basic	Problem Variables			Slack Variables		Constant / Solution
	x_1	x_2	x_3	S_1	S_2	
Z	-8	-10	-7	0	0	0
S_1	1	3	2	1	0	10
S_2	1	5	1	0	1	8

Calculating values for the index rows Z & S_1 for the Second Tableau

Row Z

$$-8 + 10\left(\frac{1}{5}\right) = -6$$

$$-10 + 10\left(\frac{5}{5}\right) = 0$$

$$-7 + 10\left(\frac{1}{5}\right) = -5$$

$$0 + 10(0) = 0$$

$$0 + 10\left(\frac{1}{5}\right) = 2$$

$$0 + 10\left(\frac{8}{5}\right) = 16$$

Row S_1

$$1 - 3\left(\frac{1}{5}\right) = \frac{2}{5}$$

$$3 - 3\left(\frac{5}{5}\right) = 0$$

$$2 - 3\left(\frac{1}{5}\right) = \frac{7}{5}$$

$$1 - 3(0) = 1$$

$$0 - 3\left(\frac{1}{5}\right) = -\frac{3}{5}$$

$$10 - 3\left(\frac{8}{5}\right) = \frac{26}{5}$$

CONT...

Second Tableau

Basic	Problem Variables			Slack Variables		Constant / Solution
	X_1	X_2	X_3	S_1	S_2	
Z	-6	0	-5	0	2	16
S_1	$\frac{2}{5}$	0	$\frac{7}{5}$	1	$-\frac{3}{5}$	$\frac{26}{5}$
X_2	$\frac{1}{5}$	1	$\frac{1}{5}$	0	$\frac{1}{5}$	$\frac{8}{5}$

Calculating values for the index rows Z & S_1 for the Third Tableau

Row Z	Row S_1
$-6 + 6(1) = 0$	$\left(\frac{2}{5}\right) - \left(\frac{2}{5}\right)(1) = 0$
$0 + 6\left(\frac{1}{5}\right) = \frac{6}{5}$	$0 - \left(\frac{2}{5}\right)\left(\frac{1}{5}\right) = \frac{2}{5}$
$-5 + 6(1) = 1$	$\frac{7}{5} - \left(\frac{2}{5}\right)(1) = 1$
$0 + 6(0) = 0$	$1 - \left(\frac{2}{5}\right)(0) = 1$
$2 + 6(1) = 8$	$-\frac{3}{5} - \left(\frac{2}{5}\right)(1) = -1$
$16 + 6(8) = 64$	$\frac{26}{5} - \left(\frac{2}{5}\right)(8) = 2$

Third Tableau

Basic	Problem Variables			Slack Variables		Constant / Solution
	X_1	X_2	X_3	S_1	S_2	
Z	0	$\frac{6}{5}$	1	0	8	64
S_1	0	$\frac{2}{5}$	1	1	-1	2
X_1	1	$\frac{1}{5}$	1	0	1	8

So, the optimal solution, $Z = 64$, $X_1 = 8$, $X_2 = 0$ and $X_3 = 0$

Checking,

$$Z = 8x_1 + 10x_2 + 7x_3$$

$$Z = 8(8) + 10(0) + 7(0)$$

$$Z = 64$$



EXAMPLE 4 :

Given the Linear Programming problem with maximum, $Z = \frac{1}{2}x + 3y$ and constraint;

$$x + y \leq 40$$

$$-2x - y \geq -10$$

$$x, y \geq 0$$

Solve the given Linear Programming problem using Simplex Method and hence find the optimal solution.

SOLUTION :

$$x + y \leq 40$$

$$2x + y \leq 10$$

Converting inequalities to Standard Form

$$Z - \frac{1}{2}x - 3y = 0$$

$$x + y + S_1 = 40$$

$$2x + y + S_2 = 10$$

Initial Tableau

Basic	Problem Variables		Slack Variables		Constant / Solution
	X	Y	S ₁	S ₂	
Z	-1/2	-3	0	0	0
S ₁	1	1	1	0	40
S ₂	2	1	0	1	10

CONT...

Calculating values for the index rows Z & S₁ for Second Tableau

Row Z	Row S ₁
$(-1/2) + 3(2) = 11/2$	$1 - 1(2) = -1$
$-3 + 3(1) = 0$	$1 - 1(1) = 0$
$0 + 3(0) = 0$	$1 - 1(0) = 1$
$0 + 3(1) = 3$	$0 - 1(1) = -1$
$0 + 3(10) = 30$	$40 - 1(10) = 30$

Second Tableau

Basic	Problem Variables		Slack Variables		Constant / Solution
	X	Y	S ₁	S ₂	
Z	11/2	0	0	3	30
S ₁	-1	0	1	-1	30
Y	2	1	0	1	10

So, the optimal solution, $Z = 30$, $Y = 10$ and $X = 0$

Checking,

$$Z = \frac{1}{2}x + 3y$$

$$Z = \frac{1}{2}(0) + 3(10)$$

$$Z = 30$$



EXAMPLE 5 :

Given the Linear Programming problem with maximum, $Z = 20x + 10y$ and constraint;

$$x + 2y \leq 10$$

$$x + y \leq 8$$

$$x + 3y \leq 15$$

$$x, y \geq 0$$

- i) Write the problem in Standard Form.
- ii) From the answer in (i), convert the standard form equations into Simplex Tableau and find the optimal solution.

SOLUTION :

$$Z - 20x - 10y = 0$$

$$x + 2y + S_1 = 10$$

$$x + y + S_2 = 8$$

$$x + 3y + S_3 = 15$$

Initial Tableau

Basic	Problem Variables		Slack Variables			Constant / Solution
	X_1	X_2	S_1	S_2	S_3	
Z	-20	-10	0	0	0	0
S_1	1	2	1	0	0	10
S_2	1	1	0	1	0	8
S_3	1	3	0	0	1	15

CONT...

Calculating values for the index rows Z, S₁ & S₃ for Second Tableau.

Row Z	Row S ₁	Row S ₃
$(-20) + 20(1) = 0$	$1 - 1(1) = 0$	$1 - 1(1) = 0$
$(-10) + 20(1) = 10$	$2 - 1(1) = 1$	$3 - 1(1) = 2$
$0 + 20(0) = 0$	$1 - 1(0) = 1$	$0 - 1(0) = 0$
$0 + 20(1) = 20$	$0 - 1(1) = -1$	$0 - 1(1) = -1$
$0 + 20(0) = 0$	$0 - 1(0) = 0$	$1 - 1(0) = 1$
$0 + 20(8) = 160$	$10 - 1(8) = 2$	$15 - 1(8) = 7$

Second Tableau

Basic	Problem Variables		Slack Variables			Constant / Solution
	X ₁	X ₂	S ₁	S ₂	S ₃	
Z	0	10	0	20	0	160
S ₁	0	1	1	-1	0	2
X	1	1	0	1	0	8
S ₃	0	2	0	-1	1	7

So, the optimal solution, $Z = 160$, $Y = 0$ and $X = 8$

Checking,

$$Z = 20x + 10y$$

$$Z = 20(8) + 10(0)$$

$$Z = 160$$

DONE

EXAMPLE 6 :

Given the Linear Programming problem with maximum, $P = 4x + 3y$ and constraint;

$$-x + y \leq 4$$

$$x + 2y \leq 14$$

$$2x + y \leq 16$$

$$x, y \geq 0$$

- i) Change the linear programming problems to Standard Form.
- ii) From the answer in (i), convert the standard form equations into Simplex Tableau and find the optimal solution.

SOLUTION :

Change the linear programming problems to Standard Form.

$$P - 4x - 3y = 0$$

$$-x + y + S_1 = 4$$

$$x + 2y + S_2 = 14$$

$$2x + y + S_3 = 16$$

Initial Tableau

Basic	Problem Variables		Slack Variables			Constant / Solution
	X	Y	S_1	S_2	S_3	
P	-4	-3	0	0	0	0
S_1	-1	1	1	0	0	4
S_2	1	2	0	1	0	14
S_3	2	1	0	0	1	16

CONT...

Calculating values for the index rows P, S₁, & S₂ for the Second Tableau

Row P	Row S ₁	Row S ₂
$(-4) + 4(1) = 0$	$-1 + 1(1) = 0$	$1 - 1(1) = 0$
$-3 + 4\left(\frac{1}{2}\right) = -1$	$1 + 1\left(\frac{1}{2}\right) = \frac{3}{2}$	$2 - 1\left(\frac{1}{2}\right) = \frac{3}{2}$
$0 + 4(0) = 0$	$1 + 1(0) = 1$	$0 - 1(0) = 0$
$0 + 4(0) = 0$	$0 + 1(0) = 0$	$1 - 1(0) = 1$
$0 + 4\left(\frac{1}{2}\right) = 2$	$0 + 1\left(\frac{1}{2}\right) = \frac{1}{2}$	$0 - 1\left(\frac{1}{2}\right) = -\frac{1}{2}$
$0 + 4\left(\frac{16}{2}\right) = 32$	$4 + 1\left(\frac{16}{2}\right) = 12$	$14 - 1\left(\frac{16}{2}\right) = 6$

Second Tableau

Basic	Problem Variables		Slack Variables			Constant / Solution
	X	Y	S ₁	S ₂	S ₃	
P	0	-1	0	0	2	32
S ₁	0	$\frac{3}{2}$	1	0	$\frac{1}{2}$	12
S ₂	0	$\frac{3}{2}$	0	1	$-\frac{1}{2}$	6
X	1	$\frac{1}{2}$	0	0	$\frac{1}{2}$	8

Calculating values for the index rows P, S₁, & X for Second Tableau

Row P	Row S ₁	Row X
$0 + 1(0) = 0$	$0 - \left(\frac{3}{2}\right)(0) = 0$	$1 - \left(\frac{1}{2}\right)(0) = 1$
$-1 + 1(1) = 0$	$\frac{3}{2} - \left(\frac{3}{2}\right)(1) = 0$	$\frac{1}{2} - \left(\frac{1}{2}\right)(1) = 0$
$0 + 1(0) = 0$	$1 - \left(\frac{3}{2}\right)(0) = 1$	$0 - \left(\frac{1}{2}\right)(0) = 0$
$0 + 1\left(\frac{2}{3}\right) = \frac{2}{3}$	$0 - \left(\frac{3}{2}\right)\left(\frac{2}{3}\right) = -1$	$0 - \left(\frac{1}{2}\right)\left(\frac{2}{3}\right) = -\frac{1}{3}$
$2 + 1\left(-\frac{1}{3}\right) = \frac{5}{3}$	$\frac{1}{2} - \left(\frac{3}{2}\right)\left(-\frac{1}{3}\right) = 1$	$\frac{1}{2} - \left(\frac{1}{2}\right)\left(-\frac{1}{3}\right) = \frac{2}{3}$
$32 + 1(4) = 36$	$12 - \left(\frac{3}{2}\right)(4) = 6$	$8 - \left(\frac{1}{2}\right)(4) = 6$

CONT...

Third Tableau

Basic	Problem Variables		Slack Variables			Constant / Solution
	X	Y	S ₁	S ₂	S ₃	
P	0	0	0	2/3	5/3	36
S ₁	0	0	1	-1	1	6
Y	0	1	0	2/3	-1/3	4
X	1	0	0	-1/3	2/3	6

So, the **optimal solution**, **P = 36**, **Y = 4** and **X = 6**

Checking,

$$P - 4x - 3y = 0$$

$$P = 4x + 3y$$

$$P = 4(6) + 3(4)$$

$$P = 36$$

DONE

~~$\operatorname{tg} y = \frac{\sin(x+y)}{\cos x \cos y}$~~

LET'S LEARN SIMPLEX METHOD



$$\begin{aligned} \operatorname{tg} y &= \frac{\sin(x+y)}{\cos x \cos y} \\ r_0 &= r^2 \end{aligned}$$

PRACTICE 1 ::

1. Solve the following linear programming by using Simplex Method.

$$\text{Maximize } z = x_1$$

$$\text{Subject to } 3x_1 + 2x_2 \leq 60$$

$$x_1 + 2x_2 \leq 28$$

$$x_1 + 4x_2 \leq 48$$

$$\text{Where } x_1, x_2 \geq 0$$

$$\text{ANSWER : } z = 20, x_1 = 20, x_2 = 0$$

2. Solve the Linear Programming problem using Simplex Method.

Maximize, $P = 7x + 8y + 10z$ subject to constraints :

$$2x + 3y + 2z \leq 1000$$

$$x + y + 2z \leq 800$$

$$x \geq 0, y \geq 0, z \geq 0$$

$$\text{ANSWER : } P = 4400, x = 200, y = 0$$

3. Solve the Linear Programming problem by using Simplex Method.

$$Z = x_1 + 2x_2$$

With the constraints

$$x_1 + 3x_2 \leq 15$$

$$2x_1 - x_2 \leq 12$$

$$x_1, x_2 \geq 0$$

$$\text{Answer : } Z = \frac{87}{7}, x = \frac{51}{7}, y = \frac{18}{7}$$

PRACTICE 1 ::

4. Solve the Linear Programming problem by using Simplex Method.

$$\text{Max, } P = 3x + 5y$$

$$3x + 2y \leq 12$$

$$x + 3y \leq 11$$

$$x \geq 0, y \geq 0$$

Answer : $P = 21, x = 2, y = 3$

5. Solve the Linear Programming problem by using Simplex Method.

$$\text{Max, } P = x_1 + 2x_2$$

Subject to the constraints of :

$$x_1 + 4x_2 \leq 8$$

$$x_1 + x_2 \leq 12$$

$$x_1, x_2 \geq 0$$

Answer : $P = 8, x_1 = 8, x_2 = 0$

6. A company produces two products, x_1 and x_2 . The profit for each unit of x_1 is RM3, and for each unit of x_2 is RM5. The production is subject to the following constraints:

$$2x_1 + x_2 \leq 100 \text{ (Resource 1 constraint)}$$

$$x_1 + 2x_2 \leq 80 \text{ (Resource 2 constraint)}$$

$$x_1, x_2 \text{ (Non-negativity)}$$

$$\text{Maximize the total profit } Z = 3x_1 + 5x_2$$

Answer : $Z = RM220, x_1 = 40, x_2 = 20$

PRACTICE 1 ::

7. Solve the following linear programming by using Simplex Method.

$$\text{Maximize } Z = 3x_1 + 2x_2$$

$$\text{Subject to } x_1 + x_2 \leq 4$$

$$x_1 + 3x_2 \leq 6$$

$$\text{Where } x_1, x_2 \geq 0$$

$$\text{ANSWER : } z = 12, x_1 = 4, x_2 = 0$$

8. Solve the following linear programming by using Simplex Method.

$$\text{Maximize } Z = 3x_1 + 5x_2$$

$$\text{Subject to } x_1 + 2x_2 \leq 6$$

$$3x_1 + 2x_2 \leq 12$$

$$\text{Where } x_1, x_2 \geq 0$$

$$\text{ANSWER : } z = 16.5, x_1 = 3, x_2 = 1.5$$

9. Solve the following linear programming by using Simplex Method.

$$\text{Maximize } Z = 4x_1 + 3x_2$$

$$\text{Subject to } 2x_1 + x_2 \leq 8$$

$$x_1 + 2x_2 \leq 6$$

$$\text{Where } x_1, x_2 \geq 0$$

$$\text{ANSWER : } z = 17.33$$

PRACTICE 1 ::

10. Solve the following linear programming by using Simplex Method.

$$\text{Maximize } Z = 6x_1 + 5x_2$$

$$\text{Subject to } x_1 + 4x_2 \geq 8$$

$$3x_1 + 2x_2 \geq 12$$

$$\text{Where } x_1, x_2 \geq 0$$

ANSWER : $z = 27.6$

11. Solve the following linear programming by using Simplex Method.

$$\text{Maximize } Z = 3x_1 + 2x_2$$

$$\text{Subject to } x_1 + x_2 \leq 4$$

$$x_1 + 2x_2 \leq 6$$

$$\text{Where } x_1, x_2 \geq 0$$

ANSWER : $z = 14$

12. Solve the following linear programming by using Simplex Method.

$$\text{Maximize } Z = 4x_1 + 3x_2$$

$$\text{Subject to } x_1 + 2x_2 \leq 8$$

$$2x_1 + x_2 \leq 6$$

$$\text{Where } x_1, x_2 \geq 0$$

ANSWER : $z = 24$

PRACTICE 2 : QUESTION RELATED TO REAL LIFE

1. A bakery produces two types of bread: **White Bread** (x_1) and **Whole Wheat Bread** (x_2). The company aims to **maximize profit**. The following information is provided:

Profit per Unit:

- White Bread (x_1) : RM5 per unit
- Whole Wheat Bread (x_2). RM7 per unit

Resource Constraints:

1. Processing Time:

- White Bread: 1 hour/unit
- Whole Wheat Bread: 2 hours/unit
- Total available time: 20 hours

2. Oven Capacity:

- White Bread: 2 units/hour
- Whole Wheat Bread: 1 unit/hour
- Maximum capacity: 22 units

Market Demand:

- Maximum demand for Whole Wheat Bread (x_2). is 8 units.

Question :

Determine the number of units of x_1 (White Bread) and x_2 (Whole Wheat Bread) to produce in order to maximize profit.

ANSWER : $x_1 = 4$ (White Bread), $x_2 = 8$ (Whole Wheat Bread) $Z = \text{Rm } 76$

PRACTICE 2 : QUESTION RELATED TO REAL LIFE

2. A factory produces two products: **Product A** (x_1) and **Product B** (x_2). The company wants to **maximize its profit**. The following details are provided:

Profit per unit:

- Product A (x_1) : RM10 per unit
- Product B (x_2). RM8 per unit

Limited Resources:

1. Material Usage:

- Product A: 3 units of material per unit produced
- Product B: 2 units of material per unit produced
- Total material available: 150 units

2. Labor Hours:

- Product A: 2 hours of labor per unit produced
- Product B: 1 hour of labor per unit produced
- Total labor hours available: 100 hours

Market Demand:

- The maximum demand for Product A is 40 units.
- The maximum demand for Product B is 50 units.

Question :

Determine the number of units of x_1 (Product A) and x_2 (Product B) that should be produced to maximize profit.

ANSWER : $x_1 = 40$ (Product A), $x_2 = 50$ (Product B) $Z = \text{Rm } 640$

PRACTICE 2 : QUESTION RELATED TO REAL LIFE

3. A company manufactures two products, **Product X** (x_1) and **Product Y** (x_2) and wants to **maximize profit**. The following information is provided:

Objective function : $Z = 6x_1 + 5x_2$

Constraints : $2x_1 + x_2 \leq 8$

$$x_1 + 2x_2 \leq 6$$

Where $x_1, x_2 \geq 0$

Solve the following linear programming by using Simplex Method.

ANSWER : The company should produce **4 units of Product X** and **2 units of Product Y** to **maximize the profit**, which will be **RM34**.

4. A small fruit shop sells two types of fruits: **Apples** (x_1) and **Oranges** (x_2). The shop wants to **maximize revenue** while considering its inventory and available sales staff.

Objective function : $Z = 3x_1 + 4x_2$

Constraints : $x_1 \leq 20$

$$x_2 \leq 15$$

$$x_1 + 1.5x_2 \leq 40$$

Where $x_1, x_2 \geq 0$

Solve the following linear programming by using Simplex Method.

ANSWER : $x_1 = 20$ (Apples), $x_2 = 15$ (Oranges) $Z = \text{Rm}100$

ADVANTAGES OF SIMPLEX METHOD

1. Efficiency in Solving Large-Scale Problems

The Simplex method can handle linear programming problems with numerous variables and constraints efficiently. Unlike graphical methods, which are limited to two-variable problems, Simplex can solve large-scale problems effectively.

2. Systematic and Iterative Approach

Simplex employs a systematic iterative process that moves from one feasible solution to another, improving the objective function at each step. This structured approach ensures accurate and reliable solutions, reducing human error.

3. Flexibility in Application

The method is versatile, applicable to both maximization and minimization problems across various fields such as economics, logistics, and marketing. It can also handle problems with equality and inequality constraints.

4. Computational Efficiency

Despite its worst-case exponential time complexity, the Simplex method often performs remarkably well in practice, solving large linear programming problems with thousands of variables and constraints in a reasonable amount of time.

5. Sensitivity Analysis

Simplex provides valuable insights through sensitivity analysis, helping understand how changes in problem parameters affect the optimal solution. This feature is crucial for decision-making in dynamic environments.

6. Foundation for Advanced Optimization Techniques

The Simplex method serves as a foundation for many advanced optimization techniques used in operations research, economics, and other fields, facilitating further research and application in optimization.

COMMON MISTAKES IN THE SIMPLEX METHOD

1. Incorrect Problem Formulation

Before applying the Simplex method, ensure that the linear programming problem is correctly formulated. This includes verifying that the objective function and constraints accurately reflect the real-world scenario you're trying to model. For example, if maximizing profit is the goal, every term in the objective function should contribute to this end. A common oversight might be neglecting to include a cost factor, which could lead to an inflated and unrealistic profit maximization.

2. Data Entry Errors

Simple data entry mistakes can lead to vastly different outcomes. For instance, confusing a '5' with a '0.5' in a constraint can alter the feasible region entirely. Always double-check your entries and consider using Excel's 'Data Validation' feature to restrict input ranges and types.

3. Incorrect Pivot Element Selection

The choice of pivot element is critical in the Simplex method. Selecting a non-positive element when pivoting can cause the algorithm to cycle indefinitely without reaching an optimal solution. To illustrate, if you mistakenly choose a pivot element of '0' in a row, the algorithm won't be able to perform the necessary row operations to find the next basic feasible solution.

4. Degeneracy and Cycling

Degeneracy occurs when more than one basic variable assumes the value of zero in a basic feasible solution. This can lead to multiple iterations without any improvement in the objective function value, known as cycling. An example of this would be if two constraints intersect at a point that doesn't correspond to an increase or decrease in the objective function, causing the algorithm to stall.

5. Unbounded Solutions

Sometimes, the Simplex algorithm may indicate that the solution is unbounded, meaning that the objective function can increase indefinitely. This usually points to a missing or incorrectly formulated constraint. For example, if you're maximizing profit without a constraint on the maximum production capacity, the algorithm will suggest producing an infinite quantity, which is not feasible in practice.

6. Numerical Stability Issues

Numerical issues can arise due to the limited precision of computer arithmetic. When dealing with very large or very small coefficients, rounding errors can accumulate, leading to incorrect solutions. It's advisable to scale the problem or use software that supports higher precision calculations to mitigate this issue.

APPLICATION OF SIMPLEX METHOD IN ENGINEERING FIELD

1. STRUCTURAL ENGINEERING

Use in optimization in material usage and design parameters of structures. Thus, it can help in minimizing the cost of materials for bridges, buildings, or frame while maintaining structural integrity and safety standards.



APPLICATION OF SIMPLEX METHOD IN ENGINEERING FIELD

2. ELECTRICAL ENGINEERING

Apply in power systems to optimize the generation and distribution on electrical power. The goal is often to minimize the cost of electricity generation while ensuring that power supply meets demand and operates with capacity limits.



APPLICATION OF SIMPLEX METHOD IN ENGINEERING FIELD

3. TRANSPORTATION & TRAFFIC ENGINEERING

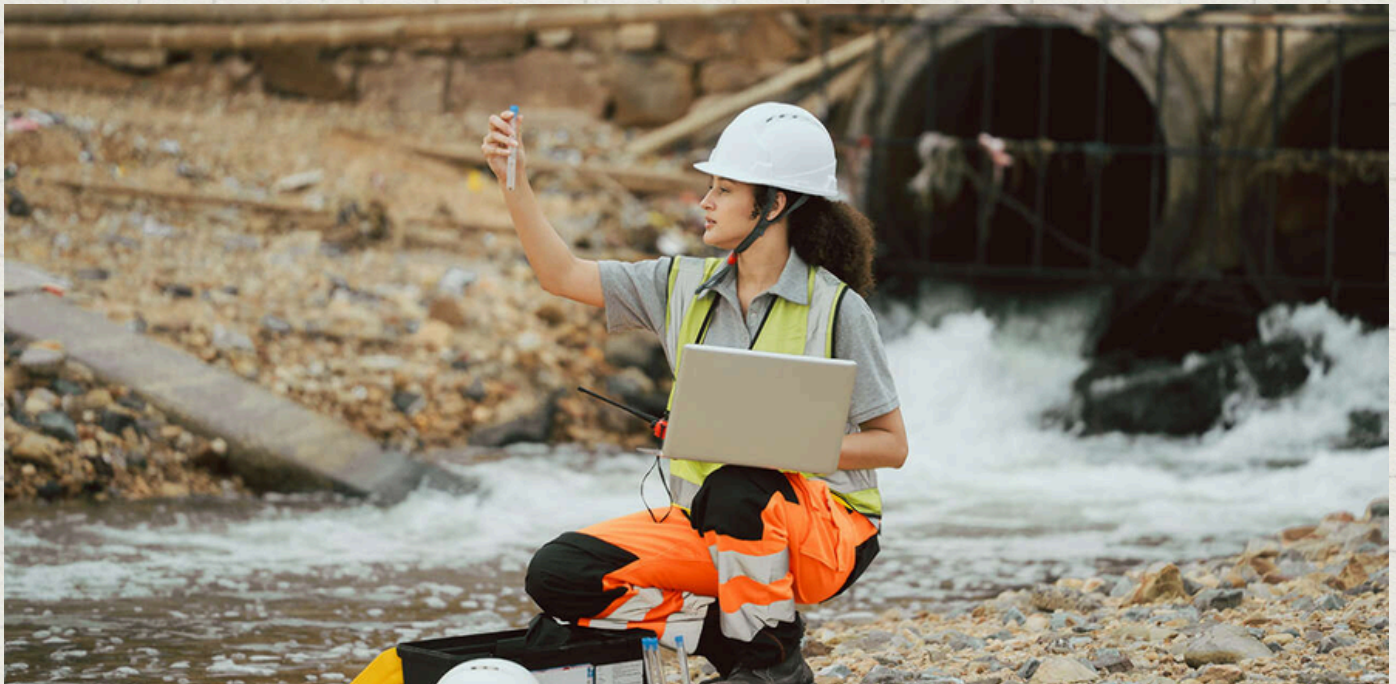
Used to optimize traffic light timing, road usage, and vehicle flow in transportation networks, aiming to minimize congestion, travel time and fuel consumption.



APPLICATION OF SIMPLEX METHOD IN ENGINEERING FIELD

4. ENVIRONMENTAL ENGINEERING

Used to optimize the collection, recycling, and disposal of waste. It helps minimize costs while considering constraints like available disposal sites, transportation costs, and recycling capabilities.



APPLICATION OF SIMPLEX METHOD IN ENGINEERING FIELD

5. AUTOMOTIVE ENGINEERING

Employed in automotive engineering to optimize vehicle designs, ensuring cost-effective manufacturing while maintaining performance and safety standards. For example, optimizing engine efficiency or fuel consumption.



APPLICATION OF SIMPLEX METHOD IN ENGINEERING FIELD

6. AEROSPACE ENGINEERING

To optimize the design of aircraft components, such as wings or fuselage, to minimize material costs while meeting safety and performance requirements.



APPLICATION OF SIMPLEX METHOD IN ENGINEERING FIELD

7. TELECOMMUNICATIONS ENGINEERING

Optimize the design of communication networks by minimizing costs, such as for cable installation or wireless transmission power, while ensuring network reliability and efficiency.



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$$\begin{aligned} \text{tg } y &= \frac{\sin(x+y)}{\cos x \cos y} \\ &= \frac{\sin x \cos y + \cos x \sin y}{\cos x \cos y} \end{aligned}$$



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