KEMENTERIAN PENDIDIKAN TINGGI
JABATAN PENDIDIKAN POLITEKNIK DAN KOLEJ KOMUNITI
POLITEKNIK MALAYSIA
TUANKU SULTANAH BAHIYAH

KEMENTERIAN PENDIDIKAN TINGGI
JABATAN PENDIDIKAN POLITEKNIK DAN KOLEJ KOMUNITI

INTERACTIVE E-BOOK

A DETAILS ABOUT..
Solving polynomial equations
FIXED POINT & NEWTON RAPHSON

MOHD FAIRUL BIN RASID
HASLIZA BINTI HALIM
NOOR AZILLA BINTI MD RADZI

Solving polynomial equations

FIXED POINT & NEWTON RAPHSON

FIRST EDITION
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PREFACE

This eBook is designed to assist engineering students in studying how to construct solutions of polynomial equations. This eBook is suitable for Engineering Mathematics 3 and Electrical Engineering Mathematics courses.

All contents in this eBook will put students on the track as it is in line with the latest syllabus specified by the Polytechnic's syllabus requirement, Ministry of Higher Education.

This eBook consists of comprehensive notes with details explanation.

Any positive feedback from lecturers and students that would improve the content of this eBook are mostly welcome and appreciated.

This eBook is one of the tiny efforts that we have made to help students mastery this course with excellence.

Author's
MOHD FAIRUL BIN RASID
HASLIZA BINTI HALIM
NOOR AZILLA BINTI MD RADZI

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JMSK
JABATAN MATEMATIK SAINS DAN KOMPUTER

POLITEKNIK MALAYSIA
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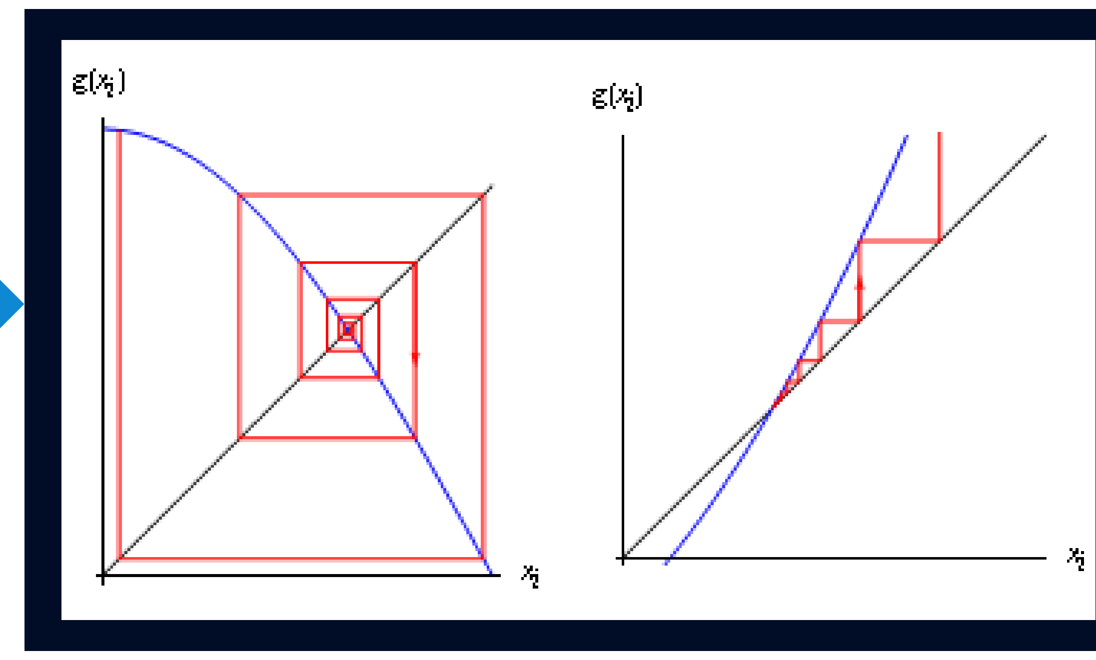
METHOD 1

FIXED POINT ITERATION METHOD



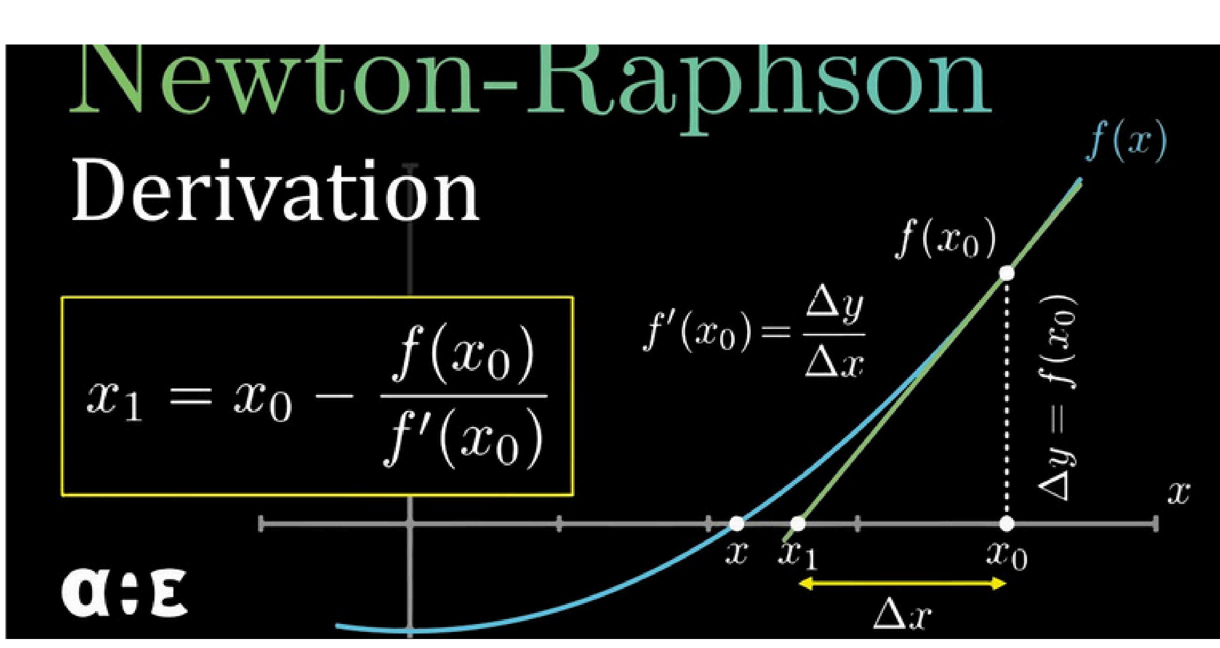
The fixed point iteration method uses the concept of a fixed point in a repeated manner to compute the solution of the given equation.

POLYNOMIAL EQUATION

equations that have multiple terms made up of numbers and variables

METHOD 2

NEWTON RAPHSON METHOD



An iterative numerical method used to find the roots of a real-valued function.

METHOD 1
FIXED POINT ITERATION METHOD

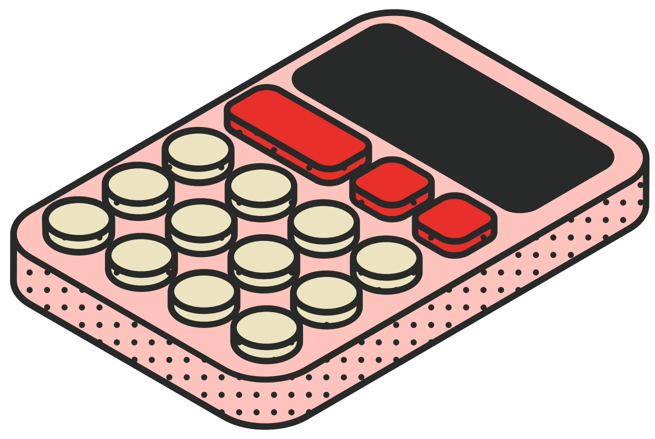
NOTES

Step in Fixed Point Iteration Method

1. Make x as a subject x = g(x)
   Initial approximate value X_0
   Use the given X_0 , If X_0 is not given, assume x = 1

2. Checking for convergence
   $|g(x)| < 0$

3. Substitute X_0 in x = g(x) to find x1
   Substitute X_1 in x = g(x) to find x2



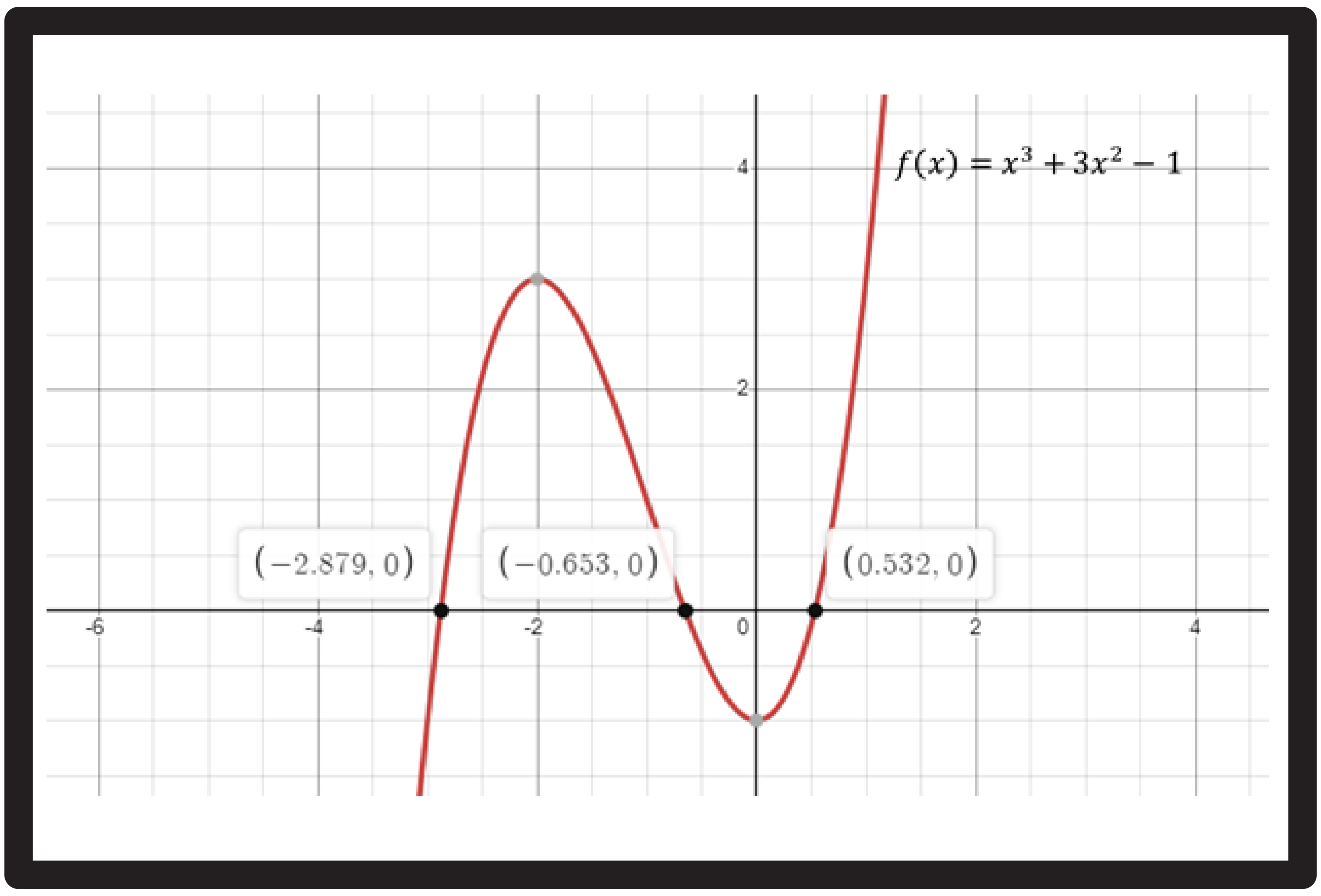
4. Find $| x_{n+1} - x_n |$

5. Repeat second and third step until
   $| x_{n+1} - x_n | = 0.001$ (depends on the required accuracy)

FIXED POINT ITERATION METHOD

EXAMPLE 1

Find the root of the equation $x^3 + 3x^2 - 1 = 0$
by using Simple Fixed Point Iteration Method,
correct to 3 decimal places.
The first approximation is 0.5



Iterative function 1

$$x^3 + 3x^2 - 1 = 0$$
$$3x^2 = 1 - x^3$$
$$x^2 = \frac{1 - x^3}{3}$$
$$x = \sqrt{\frac{1 - x^3}{3}}$$

← *Make x as a subject x = g(x)*

CHECKING!

$$x = \left(\frac{1 - x^3}{3}\right)^{1/2}$$

$$g'(x) = \frac{1}{2}\left(\frac{1 - x^3}{3}\right)^{-\frac{1}{2}}(-x^2)$$

$$= \frac{-x^2}{2}\left(\frac{1 - x^3}{3}\right)^{-\frac{1}{2}}$$

$$g'(0.5) = \frac{-(0.5)^2}{2}\left(\frac{1 - (0.5)^3}{3}\right)^{-\frac{1}{2}}$$

$$= 0.231$$

$$|g'(x)| = 0.231 < 1$$

CONVERGE ✓

SUGGESTED
CAN USE THE ITERATION

Find | x_{n+1} - x_n |

n	x_n	x_{n+1}	Error
0	$x_0 = 0.5$	$x_1 = 0.540$	0.040
1	$x_1 = 0.540$	$x_2 = 0.530$	0.010
2	$x_2 = 0.530$	$x_3 = 0.533$	0.003
3	$x_3 = 0.533$	$x_4 = 0.532$	0.001
4	$x_4 = 0.532$	$x_5 = 0.532$	0

$\therefore x_5 = 0.532$

Repeat second and third step until
| x_{n+1} - x_n |< 0.001 (depends on the required accuracy)

FIXED POINT ITERATION METHOD

EXAMPLE 1

CONTINUE...

Iterative function 2

$$x^3 + 3x^2 - 1 = 0$$
$$x^2(x+3) - 1 = 0$$
$$x^2(x+3) = 1$$
$$x = \sqrt{\frac{1}{x+3}}$$

n	x_n	x_{n+1}	Error
0	$x_0 = 0.5$	$x_1 = 0.535$	0.035
1	$x_1 = 0.535$	$x_2 = 0.532$	0.003
2	$x_2 = 0.532$	$x_3 = 0.532$	0

$$\therefore x_3 = 0.532$$

CHECKING!

$$x = \left(\frac{1}{x+3}\right)^{1/2}$$
$$x = ((x+3)^{-1})^{1/2}$$
$$x = (x+3)^{-1/2}$$
$$g'(x) = -\frac{1}{2}(x+3)^{-3/2}$$
$$g'(0.5) = -\frac{1}{2}\left((0.5)+3\right)^{-3/2}$$
$$= -0.076$$
$$|g'(x)| = 0.076 < 1$$

CONVERGE ✓

SUGGESTED

CAN USE THE ITERATION

Iterative function 3

$$x^3 + 3x^2 - 1 = 0$$
$$x^3 = 1 - 3x^2$$
$$x = \sqrt[3]{1 - 3x^2}$$

n	x_n	x_{n+1}	Error
0	$x_0 = 0.5$	$x_1 = 0.630$	0.13
1	$x_1 = 0.630$	$x_2 = -0.576$	1.206
2	$x_2 = -0.576$	$x_3 = 0.167$	0.743
3	$x_3 = 0.167$	$x_4 = 0.971$	0.804
4	$x_4 = 0.971$	$x_5 = -1.223$	2.194
5	$x_5 = -1.223$	$x_6 = -1.516$	0.293
6	$x_6 = -1.516$	$x_7 = -1.806$	0.29
7	$x_7 = -1.806$	$x_8 = -2.063$	0.257
8	$x_8 = -2.063$	$x_9 = -2.275$	0.212
9	$x_9 = -2.275$	$x_{10} = -2.440$	0.165
10	$x_{10} = -2.440$	$x_{11} = -2.564$	0.124
11	$x_{11} = -2.564$	$x_{12} = -2.655$	0.091
12	$x_{12} = -2.655$	$x_{13} = -2.721$	0.066
13	$x_{13} = -2.721$	$x_{14} = -2.768$	0.047
14	$x_{14} = -2.768$	$x_{15} = -2.801$	0.033
15	$x_{15} = -2.801$	$x_{16} = -2.825$	0.024
16	$x_{16} = -2.825$	$x_{17} = -2.841$	0.016
17	$x_{17} = -2.841$	$x_{18} = -2.853$	0.012
18	$x_{18} = -2.853$	$x_{19} = -2.861$	0.008
19	$x_{19} = -2.861$	$x_{20} = -2.867$	0.006
20	$x_{20} = -2.867$	$x_{21} = -2.871$	0.004
21	$x_{21} = -2.871$	$x_{22} = -2.874$	0.003
22	$x_{22} = -2.874$	$x_{23} = -2.876$	0.002
23	$x_{23} = -2.876$	$x_{24} = -2.877$	0.001
24	$x_{24} = -2.877$	$x_{25} = -2.878$	0.001
25	$x_{25} = -2.878$	$x_{26} = -2.878$	0

$$\therefore x_{26} = -2.878$$

CHECKING!

$$x = (1 - 3x^2)^{1/3}$$
$$g'(x) = \frac{1}{3}\left(1 - 3x^2\right)^{-\frac{2}{3}}(-6x)$$
$$= -2x\left(1 - 3x^2\right)^{-\frac{2}{3}}$$
$$g'(0.5) = -2(0.5)\left(1 - 3(0.5)^2\right)^{-\frac{2}{3}}$$
$$= -2.52$$
$$|g'(x)| = 2.52 > 1$$

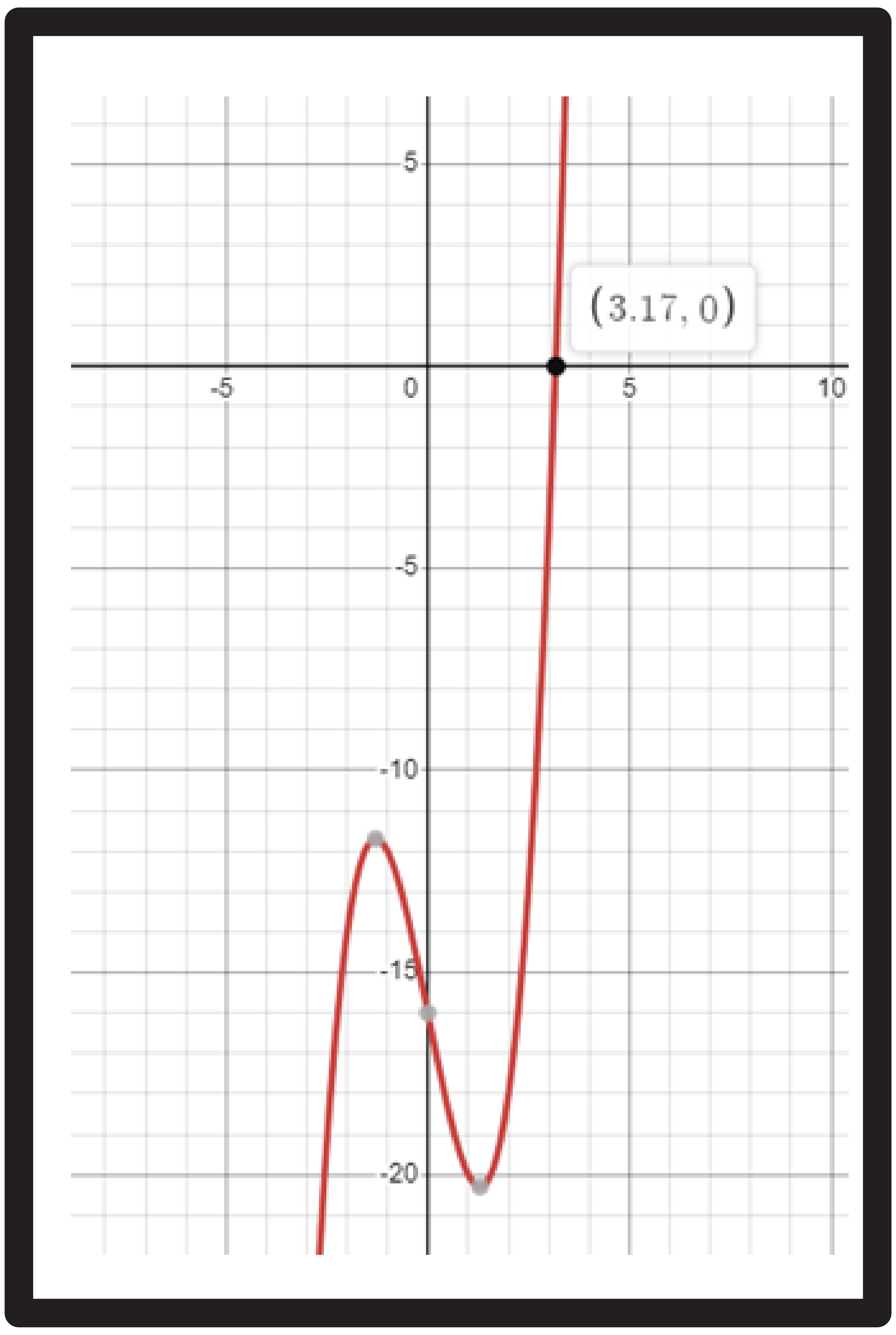
DIVERGE ✗

NOT SUGGESTED

FIXED POINT ITERATION METHOD

EXAMPLE 2

Find the root of the equation $f(x) = x^3 - 5x - 16$ correct to 4 decimal places with an initial approximation is 3.5 by using Simple Fixed Point Iteration Method.



Iterative function 1

$$x^3 - 5x - 16 = 0$$
$$x^3 = 5x + 16$$
$$x = \sqrt[3]{5x + 16}$$

n	x_n	x_{n+1}	Error
0	$x_0 = 3.5$	$x_1 = 3.2237$	0.2763
1	$x_1 = 3.2237$	$x_2 = 3.1787$	0.0450
2	$x_2 = 3.1787$	$x_3 = 3.1713$	0.0074
3	$x_3 = 3.1713$	$x_4 = 3.1700$	0.0013
4	$x_4 = 3.1700$	$x_5 = 3.1698$	0.0002
5	$x_5 = 3.1698$	$x_6 = 3.1698$	0

$$\therefore x_6 = 3.1698$$

CHECKING!

$$x = (5x + 16)^{1/3}$$

$$g'(x) = \frac{1}{3}(5x + 16)^{-\frac{2}{3}}(5)$$

$$= \frac{5}{3}(5x + 16)^{-\frac{2}{3}}$$

$$g'(3.5) = \frac{5}{3}(5(3.5) + 16)^{-\frac{2}{3}}$$

$$= 0.16$$

$$|g'(x)| = 0.16 < 1$$

CONVERGE ✓

SUGGESTED
CAN USE THE ITERATION

FIXED POINT ITERATION METHOD

EXAMPLE 2

CONTINUE...

Iterative function 2

$x^3 - 5x - 16 = 0$

$5x = x^3 - 16$

$x = \dfrac{x^3 - 16}{5}$

n	x_n	x_{n+1}	Error
0	$x_0 = 3.5$	$x_1 = 5.3750$	1.875
1	$x_1 = 5.3750$	$x_2 = 27.8574$	22.4824
2			

CHECKING!

$x = \dfrac{x^3}{5} - \dfrac{16}{5}$

$g'(x) = \dfrac{3}{5}x^2$

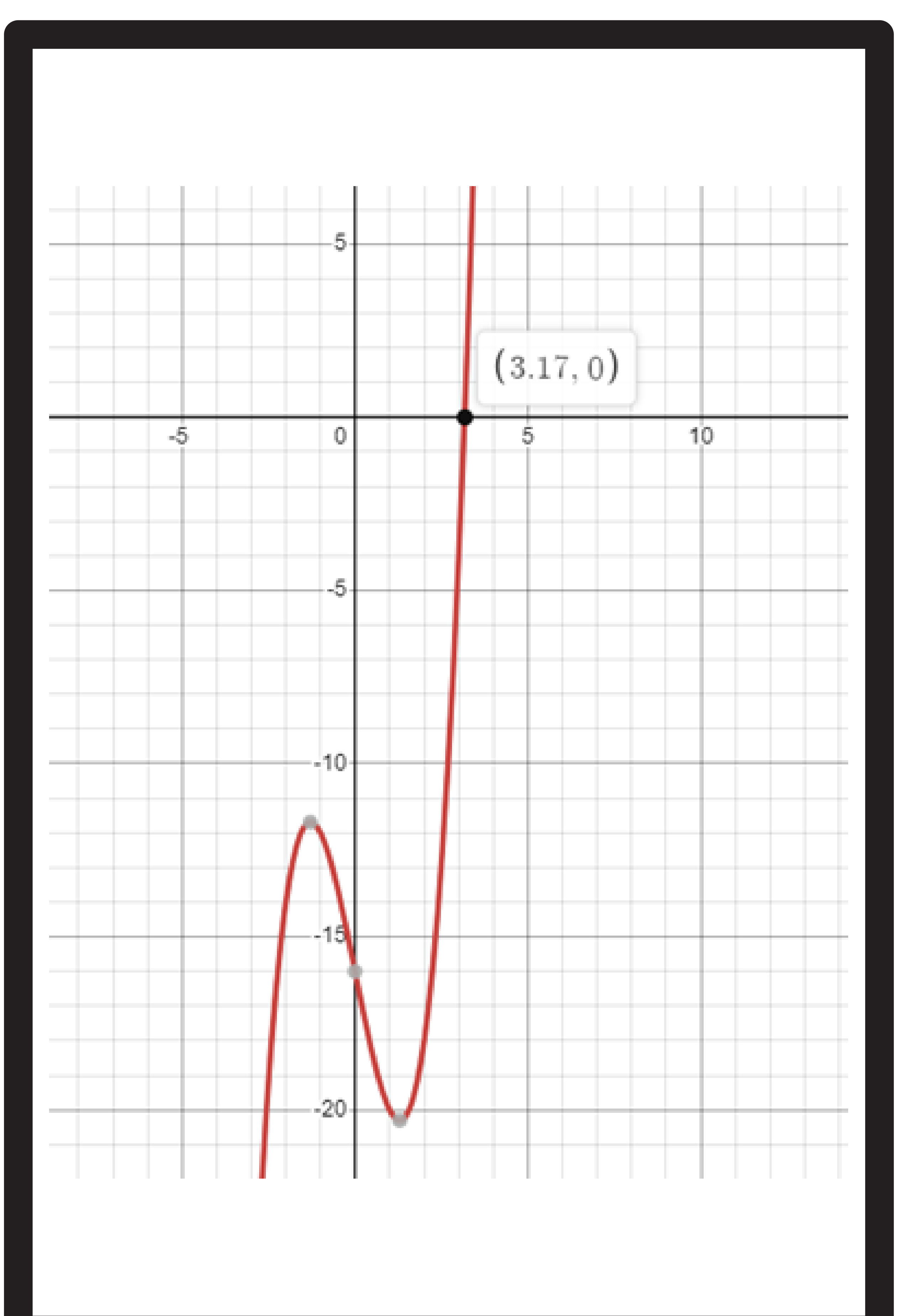
$g'(3.5) = \dfrac{3}{5}(3.5)^2$

$= 7.35$

$|g'(x)| = 7.35 > 1$

DIVERGE ✗

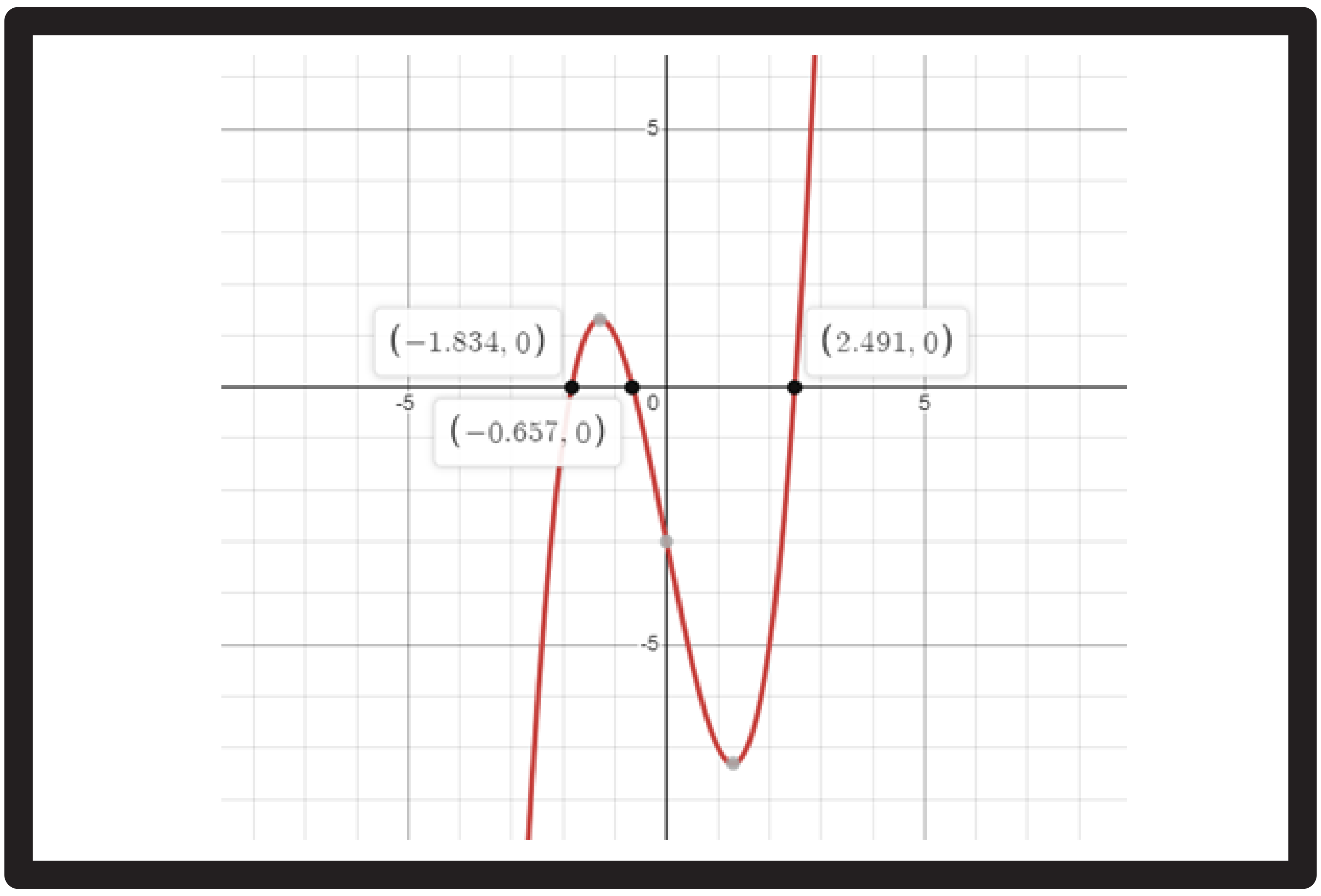
NOT SUGGESTED
ALSO NOT APPROPRIATE ITERATION
BECAUSE THERE IS ONLY ONE
POINT CAN BE TRUE
(REFER GRAPH)



FIXED POINT ITERATION METHOD

EXAMPLE 3

Find the root of the equation
$f(x) = x^3 - 5x - 3$ correct to 3 decimal places
with an initial approximation is 2.4 by
using Simple Fixed Point Iteration Method.



Iterative function 1

$$x^3 - 5x - 3 = 0$$
$$x^3 = 5x + 3$$
$$x = \sqrt[3]{5x + 3}$$

n	x_n	x_{n+1}	Error
0	$x_0 = 2.4$	$x_1 = 2.466$	0.066
1	$x_1 = 2.466$	$x_2 = 2.484$	0.018
2	$x_2 = 2.484$	$x_3 = 2.489$	0.005
3	$x_3 = 2.489$	$x_4 = 2.490$	0.001
4	$x_4 = 2.490$	$x_5 = 2.491$	0.001
5	$x_5 = 2.491$	$x_6 = 2.491$	0

$$\therefore x_6 = 2.491$$

CHECKING!

$$x = (5x + 3)^{1/3}$$

$$g'(x) = \frac{1}{3}(5x + 3)^{-\frac{2}{3}}(5)$$

$$= \frac{5}{3}(5x + 3)^{-\frac{2}{3}}$$

$$g'(2.4) = \frac{5}{3}(5(2.4) + 3)^{-\frac{2}{3}}$$

$$= 0.274$$

$$|g'(x)| = 0.274 < 1$$

CONVERGE ✓

SUGGESTED
CAN USE THE ITERATION

FIXED POINT ITERATION METHOD

EXAMPLE 3

CONTINUE...

Iterative function 2

$$x^3 - 5x - 3 = 0$$
$$x(x^2 - 5) - 3 = 0$$
$$x(x^2 - 5) = 3$$
$$x = \frac{3}{x^2 - 5}$$

n	x_n	x_{n+1}	Error
0	$x_0 = 2.4$	$x_1 = 3.947$	1.547
1	$x_1 = 2.466$	$x_2 = 2.484$	0.018
2	$x_2 = 2.484$	$x_3 = -0.610$	3.094
3	$x_3 = 2.489$	$x_4 = -0.648$	3.137
4	$x_4 = 2.490$	$x_5 = -0.655$	3.145
5	$x_5 = 2.491$	$x_6 = -0.656$	3.147
6	$x_6 = -0.656$	$x_7 = -0.657$	0.001
7	$x_7 = -0.657$	$x_8 = -0.657$	0

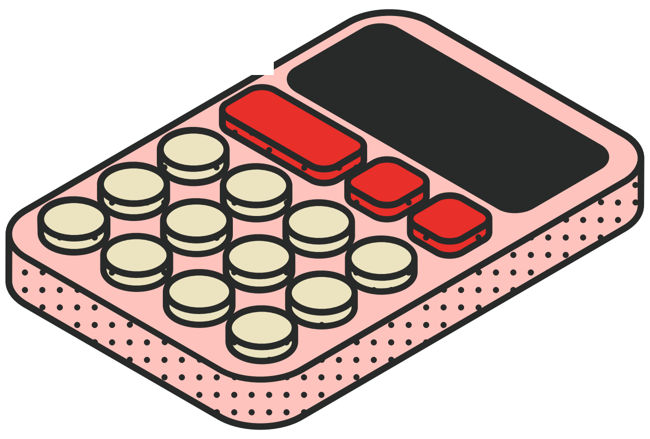
$$\therefore x_8 = -0.657$$

CHECKING!

$$x = 3(x^2 - 5)^{-1}$$
$$g'(x) = -3(x^2 - 5)^{-2}(2x)$$
$$= -6x(x^2 - 5)^{-2}$$
$$g'(2.4) = -6(2.4)((2.4)^2 - 5)^{-2}$$
$$= -24.93$$
$$|g'(x)| = 24.93 > 1$$

DIVERGE ✗

NOT SUGGESTED



Iterative function 3

$$x^3 - 5x - 3 = 0$$
$$5x = x^3 - 3$$
$$x = \frac{x^3 - 3}{5}$$

n	x_n	x_{n+1}	Error
0	$x_0 = 2.4$	$x_1 = 2.165$	0.235
1	$x_1 = 2.165$	$x_2 = 1.430$	0.735
2	$x_2 = 1.430$	$x_3 = -0.015$	1.445
3	$x_3 = -0.015$	$x_4 = -0.600$	0.585
4	$x_4 = -0.600$	$x_5 = -0.643$	0.043
5	$x_5 = -0.643$	$x_6 = -0.653$	0.010
6	$x_6 = -0.653$	$x_7 = -0.656$	0.003
7	$x_7 = -0.656$	$x_8 = -0.656$	0

$$\therefore x_8 = -0.656$$

CHECKING!

$$x = \frac{x^3}{5} - \frac{3}{5}$$
$$g'(x) = \frac{3}{5}x^2$$
$$g'(2.4) = \frac{3}{5}(2.4)^2$$
$$= 3.456$$
$$|g'(x)| = 3.456 > 1$$

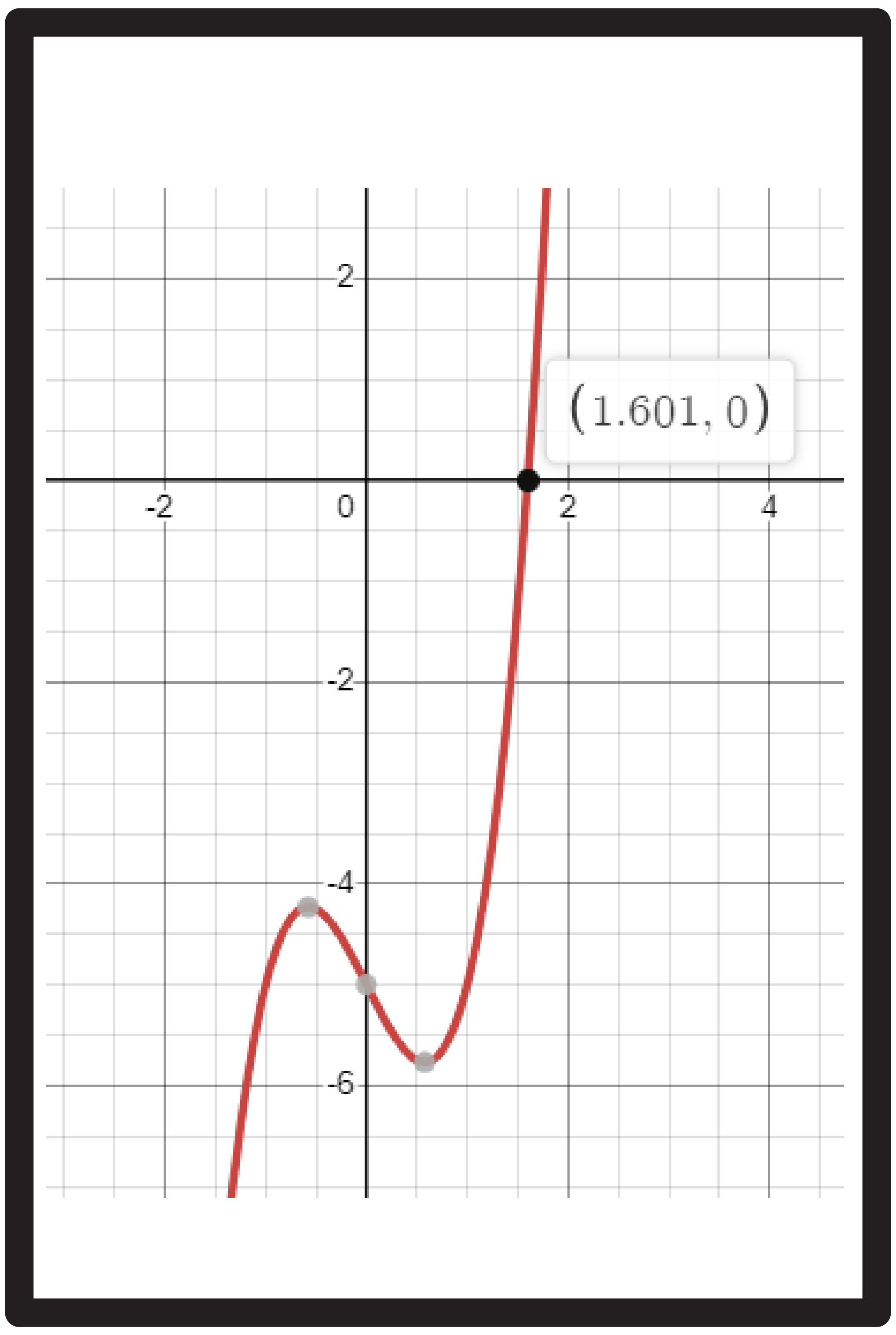
DIVERGE ✗

NOT SUGGESTED

FIXED POINT ITERATION METHOD

EXAMPLE 4

Find the first approximate root of the equation $2x^3 - 2x - 5 = 0$ up to 4 decimal places by using Simple Fixed Point Iteration Method.



Iterative function 1

$$2x^3 - 2x - 5 = 0$$
$$2x^3 = 2x + 5$$
$$x^3 = \frac{2x+5}{2}$$
$$x = \sqrt[3]{\frac{2x+5}{2}}$$

n	x_n	x_{n+1}	Error
0	$x_0 = 1$	$x_1 = 1.5183$	0.5183
1	$x_1 = 1.5183$	$x_2 = 1.5898$	0.0715
2	$x_2 = 1.5898$	$x_3 = 1.5991$	0.0093
3	$x_3 = 1.5991$	$x_4 = 1.6004$	0.0013
4	$x_4 = 1.6004$	$x_5 = 1.6006$	0.0002
5	$x_5 = 1.6006$	$x_6 = 1.6006$	0

$$\therefore x_6 = 1.6006$$

CHECKING!

$$x = \left(\frac{2x+5}{2}\right)^{1/3}$$

$$g'(x) = \frac{1}{3}\left(\frac{2x+5}{2}\right)^{-\frac{2}{3}}(1)$$

$$= \frac{1}{3}\left(\frac{2x+5}{2}\right)^{-\frac{2}{3}}$$

$$g'(1) = \frac{1}{3}\left(\frac{2(1)+5}{2}\right)^{-\frac{2}{3}}$$

$$= 0.1446$$

$$|g'(x)| = 0.1446 < 1$$

CONVERGE ✓

SUGGESTED

CAN USE THE ITERATION

FIXED POINT ITERATION METHOD

EXAMPLE 4
CONTINUE...

Iterative function 2

$$2x^3 - 2x - 5 = 0$$
$$2x = 2x^3 - 5$$
$$x = \frac{2x^3 - 5}{2}$$

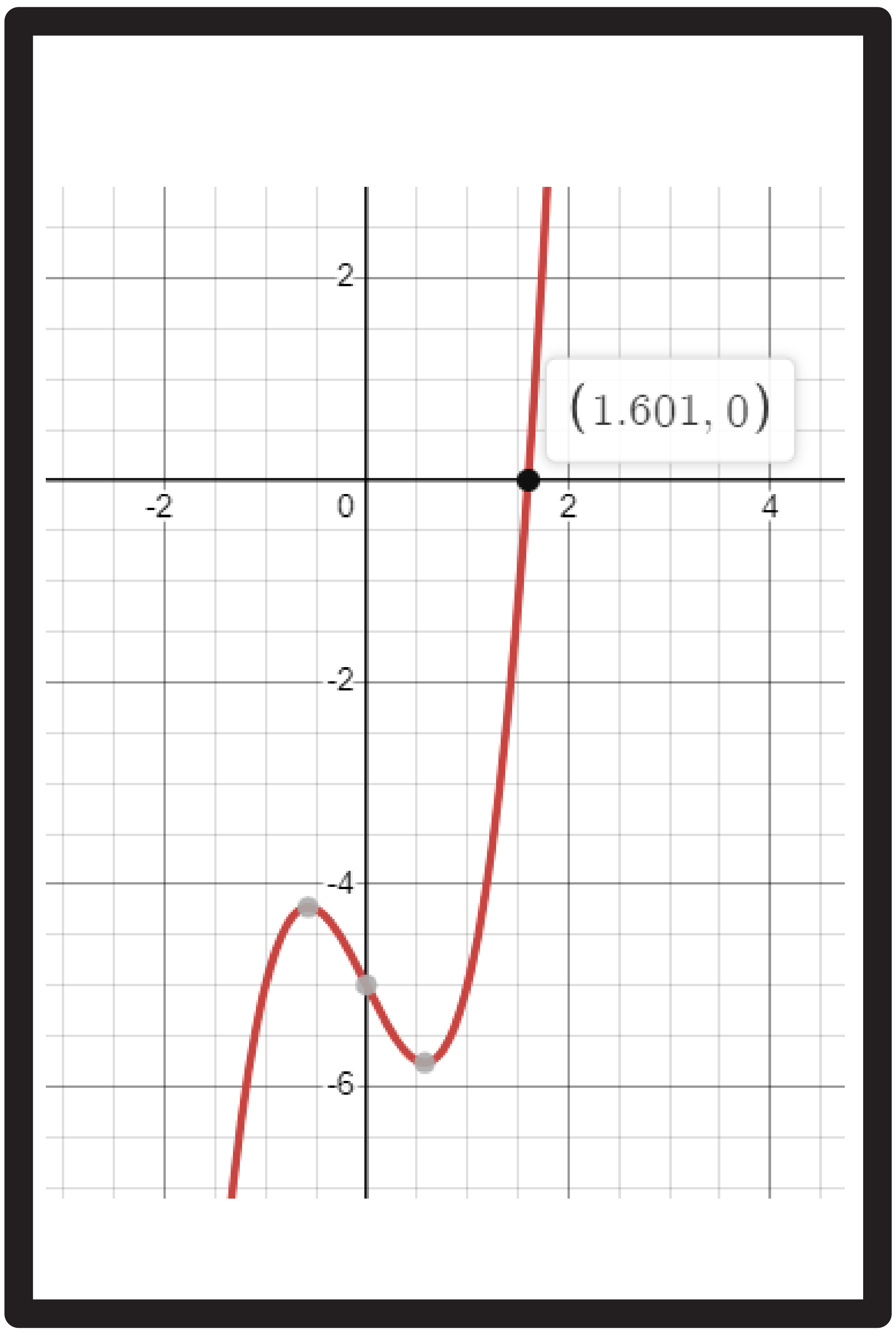
n	x_n	x_{n+1}	Error
0	$x_0 = 1$	$x_1 = -2$	3
1	$x_1 = -2$	$x_2 = -10.5$	8.5
2			

CHECKING!

$$x = x^3 - \frac{5}{2}$$
$$g'(x) = 3x^2$$
$$g'(1) = 3(1)^2$$
$$= 3$$
$$|g'(x)| = 3 > 1$$

DIVERGE ✗

NOT SUGGESTED
ALSO NOT APPROPRIATE ITERATION
BECAUSE THERE IS ONLY ONE POINT CAN BE TRUE
(REFER GRAPH)



EXERCISE

QUESTION 1

Find the root of the equation $2x^2 - 3 = \frac{1}{x}$ with $x_0 = 1.5$ by using Fixed Point Iteration Method, correct to 3 decimal places.

ANSWER

Step 1 : Make x as subject

Step 2 : Convergence checking

Step 3 : Substitute X_0 in $x = g(x)$ to find x1

Step 4 : Find $| x_{n+1} - x_n |$

Step 5 : Repeat second and third step until $| x_{n+1} - x_n | = 0.001$

FIXED POINT ITERATION METHOD

EXERCISE

QUESTION 2

Find the root of the equation $f(x) = x^2 - 5x + 1$ by using Fixed Point Iteration Method, correct to 3 decimal places. The first approximation is 1.

ANSWER

Step 1 : Make x as subject

Step 2 : Convergence checking

Step 3 : Substitute X_0 in x = g(x) to find x1

Step 4 : Find $| x_{n+1} - x_n |$

Step 5 : Repeat second and third step until $| x_{n+1} - x_n | = 0.001$

FIXED POINT ITERATION METHOD

EXERCISE

QUESTION 3

Find the root of the equation $f(x) = x^2 + 5x - 3$ by using Fixed Point Iteration Method, correct to 3 decimal places. The first approximation is 1.

ANSWER

Step 1 : Make x as subject

Step 2 : Convergence checking

Step 3 : Substitute X_0 in $x = g(x)$ to find x1

Step 4 : Find $| x_{n+1} - x_n |$

Step 5 : Repeat second and third step until $| x_{n+1} - x_n | = 0.001$

FIXED POINT ITERATION METHOD

METHOD 2

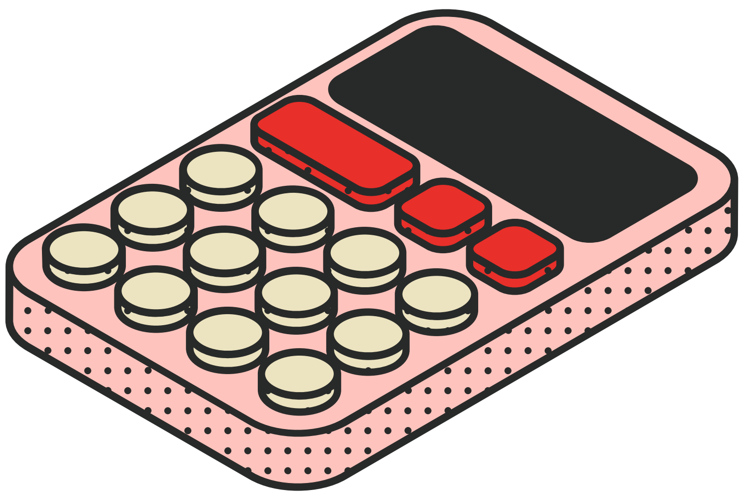
NEWTON RAPHSON METHOD

NOTES

Step in Newton Raphson Method

X_0 is not given

1. Find X_0 (change in sign) $x_0 = \frac{1}{y_2 - x_1} \begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \end{vmatrix}$
2. Differentiate f(x)
3. Substitute into table
4. Repeat second and third step until $| x_{n+1} - x_n | < 0.001$ (depends on the required accuracy)



X_0 given

1. Differentiate f(x)
2. Substitute into table
3. Repeat second and third step until $| x_{n+1} - x_n | < 0.001$ (depends on the required accuracy)

NEWTON RAPHSON METHOD

NOTES

The method requires you to differentiate the equation you're trying to find a root of, so before revising this topic you may want to look back at differentiation to refresh your mind.

1.	$\frac{d}{dx}[k] = 0$, k is constant	2.	$\frac{d}{dx}[ax^n] = nax^{n-1}$ [Power Rule]
3.	$\frac{d}{dx}[f(x) \pm g(x)] = f'(x) \pm g'(x)$	4.	$\frac{d}{dx}[a\{u(x)\}^n] = na\{u(x)\}^{n-1} \cdot u'(x)$ [Composite]

EXAMPLE

$$f(x) = x^2 - 8x + 11$$

$$f'(x) = 2x - 8$$

$$f(x) = -x^2 + x + 12$$

$$f'(x) = -2x + 1$$

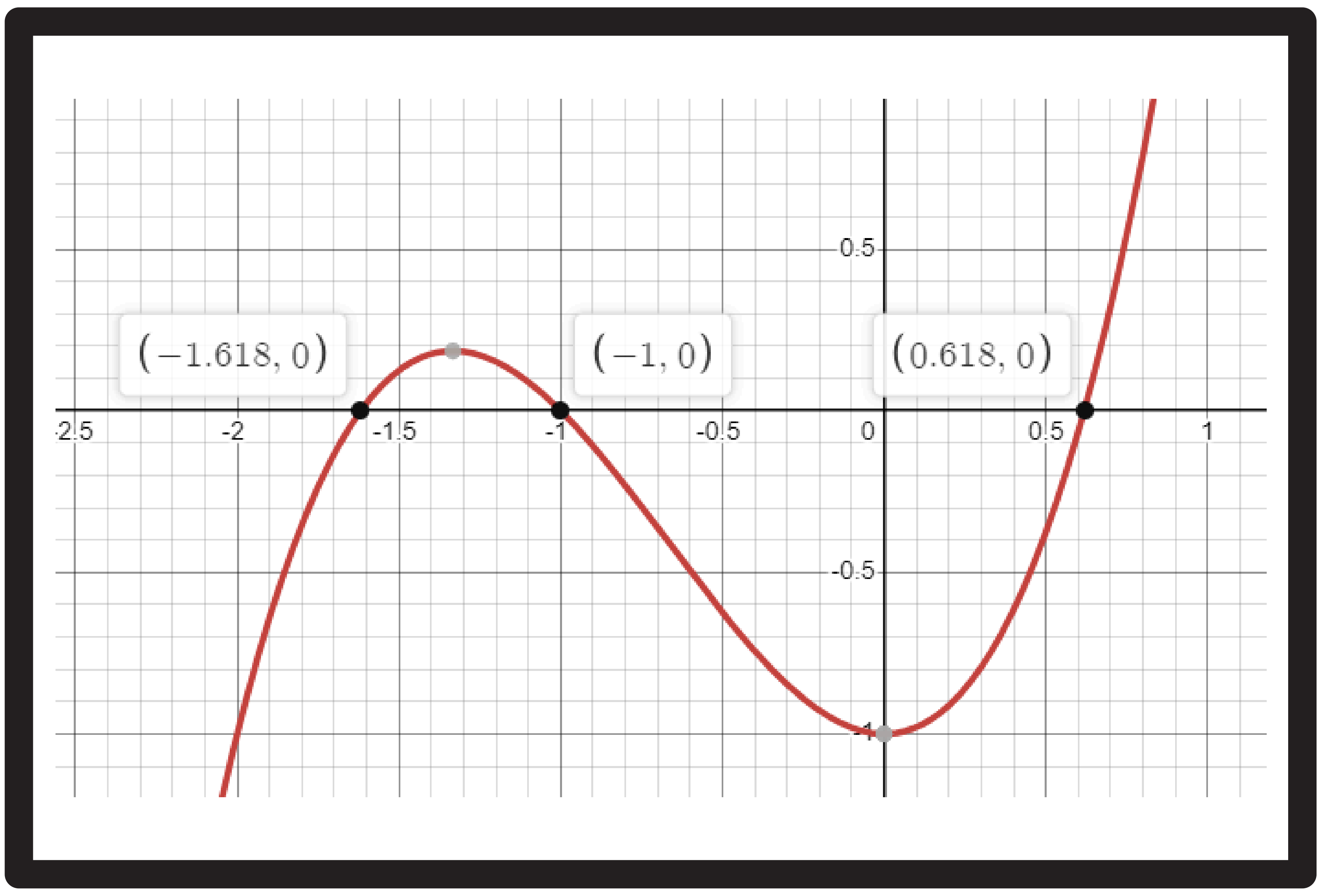
$$f(x) = x^3 - 2x^2 - 5x + 8$$

$$f'(x) = 3x^2 - 4x - 5$$

NEWTON RAPHSON METHOD

EXAMPLE 1

Use Newton Raphson Method to find the root of the equation $f(x) = x^3 + 2x^2 - 1$, starting with $x_0 = 0.5$. Give your answer correct to 4 decimal places.



SOLUTION

$$f(x) = x^3 + 2x^2 - 1$$

$$f'(x) = 3x^2 + 4x$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_{n+1} = x_n - \frac{x^3 + 2x^2 - 1}{3x^2 + 4x}$$

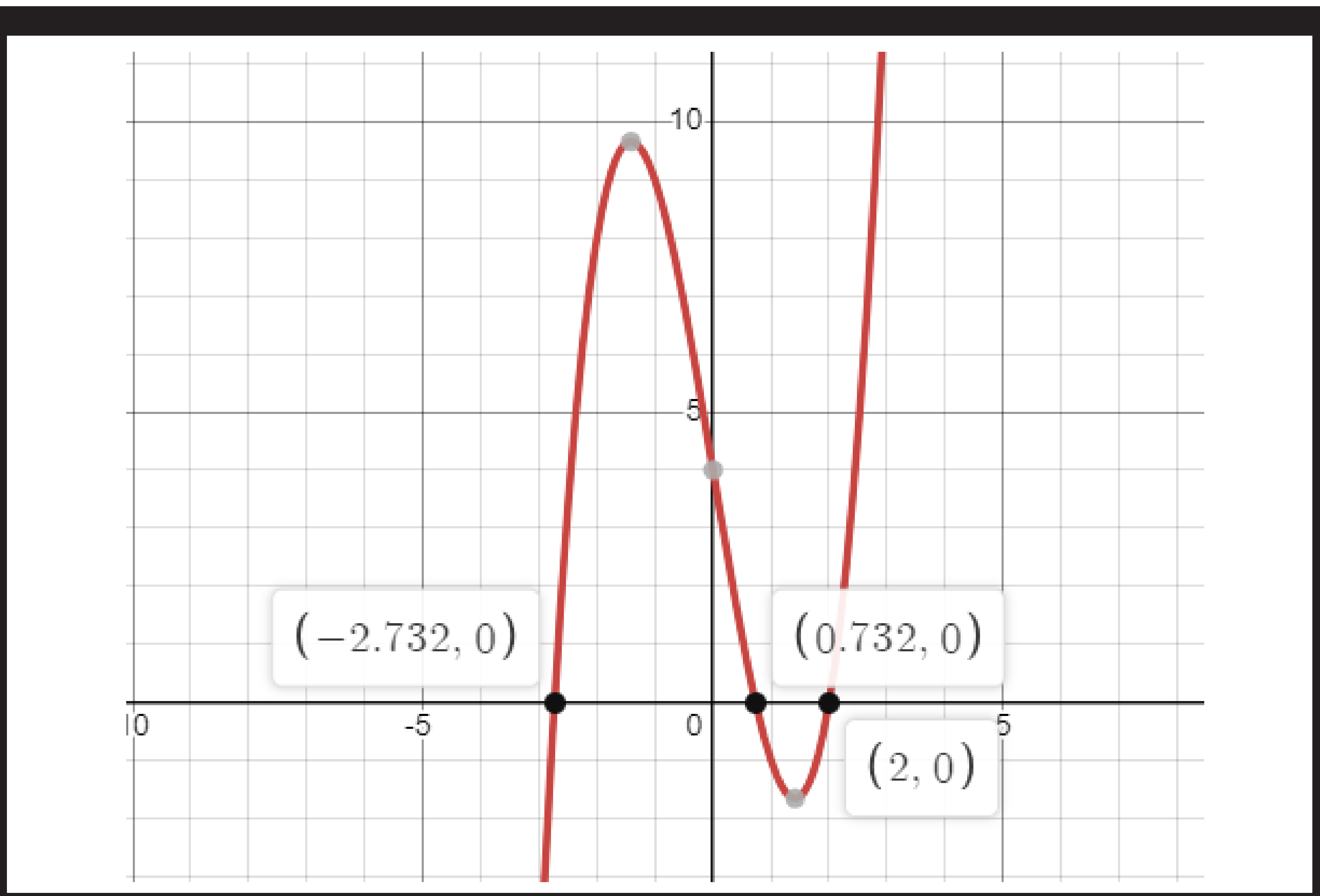
n	x_n	$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$	Error
0	$x_0 = 0.5$	$x_1 = 0.6364$	0.1364
1	$x_1 = 0.6364$	$x_2 = 0.6183$	0.0181
2	$x_2 = 0.6183$	$x_3 = 0.6180$	0.0003
3	$x_3 = 0.6180$	$x_4 = 0.6180$	0.0000

$$x_4 = 0.6180$$

NEWTON RAPHSON METHOD

EXAMPLE 2

Solve the equation of $x^3 - 6x + 4 = 0$ using Newton Raphson Method, state your answer to 4 decimal places with an initial guess $x_0 = 1$.



SOLUTION

$$f(x) = x^3 - 6x + 4$$

$$f'(x) = 3x^2 - 6$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_{n+1} = x_n - \frac{x^3 - 6x + 4}{3x^2 - 6}$$

n	x_n	$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$	Error
0	$x_0 = 1$	$x_1 = 0.6667$	0.3333
1	$x_1 = 0.6667$	$x_2 = 0.7302$	0.0635
2	$x_2 = 0.7302$	$x_3 = 0.7320$	0.0018
3	$x_3 = 0.7320$	$x_4 = 0.7320$	0.0000

$x_4 = 0.7320$

NEWTON RAPHSON METHOD

EXAMPLE 3

Determine a real root of $f(x) = x^2 - 4$ using Newton Raphson Method, give your answer correct to 2 decimal places.

If the first approximate (x_0) to the root of the question **is not given.** You must use **False Position Method** to find the first approximate (x_0).

$$\mathbf{x_0 = \frac{1}{y_2 - y_1}\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \end{vmatrix}}$$

$$y = x^2 - 4$$

$$x_1 = 0, \ y_1 = -4$$

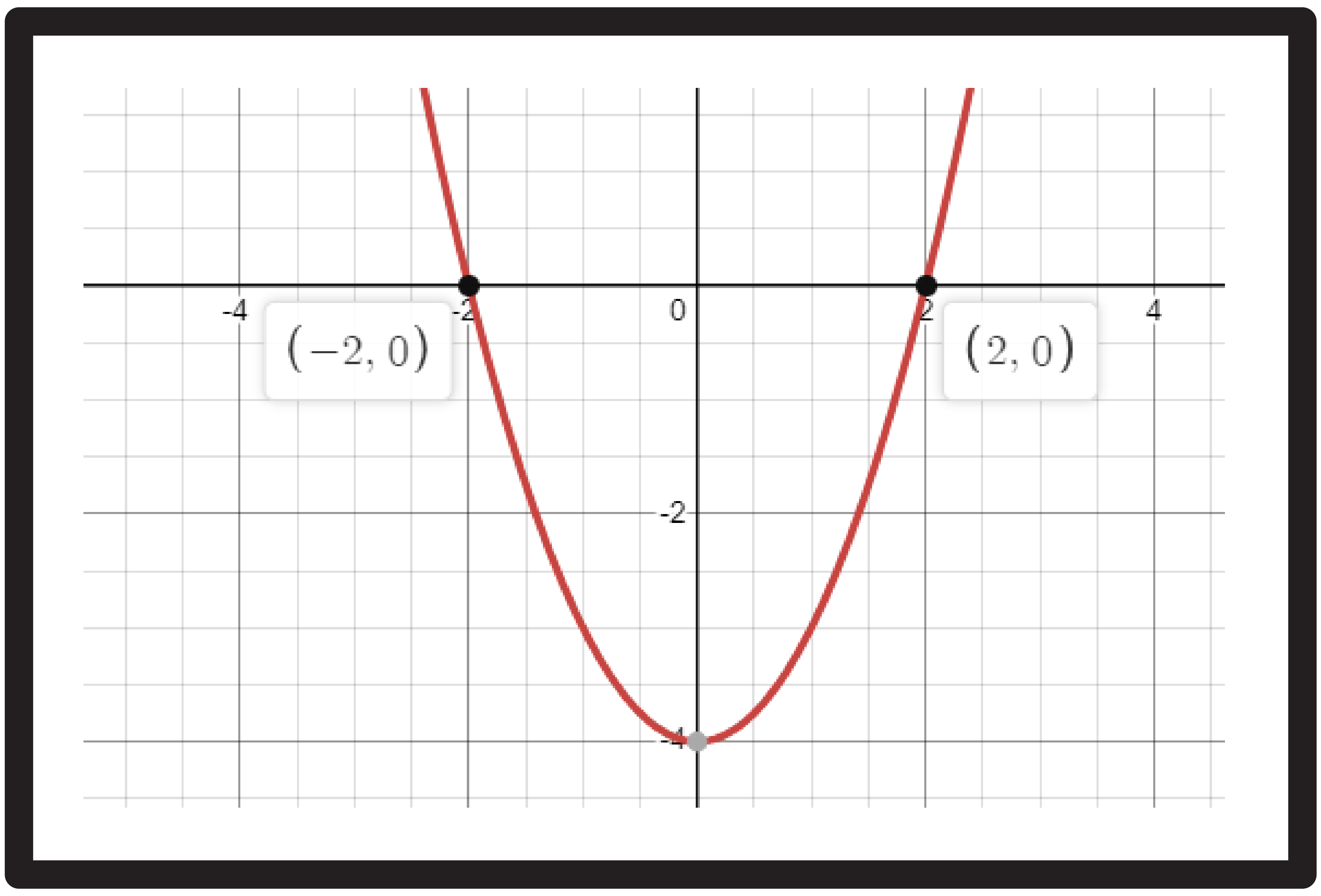
$$x_2 = 1, \ y_2 = -3$$

$$\mathbf{x_0 = \frac{1}{y_2 - y_1}\begin{vmatrix} x_1 & y_1 \\ x_2 & y_2 \end{vmatrix}}$$

$$x_0 = \frac{1}{-3 - (-4)}\begin{vmatrix} 0 & -4 \\ 1 & -3 \end{vmatrix}$$

$$= \frac{1}{1}(0 - (-4))$$

$$x_0 = 4$$



SOLUTION

$$f(x) = x^2 - 4$$

$$f'(x) = 2x$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_{n+1} = x_n - \frac{x^2 - 4}{2x}$$

n	x_n	$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$	Error
0	$x_0 = 4$	$x_1 = 2.5$	1.5
1	$x_1 = 2.5$	$x_2 = 2.05$	0.45
2	$x_2 = 2.05$	$x_3 = 2.00$	0.05
3	$x_3 = 2.00$	$x_4 = 2.00$	0.00

$$x_4 = 2.00$$

NEWTON RAPHSON METHOD

EXAMPLE 4

Given the equation $x^4 - 2x^3 - x + 1 = 0$. Find the root of the equation by using Newton Raphson Method where the root is between $x = 0$ and $x = 1$. Give the answer correct to three decimal places.

If the first approximate (x_0) to the root of the question **is given between x_1 and x_2**. You must use

$$x_0 = \frac{x_1 + x_2}{2}$$

to find the first approximate (x_0).

$$x_0 = \frac{0 + 1}{2} = 0.5$$

n	x_n	$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$	Error
0	$x_0 = 0.5$	$x_1 = 0.657$	0.157
1	$x_1 = 0.657$	$x_2 = 0.642$	0.015
2	$x_2 = 0.642$	$x_3 = 0.642$	0.000

$x_3 = 0.642$

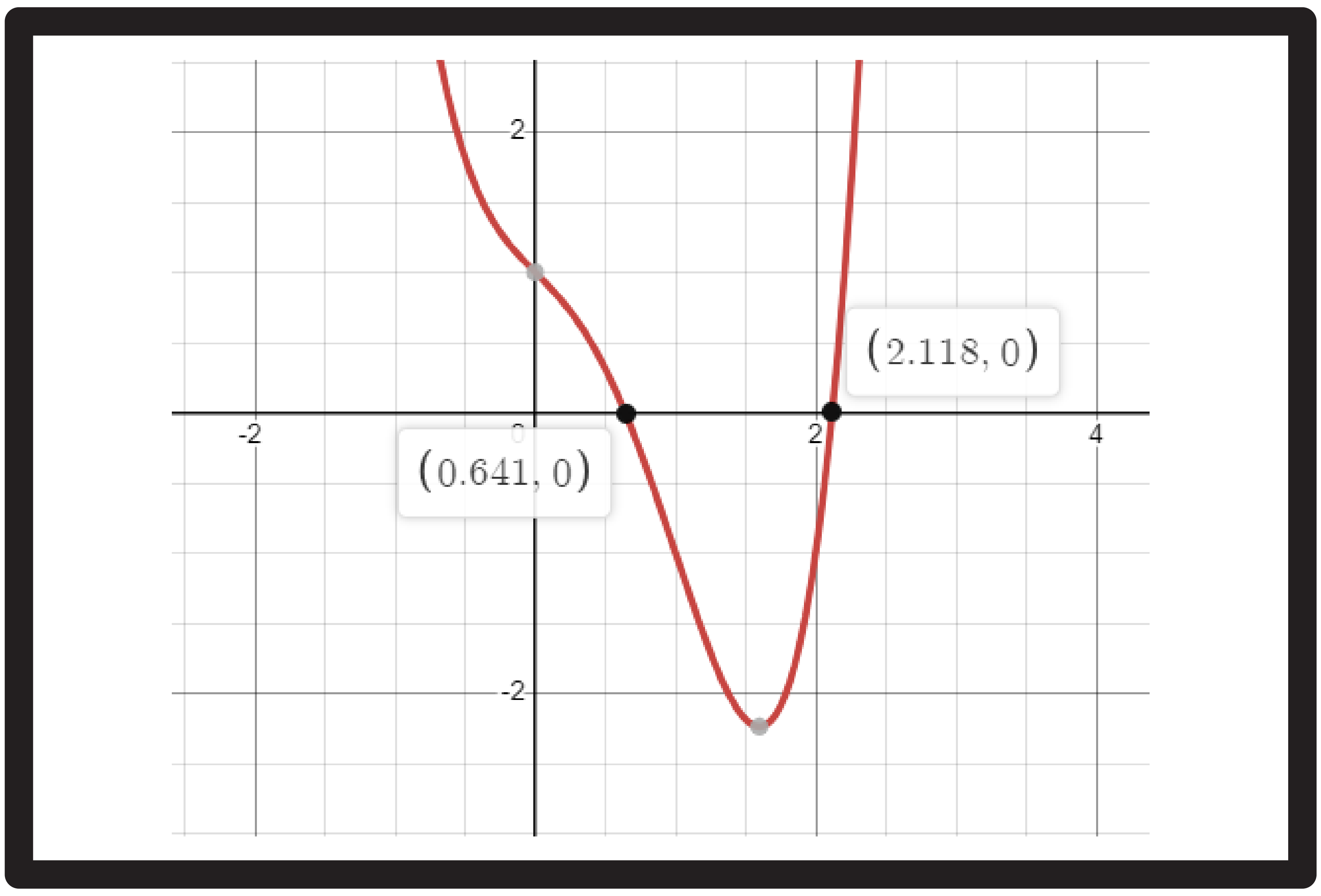
SOLUTION

$f(x) = x^4 - 2x^3 - x + 1$

$f'(x) = 4x^3 - 6x^2 - 1$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$x_{n+1} = x_n - \frac{x^4 - 2x^3 - x + 1}{4x^3 - 6x^2 - 1}$$



EXERCISE

QUESTION 1

Solve the equation of $5x^2 + 11x - 17 = 0$ using Newton Raphson Method, state your answer correct to 4 decimal places when $x_0 = 1$.

ANSWER

Step 1 : Differentiate f(x)

Step 2 : Substitute into table

Step 3 : Repeat second and third step until $|x_{n+1} - x_n| < 0.001$

NEWTON RAPHSON METHOD

EXERCISE

QUESTION 2

Solve the equation of $x^3 + 3x^2 - 2 = 0$ using Newton Raphson Method, state your answer to 3 decimal places with an initial guess $x_0 = 1$.

ANSWER

Step 1 : Differentiate f(x)

Step 2 : Substitute into table

Step 3 : Repeat second and third step until $|x_{n+1} - x_n| < 0.001$

NEWTON RAPHSON METHOD

EXERCISE

QUESTION 3

Determine an approximate of the equation $f(x) = x^2 - 5x + 1$ using Newton Raphson Method. Given $x_0 = 1$, and give your answer correct to 3 decimal places.

ANSWER

Step 1 : Differentiate f(x)

Step 2 : Substitute into table

Step 3 : Repeat second and third step until $|x_{n+1} - x_n| < 0.001$

NEWTON RAPHSON METHOD

EXERCISE

QUESTION 4

Use Newton Raphson Method to determine an approximation of the question $f(x) = x^2 - x - 1$. Let $x_0 = 2$ and give your answer correct to 4 decimal places.

ANSWER

Step 1 : Differentiate f(x)

Step 2 : Substitute into table

Step 3 : Repeat second and third step until $|x_{n+1} - x_n| < 0.001$

NEWTON RAPHSON METHOD

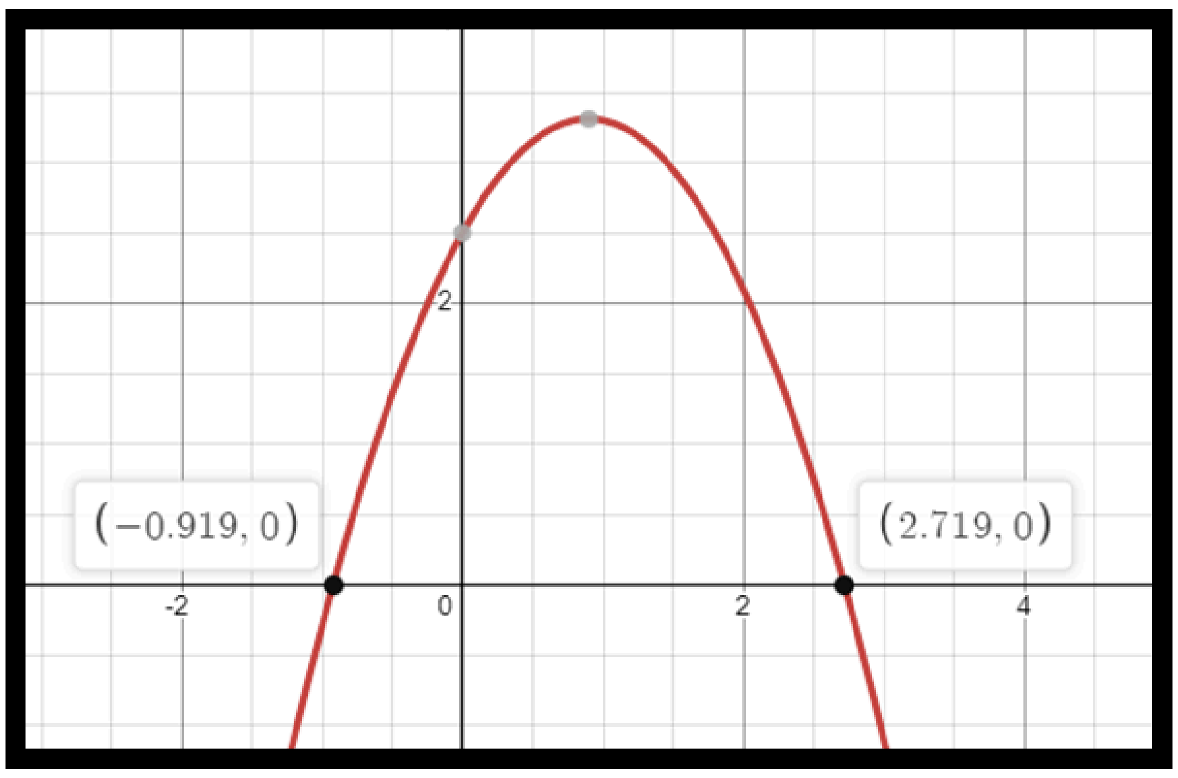
NEWTON RAPHSON METHOD VS FIXED POINT ITERATION METHOD

Find the first approximate root of the equation

$$f(x) = -x^2 + 1.8x + 2.5$$

up to 4 decimal places

by using both methods.



$f(x) = -x^2 + 1.8x + 2.5$

$f'(x) = -2x + 1.8$

$$x_{n+1} = x_n - \frac{f(x)}{f'(x)}$$

$$x_{n+1} = x_n - \frac{-x^2 + 1.8x + 2.5}{-2x + 1.8}$$

n	x_n	$x_{n+1} = x_n - \frac{f(x)}{f'(x)}$	Error
0	$x_0 = 5$	$x_1 = 3.3537$	1.6463
1	$x_1 = -2$	$x_2 = 2.8013$	0.8013
2	$x_2 = 2.8013$	$x_3 = 2.7211$	0.0802
3	$x_3 = 2.7211$	$x_4 = 2.7193$	0.0018
4	$x_4 = 2.7193$	$x_5 = 2.7193$	0

$x_5 = 2.7193$

CHECKING!

Iterative function 1	**Iterative function 2**
$-x^2 + 1.8x + 2.5 = 0$ $-x^2 - 2.5 = -1.8x$ $x = \frac{-x^2 - 2.5}{-1.8}$ $x = \frac{x^2 + 2.5}{1.8}$	$-x^2 + 1.8x + 2.5 = 0$ $-x^2 = -1.8x - 2.5$ $x^2 = 1.8x + 2.5$ $x = \sqrt{1.8x + 2.5}$
$x = \frac{x^2}{1.8} + \frac{2.5}{1.8}$ $g'(x) = \frac{1}{0.9}x$ $g'(5) = \frac{1}{0.9}(5)$ $= 5.55$ $\lvert g'(x)\rvert = 5.55 > 1$	$x = (1.8x + 2.5)^{1/2}$ $g'(x) = \frac{1}{2}(1.8x + 2.5)^{-\frac{1}{2}}(1.8)$ $= 0.9(1.8x + 2.5)^{-\frac{1}{2}}$ $g'(5) = 0.9(1.8(5) + 2.5)^{-\frac{1}{2}}$ $= 0.265$ $\lvert g'(x)\rvert = 0.265 < 1$

DIVERGE ✗ CONVERGE ✓

n	x_n	x_{n+1}	Error
0	$x_0 = 5$	$x_1 = 3.3912$	1.6088
1	$x_1 = 3.3912$	$x_2 = 2.9333$	0.4579
2	$x_2 = 2.9333$	$x_3 = 2.7893$	0.1440
3	$x_3 = 2.7893$	$x_4 = 2.7424$	0.0469
4	$x_4 = 2.7424$	$x_5 = 2.7270$	0.0154
5	$x_5 = 2.7270$	$x_6 = 2.7219$	0.0051
6	$x_6 = 2.7219$	$x_7 = 2.7202$	0.0017
7	$x_7 = 2.7202$	$x_8 = 2.7196$	0.0006
8	$x_8 = 2.7196$	$x_9 = 2.7194$	0.0002
9	$x_9 = 2.7194$	$x_{10} = 2.7194$	0

$x_{10} = 2.7194$

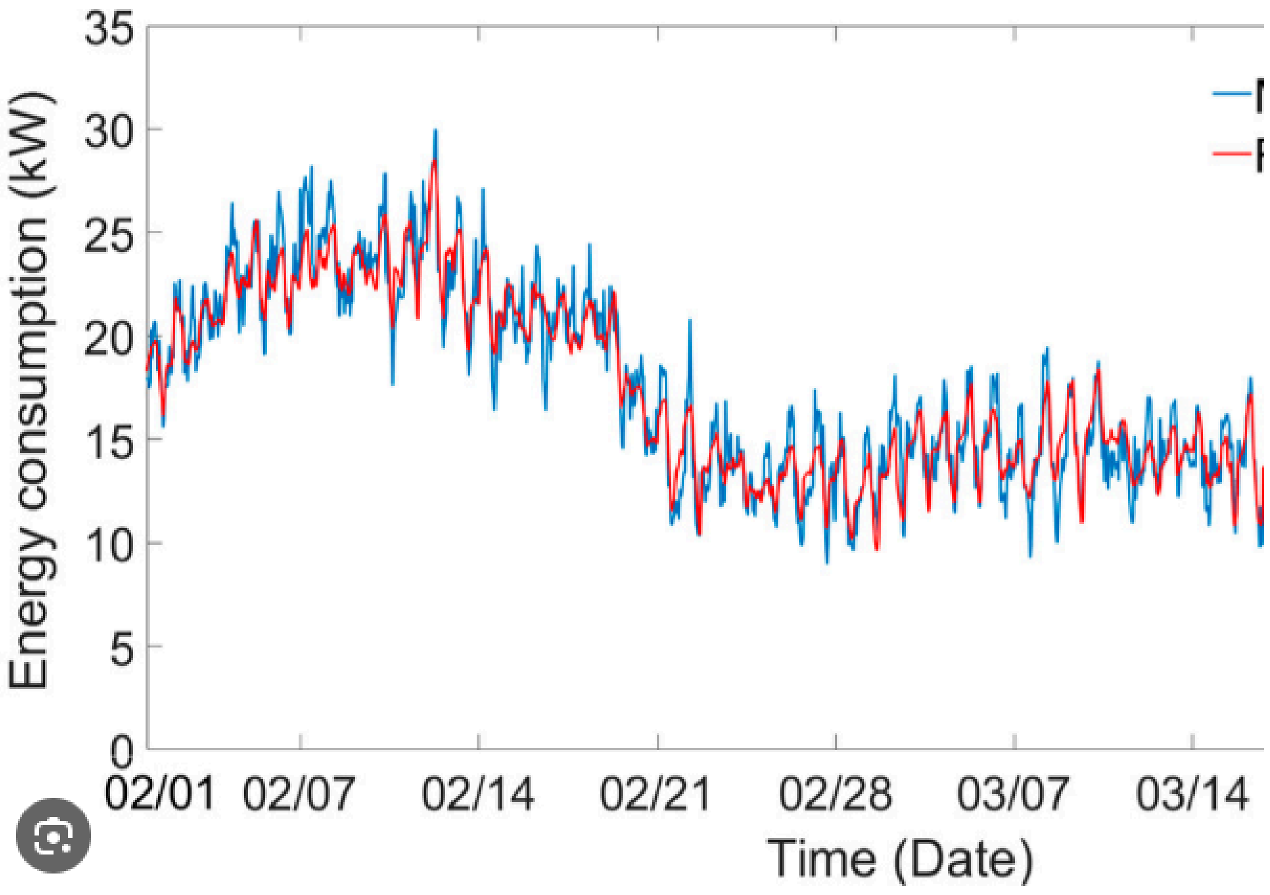
EXAMPLE OF AN ITERATIVE PROCESS IN MUSIC

Practicing an instrument requires the musician to play a song multiple times, improving on each attempt and learning from their mistakes. This is a common example of real-world iterative processes, as it represents how repetition leads to perfection.

EXAMPLE OF AN ITERATIVE PROCESS IN BUSINESS

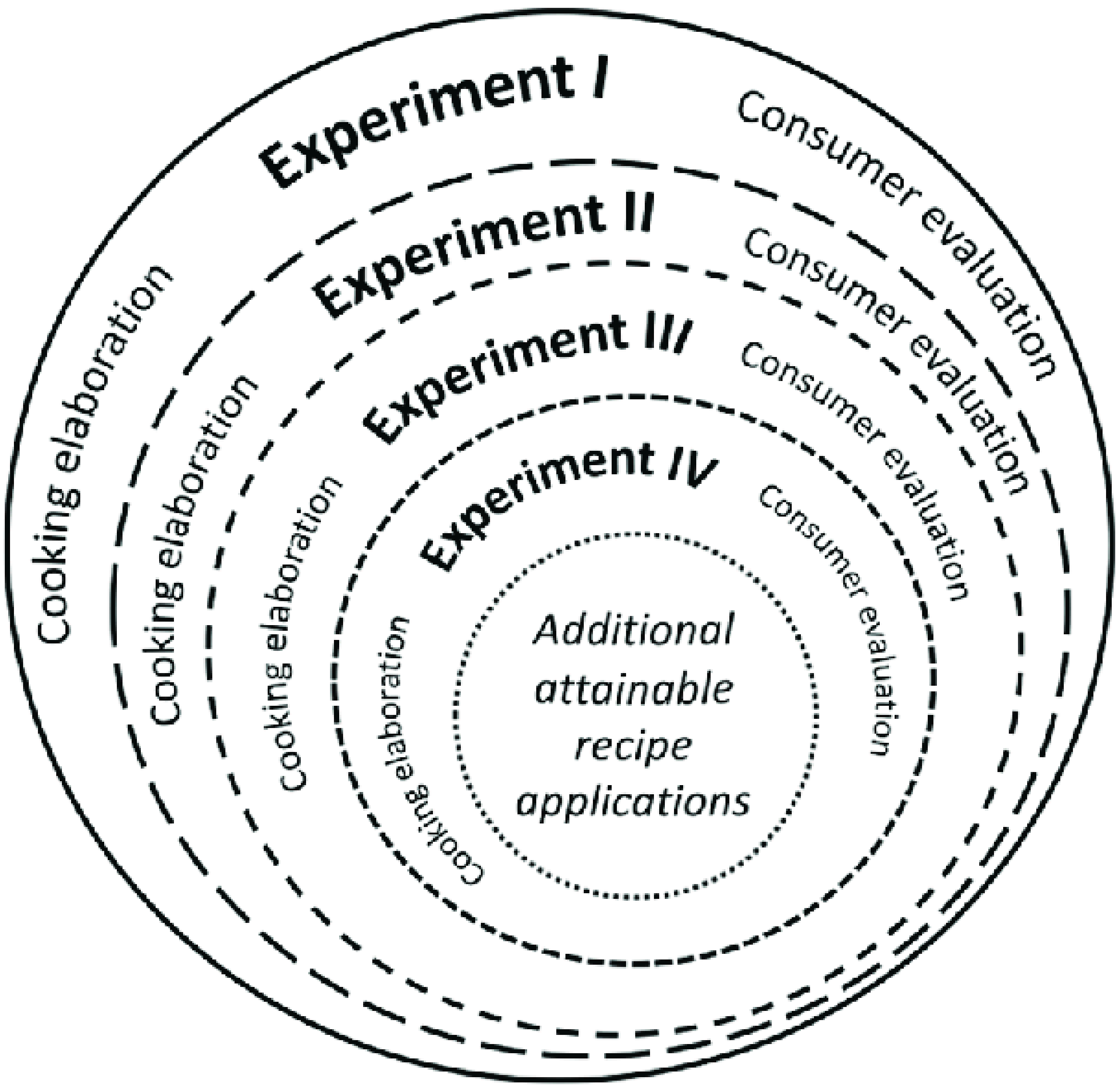
For example, when companies plan to hire a new link building agency or a new vendor, they often start with a small test campaign. They will then use the feedback from this test market to make changes to their marketing strategy. This process is repeated until the product is launched in all markets.

EXAMPLE OF AN ITERATIVE PROCESS IN WEATHER



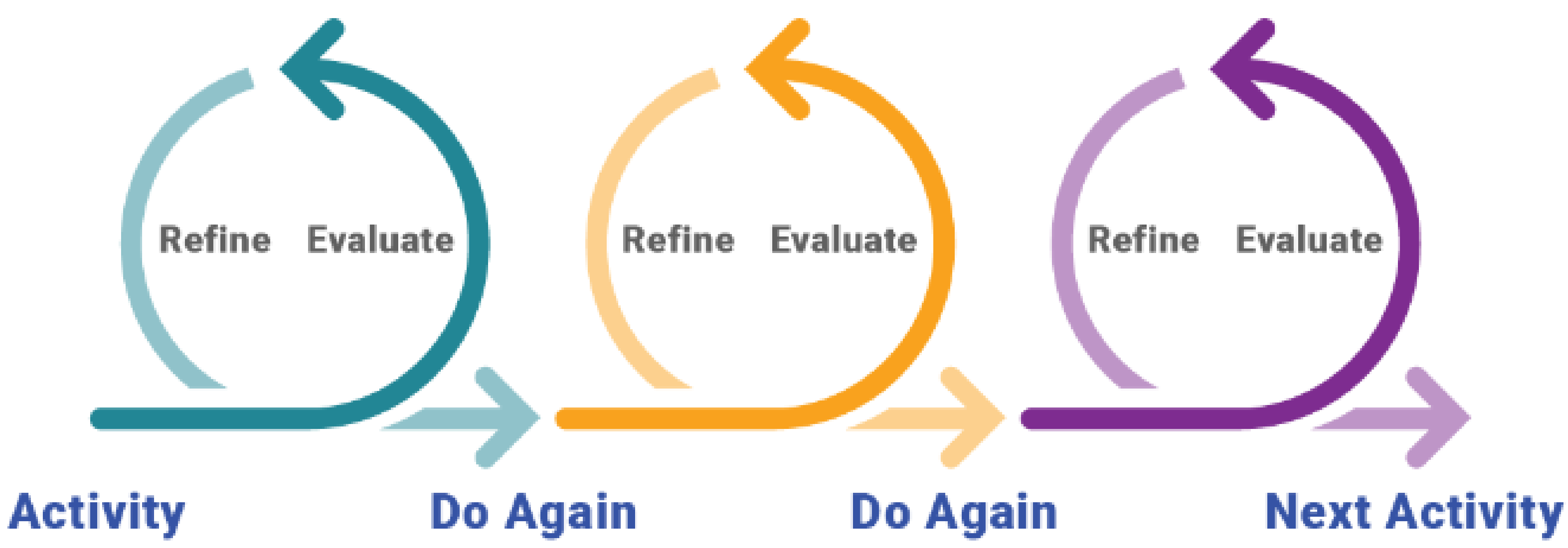
Energy consumption data are influenced by the weather conditions. The use of weather unadjusted energy data in analysis and forecasting could give misleading and erroneous results. In this paper a new iterative econometric technique is developed to correct for abnormal weather conditions in published energy consumption data.

EXAMPLE OF AN ITERATIVE PROCESS IN COOKING AREA



Professional chefs use the iterative process by working on a single entree many times before being satisfied. By repeating the process of cooking a single meal repeatedly, chefs use iterative processes to perfect their dishes' flavor and appearance.

EXAMPLE OF AN ITERATIVE PROCESS IN EDUCATION



Iterating allows you to make mistakes quickly and learn from them in order to change or improve what you've done.

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Iterative function in real life
https://www.yourdictionary.com/articles/iteration-examples-real-life

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